Specific Human Capital Accumulation and Job Match Quality –
Implications for Measuring Returns to Tenure

Tim Barmby\(^1\) and Barbara Eberth\(^2\) (University of Aberdeen)\(^3\)

May 2006

Abstract

This paper uses the theoretical argument presented by Stevens (2003) that suggests that the measured returns to tenure will unambiguously be biased downwards. We illustrate this effect for data from a UK internal labour market using the counterfactual methodology outlined by DiNardo, Fortin and Lemieux (1996). Finally we argue that Stevens’s theory offers a possible explanation for the apparent puzzle presented by Medoff and Abraham (1980) who find that their estimated coefficient on tenure did not fall when direct measures of productivity were introduced into the wage equation.

JEL: J24

Keywords: Specific Human Capital, Tenure

---

\(^1\) Address for correspondence:- Department of Economics, University of Aberdeen, Old Aberdeen, Scotland AB24 3SQ tim.barmby@abdn.ac.uk
\(^2\) Health Economics Research Unit, University of Aberdeen, Aberdeen, Scotland AB25 2ZD b.eberth@abdn.ac.uk
\(^3\) We would like to acknowledge useful comments received from John Heywood and Keith Bender, also seminar audiences at Aberdeen and the 2006 Scottish Economic Society Conference at Perth.
I Introduction

Studying the way earnings distributions evolve for a given cohort of workers within an internal labour market reflects a number of processes. Workers may accumulate specific human capital within the firm thereby increasing their productivity and their remuneration. Also reflected in these distributions will be aspects of workers’ behaviour in terms of their decisions to stay with or leave the organisation. If a worker’s external market offer dominates existing remuneration, the worker will leave. Therefore, the decision to stay with or leave the firm will result in non-random attrition from the distribution of incumbent earnings. This is the focus of this paper. To fix initial ideas consider the following distributions in Figure 1

![Figure 1](image-url)

These distributions are drawn using data from a large UK financial sector firm. The data is fully described in Treble, Van Gameren, Bridges and Barmby (2001). For the purposes of this paper we have access to earnings for all workers observed between 1989 and 2001. Figure 1 plots the distribution of real average log hourly wages (hourly wages being computed by dividing base earnings by annual hours) for a cohort of employees who entered the firm for the first time in the year 1989 at three points in their employment history with the firm: in the year of entry 1989, five years after entry in 1994, and ten years after entry in...
1999. The observed earnings growth in figure 1 will be, as we have noted, be partly the result of workers accumulating specific human capital. However workers will leave the firm and will therefore not be represented in subsequent earnings distributions. In this data of the 4475 workers who entered the firm in 1989, 2399 remained in 1994, and 1389 in 1999. It is clear however that we cannot infer the returns to tenure by observing the shift in the mean of the above distribution, as the attrition from the distribution is likely not to be random. This suggests that the way in which workers receive, and either accept job offers and leave, or decline job offers and stay is important and we will return to this in the description of the theoretical framework in the next section. It also gives us the motivation for our empirical approach since if workers did leave the firm randomly then comparison of the means of the above distributions would be informative about the returns to tenure. We return to this in section III.

II Stevens’s Model

Stevens (2003) proposes an auction model of wage determination. There are \( n \) firms in the market and they each privately form a valuation of the match component of a worker’s productivity, \( \varepsilon_i > 0 \) for \( i \geq 0 \). These match qualities are independently distributed
\[
\varepsilon_i \sim f(\varepsilon_i); \quad \varepsilon_i \in (\underline{\varepsilon}, \bar{\varepsilon}) \quad i \geq 0.
\]
Realised productivity within the organisation the worker is presently employed at will be of the form
\[
\ln v_0 = \ln g + \ln k + \ln \varepsilon_0
\]
however if he/she were to move then the specific component is lost and
\[
\ln v_i = \ln g + \ln \varepsilon_i \quad i > 0
\]
where $v$ is productivity, $g$ general human capital, $k$ specific human capital and $\varepsilon_0$ match quality. There are some parallels with Lazear’s (1986) Raids and Offer Matching paper, but Stevens considers the wage setting in this framework explicitly as a private value auction.

