

## Closed-Loop Control of Complex Networks: A Trade-Off between Time and Energy

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Controlling complex nonlinear networks is largely an unsolved problem at the present. Existing works focus either on open-loop control strategies and their energy consumptions or on closed-loop control schemes with an infinite-time duration. We articulate a finite-time, closed-loop controller with an eye toward the physical and mathematical underpinnings of the trade-off between the control time and energy as well as their dependence on the network parameters and structure. The closed-loop controller is tested on a large number of real systems including stem cell differentiation, food webs, random ecosystems, and spiking neuronal networks. Our results represent a step forward in developing a rigorous and general framework to control nonlinear dynamical networks with a complex topology.

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Recent years have witnessed a growth of interest in controlling complex networks. A vast majority of the existing works in this area dealt with the controllability and control of linear dynamical networks [1–27]. Controlling complex networks with nonlinear dynamics has been limited to brute force strategies such as local pinning [28–31] or to specific systems exhibiting a simple kind of multistability [32–36]. Most existing methods of controlling nonlinear networks were of the open-loop type; i.e., one selects a suitable subset of nodes and applies *predefined* control signals or parameter perturbations, which are state independent, to drive the system from an initial state to a desired target. It is, however, difficult to formulate a general and robust open-loop control framework. It is thus of interest to investigate closed-loop control for complex nonlinear dynamical networks, in which a predesigned feedback loop generates control signals according to the instantaneous state of the system. Closed-loop control thus provides a theoretically relevant and significant alternative to controlling complex nonlinear networks.

In controlling chaos in low-dimensional dynamical systems, both open- and closed-loop controls were extensively investigated. The Ott-Grebogi-Yorke [37] principle, in which small, deliberate, and time-dependent perturbations calculated from measured time series are applied to a parameter or a dynamical variable to keep the system in the vicinity of a target periodic orbit, belongs to the open-loop category. Because of the hallmark of chaos, i.e., sensitive dependence on initial conditions, the control perturbation can be small, and there is great flexibility to switch the target orbit. However, real-time observations of the system are needed, and control can be fragile to external

disturbances. The method of Pyragas [38,39] is a closed-loop type of control in which a delayed feedback term is added to the system equations. It does not require real-time observation and analysis of the system, so experimental implementation is greatly facilitated and control can be robust against noise, but the time for control realization is infinite and control flexibility is limited. The developmental history of the field of chaos control provides another motivation for us to consider frameworks as an alternative to open-loop methods for nonlinear network control.

In this Letter, we articulate and analyze a global, finite-time, and closed-loop control framework for complex nonlinear dynamical networks. To ensure that our framework is physically significant, we focus on the control energy and the time required to achieve control and investigate their trade-off. We study how network parameters and structure affect the control time and energy and test the control framework using a variety of real biophysical systems including stem cell differentiation, food webs, random ecosystems, and neuronal networks. Analytically, we derive rigorous upper bounds for both the control energy and time. These results suggest that to develop closed-loop control with optimized control time and energy not only is fundamental to the network control field but also has applied values.

We consider nonlinear dynamical networks described by  $\dot{x}_i = f(x_i) + \sum_{j=1}^N c_{ij} \Gamma x_j(t) + \mathbf{u}[\mathbf{x}(t)] B_i$ ,  $1 \leq i \leq N$ , where  $N$  is the network size,  $x_i = [x_{i1}, \dots, x_{id}]^T \in \mathbb{R}^d$  denotes the  $d$ -dimensional state variable of the  $i$ th node,  $\mathbf{x}$  represents the state variables of the whole network,  $f: \mathbb{R}^d \rightarrow \mathbb{R}^d$  is a nonlinear velocity field governing the nodal dynamics and satisfying  $\|f(x)\| \leq l\|x\|$  or  $|x^T f(x)| \leq l\|x\|^2$  ( $\forall x \in \mathbb{R}^d$ )