Stevens draws on the results of Maskin and Riley (2000) to describe the process by which firms will set wages, $w$, in this situation. Expected profit for firm 0 (the firm the worker is presently at will be)

$$\Pi_0(\varepsilon_0, w) = (v_0 - w) \Pr(w_0(\varepsilon_j) \leq w \forall j > 0)$$  \hspace{1cm} (3)

This is saying that firm 0’s (expected) profit will be the excess of productivity over wage offered multiplied by the probability that the worker doesn’t receive a better wage offer. For firm $i > 0$ the expected profit function is similar.

$$\Pi_i(\varepsilon, w) = (v_i - w) \Pr(w_i(\varepsilon_j) \leq w \forall j > 0, j \neq i, w_0(\varepsilon_0) \leq w)$$  \hspace{1cm} (4)

The only difference here that the probability of the firm realising the surplus $(v_i - w)$ is the probability of the event that no other firm outbids the firm and that the firm which the worker is at doesn’t make a higher wage offer.

The wage offer $w_i = w_i(\varepsilon_i, g, k)$ functions for firms will be defined in equilibrium. Stevens notes that since general human capital will multiply productivity in all firms, therefore the elasticity of wage offers with respect to $g$ will be unity. Thus defining $W_i(\varepsilon_i, k) = w_i(\varepsilon_i, 1, k)$ wage offers can be written as $w_i = gW_i(\varepsilon_i, k)$. The solution to the auction is characterised by
two differential equations which are the first order conditions that result from the
maximisation\(^4\) of (3) and (4).

\[
\frac{n f(\phi_i) \frac{\partial \phi_i}{\partial w}}{F(\phi_i)} \frac{1}{\phi_i - w} = \frac{1}{k\phi_0 - w}
\]

(5)

\[
\frac{f(\phi_0) \frac{\partial \phi_0}{\partial w}}{F(\phi_0)} + (n - 1)\frac{f(\phi) \frac{\partial \phi}{\partial w}}{F(\phi)} \frac{1}{\phi - w} = \frac{1}{\phi_i - w}
\]

(6)

Where \(\phi_i(w, k) \equiv W_i^{-1}(w, k)\) are the inverse wage offer functions. The solution of these
differential equations for the asymmetric auction (as in this case because of specific human
capital \(k\)) gives equilibrium wage offers by the firms in the market. Focussing specifically on
the firm that the worker presently works for, in part because our empirical application
involves a single cohort in the firm. Since we know that the wage offer function will be of the
form \(w_0 = gW_0(\epsilon, k)\) this will allow an approximation of the log wage offer of the current
employer to be written as

\[
\ln w_0 = \ln g + \alpha(\epsilon_0) \ln k + \eta(\epsilon_0)
\]

(7)

This expression follows from a Taylor series expansion of \(\ln W_0(\epsilon, k)\)\(^5\) around the point \(k=1,\)
and so will be valid for \(k\) close to 1, which models the situation when specific human capital
is “small” relative to general human capital. In the above expression
\[
\alpha(\epsilon) = \frac{1}{W_0(\epsilon, 1)} \left. \frac{\partial W_0}{\partial k} \right|_{k=1}
\]
\(\eta(\epsilon) = \ln W_0(\epsilon, 1)\)\(^6\). A number of points are of interest at this point
in the analysis; firstly that a standard (Mincerian) human capital earnings equation has been
obtained in a different theoretical framework, namely that of a private value auction. Stevens

\(^4\) Take logs and differentiate by \(w\)
\(^5\) Writing \(\ln W_0 = \ln g + \ln W_0(\epsilon, k)\) and expanding \(\ln W_0(\epsilon, k)\) to get (7)
\(^6\) \(\ln k = (k - 1)\) for \(k\) close to 1 has also been used
develops this equation in two directions. Firstly she takes the expectation of (7) with respect to the distribution of match values \( f(\varepsilon) \), so as to be able to assess the effect of the auction market on the coefficient on tenure for an observed sample. Secondly she takes account of selectivity on the sample by recognizing that an empirical analyst will not observe all wage offers but only observe accepted offers. She concludes in her proposition 2 that there will be a negative bias in the coefficient on specific human capital. The reader is referred to Stevens (2003) for specific analytical details. The intuition is however clear: those workers who stay with their current employers, are rejecting outside offers, those workers who have high \( k \) can “carry” a lower value of a match with their current employer and still reject a given outside offer. As a consequence \( \ln k \) and \( \ln \varepsilon \) will be negatively correlated, and negative correlation between an included regressor in an (estimating) equation and the error in the equation biases the estimated coefficient downwards.

Figure 2 gives the joint distribution of specific capital, \( \ln k \) and match quality with the current employer, \( \ln \varepsilon_0 \), the diagram is drawn with \( \text{cov}(\ln k, \ln \varepsilon_0) = 0 \). That is, the initial distribution of \( \ln k \) and \( \ln \varepsilon_0 \) are independent, however this isn’t vital. The workers in the area above and to the right of the outside offer line reject outside offers, the workers in the area below and to
the left will accept. This process is a non-random selection from the joint distribution of $\ln k$ and $\ln \varepsilon_0$ and will induce a negative correlation between $\ln k$ and $\ln \varepsilon_0$ relative to the initial distribution. In other words, it demonstrates that within the group of *stayers*, the effect of non-random exits generates a negative correlation between $\ln k$ and $\ln \varepsilon_0$. This negative correlation will generate a negative bias in the coefficient on tenure from a regression of earnings on tenure. Figure 2 also visualises Stevens (2003) assertion that employees with high specific capital tend to stay with their current employer even though their match quality can be relatively low.

Many studies still assert that the bias on tenure could go either way, Altonji and Williams (2005) for one, and a persuasive idea regarding this comes from Topel (1991) where it is argued that since high quality workers are likely to move more readily and also be paid more (in the alternative job), you are more likely to observe high pay in low tenure jobs, but as Stevens analysis makes clear this will only increase the intercept on these wage schedules not the slope.

### III Empirical Illustration

To give an illustration as to whether the negative bias on the returns to tenure exists we analyse the data briefly described in the introduction. We exploit the fact that the process in Stevens’s model in empirical terms is essentially describing a non-random selection from the distribution of match quality. In short we ask whether it is possible, in analogy to figure 1, to draw the distribution of earnings that would have existed if exits from the cohort had not been systematic but had been random? We seek to do this by constructing the distribution of earnings that would have prevailed at subsequent time periods if the distribution of

---

7 This is a standard result in OLS, Johnston and DiNardo (1997) page 155 call this attenuation bias.
characteristics of workers had remained as it was on entry to the firm. Since if exits are random, the distribution of characteristics will, in expectation, remain the same.

The DiNardo, Fortin and Lemieux (1996) methodology allows for a visual presentation of wage densities of the 1989 cohort that would have prevailed at time periods subsequent to entry to the firm if the distribution of characteristics of workers had prevailed as at entry. This approach allows for a graphical comparison of the actual and counterfactual wage distributions of a cohort at a given point in time. The counterfactual distribution being the distribution showing the extent to which the observed growth in earnings of a cohort in years after entry under- or overstates earnings growth that would have been observed if those workers who left the firm over time did so in a random fashion. The DiNardo et al methodology is described in Appendix 1.

Applying the methodology outlined above, and described in more detail in Appendix 1 to the cohort of workers who entered the firm in 1989 five and ten years after entry to the firm yields the counterfactual real log hourly wage densities as shown in figure 3. The counterfactual wage densities are the adjusted densities that would have prevailed five and ten years after the respective workers in each cohort joined the firm if the characteristics of cohort individuals had stayed as on entry, or as was argued earlier if exits from the firm had been random.

---

8 Of course this is similar to the Oaxaca (1973) decomposition. Since the Oaxaca decomposition focuses entirely on the mean, constructing counterfactual wage densities allows for making inferences about how wages evolve across the distribution of wages over time.
The empirical evidence embodied in figure 3 suggest for all three cohorts that the actual ln wage distributions consistently understate the growth in wages that would have occurred if cohort individuals had left the firm randomly. In other words, figure 3 supports Stevens’s assertion. The DiNardo methodology relies on reweighting observations based on a probit estimated on observables whereas the Stevens argument implies a non-random selection from the distribution of match quality which of course is unobservable. Note however that we are not interested in the coefficients of the probit equation directly. These coefficients could of course be biased by the exclusion of relevant unobserved variables. What we want is to be able to unbiasedly estimate the weighting function $\psi(z)$ which we assert we will be able to do as long as some of the included regressors will be correlated with unobserved match quality.

In order to get a numerical understanding of the observed empirical negative biases, consider table 1. Approximations of the expected hourly wages in pound sterling for the actual and counterfactual hourly wage distributions are computed at the mean, the 25th and the 75th percentile corresponding to figure 3. For the 1989 cohort, actual wages after 5 and 10 years of entry fall short of counterfactual wages at the mean, the 25th and the 75th percentile. Table 1
also computes the implied annual wage growth for the respective cohort individuals over time. Actual average annual wage growth over ten years for cohort 89 is approximated to be 4.3%. If characteristics of cohort 89 employees had remained as they were on entry, implied average annual wage growth would have been 5.6% suggesting a downward bias in average annual wage growth of 1.3 percentage points.

Table 1

<table>
<thead>
<tr>
<th>Approximated hourly wages (£) and percentage annual hourly wage growth</th>
<th>Actual 89</th>
<th>Actual 94</th>
<th>Counterfactual 94</th>
<th>Actual 99</th>
<th>Counterfactual 99</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average</td>
<td>5.12</td>
<td>6.32</td>
<td>6.56</td>
<td>7.80</td>
<td>8.80</td>
</tr>
<tr>
<td>25th percentile</td>
<td>2.785</td>
<td>3.912</td>
<td>3.989</td>
<td>5.195</td>
<td>5.461</td>
</tr>
<tr>
<td>75th percentile</td>
<td>5.534</td>
<td>6.277</td>
<td>7.220</td>
<td>8.632</td>
<td>10.149</td>
</tr>
</tbody>
</table>

Implied annual wage growth

| Average | 4.3% | 5.1% | 4.3% | 5.6% |
| 25th percentile | 7.0% | 7.5% | 6.4% | 7.0% |
| 75th percentile | 2.6% | 5.5% | 4.5% | 6.3% |

IV The Medoff-Abraham Puzzle

The Stevens model provides a potential explanation of the apparent puzzle presented by Medoff and Abraham (1980) who empirically test the human capital on-the-job training hypothesis that increasing experience earnings profiles can be explained by increasing productivity profiles using single firm data. At the heart of their empirical test lies the assumption that performance indicators are an appropriate instrument to estimate relative within grade employee productivity. Further, if these performance indicators perfectly
captured productivity then adding these performance indicators as regressors into a human capital type log earnings equation should drive the effect of tenure on wages to zero. Medoff and Abraham find that wages increase with tenure but that the introduction of performance indicators into an OLS earnings equation results in the coefficient on tenure to either increase or remain unchanged. Medoff and Abraham conclude that the human capital on-the-job training hypothesis must therefore be questioned as a candidate to explain why wages increase with seniority.

The Stevens results that we have discussed offer another interpretation of what Medoff and Abrahams find. Consider if the coefficient on tenure was already biased downwards, because of the negative correlation between tenure and match quality, then introducing performance indicators which potentially could act as a proxy for match quality would reduce the extent of the negative correlation and therefore reduce the negative bias. This seems consistent with what Medoff an Abraham observe.

V Conclusion

This paper contributes to the ongoing debate in the literature surrounding the question of the bias in the returns to tenure by testing empirically the theoretical prediction of Stevens (2003) that the returns to tenure are unambiguously negatively biased. The model shows that employees accumulating higher levels of specific human capital have a higher propensity of staying with the firm even if their match quality is low resulting in a negative correlation between seniority and match quality for a sample of accepted wages.
We illustrate the validity of the Stevens assertion by constructing counterfactual wage densities in the spirit of DiNardo et al. (1996). Here the question was asked how the distribution of wages would have looked like five and ten years after entry for a cohort who entered the firm in 1989 if characteristics had remained as they were on entry. The results clearly suggest a negative bias in the returns to tenure.

The results also suggest reinterpretation of the apparent puzzle in Medoff and Abraham’s (1980) work. They find that the introduction of performance indicators into an OLS earnings equation appear to either increase or leave the estimated coefficient on tenure unchanged. If performance ratings are measures of productivity then their inclusion should reduce the estimated coefficient on tenure. However if there is an existing negative bias on tenure, adding performance ratings to an OLS equation would serve as a proxy for match quality and therefore reduce some of the existing negative bias on tenure.
Appendix 1: The DiNardo et al counterfactual methodology

The DiNardo et al (1996) methodology is based on the idea of considering each observation as a realisation of a random vector \((w, z, t)\) that belongs to a joint distribution \(F(w, z, t)\) of wages \(w\), individual characteristics \(z\), and date \(t\). Then considering the marginal distribution of wages of, say, cohort 89 on entry, \(t = 1989\),

\[
f(w|t = 1989) = \int f(w, z|t = 1989)dz
\]  

(1A)

Since \(f(w|z, t = 1989) = \frac{f(w, z|t = 1989)}{f(z|t = 1989)}\) and \(\frac{dF(z)}{dz} = f(z)\) equation (1A) can be written as,

\[
f(w|t = 1989) = \int f(w|z, t = 1989)f(z|t = 1989)dz = \int f(w|z, t = 1989)dF(z|t = 1989)
\]  

(2A)

Equation (2A) clearly refers to time period 1989. However, if we write the right hand side of equation (2A) as

\[
\int f(w|z, t = 1994)dF(z|t = 1989)
\]  

(3A)

we would be constructing a counterfactual density of wages that would have prevailed in 1994 (five years after the cohort entered the firm) if the distribution of characteristics \((z)\) would have remained as of 1989. Following DiNardo et al. (1996) this can be denoted by

\[
\int f(w|z, t = 1994)dF(z|t = 1989) = f(w|t_w = 1994, t_z = 1989)
\]  

(4A)

The density in (4A) is what we would like to estimate. The question is simply how to estimate it. Note that (4A) can be written as

\[
f(w|t_w = 1994, t_z = 1989) = \int f(w|z, t = 1994)\frac{dF(z|t = 1989)}{dF(z|t = 1994)}dF(z|t = 1994)
\]  

(5A)

\[
= \int f(w|z, t = 1994)\psi(z)dF(z|t = 1994)
\]
From this, the counterfactual density that would have prevailed in 1994 if the characteristics of workers who entered the firm in 1989 had remained as on entry is the density of wages in 1994 reweighted by the function

$$\psi(z) = \frac{dF(z | t = 1989)}{dF(z | t = 1994)}$$

(6A)

Reweighting the 1994 density allows for the simulation of randomness and thus provides a mechanism to model non-random attrition, \(\psi(z)\) can be estimated by observing that

$$\psi(z) = \frac{dF(z | t = 1989)}{dF(z | t = 1994)} = \frac{dF(t = 1989 | z)dF(z)}{dF(t = 1994 | z)dF(z)} = \frac{\Pr(t = 1989 | z)\Pr(t = 1994)}{\Pr(t = 1994 | z)\Pr(t = 1989)}$$

(7A)

The first terms in both numerator and denominator \(\Pr(t = 1989 | z)\) and \(\Pr(t = 1994 | z)\) can be estimated by conventional probits, these are reported in Appendix 2. To aid with the intuition behind \(\psi(z)\) note that if there was no systematic aspect to the attrition \(\Pr(t = 1989 | z) = \Pr(t = 1989)\) and similarly for 1994 this would imply that \(\psi(z) = 1\).
### Appendix 2: Probit estimations for the reweighting function $\psi(z)$

#### Probit estimations for the reweighting function $\psi(z)$ for cohort 89

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Gender</td>
<td>-0.091</td>
<td>0.027</td>
<td>0.231</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Age</td>
<td>-0.079</td>
<td>0.006</td>
<td>0.049</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Grade on entry$^a$</td>
<td>-0.665</td>
<td>0.060</td>
<td>0.514</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Higher Education</td>
<td>-0.549</td>
<td>0.145</td>
<td>0.405</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>A-level</td>
<td>-0.362</td>
<td>0.060</td>
<td>0.283</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>O-level</td>
<td>-0.623</td>
<td>0.141</td>
<td>0.387</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Children aged 0 to 4</td>
<td>-0.302</td>
<td>0.074</td>
<td>0.210</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Asian/Asian British</td>
<td>-5.724</td>
<td>0.285</td>
<td>0.247</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Black/Black British</td>
<td>0.103</td>
<td>-0.033</td>
<td>-0.194</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Chinese/Ethnic</td>
<td>0.258</td>
<td>(0.213)</td>
<td>0.417</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Gender*Grade on entry</td>
<td>0.415</td>
<td>-0.038</td>
<td>-0.080</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Gender*Further Education</td>
<td>-0.293</td>
<td>0.019</td>
<td>0.179</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Gender*A-level</td>
<td>-0.321</td>
<td>0.051</td>
<td>0.256</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Gender*O-level</td>
<td>-0.295</td>
<td>0.035</td>
<td>0.167</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Gender*Children aged 0-4</td>
<td>-0.223</td>
<td>-0.106</td>
<td>0.282</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Gender*Asian/Asian British</td>
<td>4.266</td>
<td>0.101</td>
<td>-0.061</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Gender*Black/Black British</td>
<td>-0.339</td>
<td>0.047</td>
<td>0.299</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Gender*Chinese/Ethnic</td>
<td>0.168</td>
<td>0.130</td>
<td>0.242</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Constant</td>
<td>1.844</td>
<td>-1.737</td>
<td>-3.770</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Observations</td>
<td>30656</td>
<td>30778</td>
<td>30778</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$X^2$</td>
<td>3179.64</td>
<td>77.88</td>
<td>1068.74</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Log-likelihood</td>
<td>-11152.74</td>
<td>-8385.67</td>
<td>-5126.23</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

a) If staff on entry, grade on entry = 1, 0 otherwise.
b) * significant at the 10% level, ** significant at the 5% level, *** significant at the 1% level.
c) The omitted category for the qualification groups is degree holders. The omitted category for ethnic background is whites.
References


Oaxaca, Ronald. (1973)."Male-Female Wage Differentials in Urban Labor Markets."