with a positive constant  $l$ ,  $\mathbf{C} = (c_{ij}) \in \mathbb{R}^{N \times N}$  is the coupling matrix determined by the network structure,  $\mathbf{\Gamma} \in \mathbb{R}^{d \times d}$  describes the internal coupling configuration at each node,  $\mathbf{u}[\mathbf{x}(t)] = [u_i(t)]_{1 \leq i \leq M} \in \mathbb{R}^{d \times M}$  ( $M \leq N$ ) is the closed-loop control protocol to be designed, and  $\mathbf{B}_i = [b_{i1}, \dots, b_{iM}]^T \in \mathbb{R}^M$  ( $b_{im} = 0, 1$ ) characterizes the driving by the controller  $\mathbf{u}$  to the  $i$ th node. Going beyond the existing works on open-loop control of complex networks, where the goal is to drive the system to an instantaneous state, we set the control target to be an *unstable* steady state, which, for mathematical convenience, is assumed to be  $x_i = 0$  for all  $i$ . For any nontrivial target state, a direct translation can be used to transfer the state to  $x_i = 0$ .

For a general nonlinear dynamical system, a straightforward approach to realizing closed-loop control [40–42] is to set each component of  $\mathbf{u}$  as  $u_i = -kx_i \triangleq u_i^L$  ( $1 \leq i \leq M \leq N$ ). In principle, this linear feedback controller of strength  $k$  is able to steer the dynamics to converge to the target  $x_i = 0$ , but the time required for convergence is infinite. We thus seek alternative methods [43–46] to achieve a finite control time and robustness against disturbances. A typical form of the feedback controller is  $u_i = -k \text{sig}(x_i)^\alpha \triangleq u_i^F$ , which can drive the system to  $x_i = 0$  for all  $t \geq T_f^F$  with  $T_f^F < \infty$ , where  $\text{sig}(x_i)^\alpha = [\text{sgn}(x_{i1})|x_{i1}|^\alpha, \dots, \text{sgn}(x_{id})|x_{id}|^\alpha]^T$ ,  $\text{sgn}(\cdot)$  is a sign function,  $k$  is the control strength, and  $\alpha \in (0, 1)$  is the steepness exponent. The mathematical underpinning of the controller  $u_i^F$  lies in that the non-Lipschitzian  $|\cdot|^\alpha$  at  $x_i = 0$  violates the solution uniqueness of the system of coupled differential equations.

To gain physical insights into the control process, we consider the potential function  $\mathcal{E}_p^{L,F}(x_i) = \int_0^{x_i} u_i^{L,F} dx_i$ , which can be determined from the closed-loop feedback controller. We find that  $u_i^L$  is located higher than  $u_i^F$  for  $|x_i| > 1$ , while the opposite occurs for  $|x_i| < 1$ , as shown in Fig. 1(a). On the potential landscape, the controlled system trajectory can be regarded as a particle moving along some optimal path towards the target  $x_i = 0$ , the minimum of the potential. The particle experiences a stronger potential force along a path determined by  $u_i^L$  ( $u_i^F$ ) for  $|x_i| > 1$  ( $|x_i| < 1$ ). The maximum force occurs for  $|x_i| < 1$  and  $\alpha \rightarrow 0$ . In this case,  $u_i^F|_{\alpha=0}$  corresponds to a double-valued and closed-loop controller, similar to the classical bang-bang control [47]. The basic principle is then to design two controllers in complementary regions of the phase space. This consideration leads us to propose the following global, compound controller:  $u_i = u_i^F \mathcal{I}_U + u_i^L \mathcal{I}_{U^c} \triangleq u_i^S$ , where  $1 \leq i \leq M$ , the unit ball is defined by  $\mathcal{U} = \{\|\mathbf{x}\| < 1\}$ ,  $\mathbf{x} = [x_1^T, \dots, x_N^T]^T$ ,  $\|\cdot\|$  denotes an appropriate norm of the underlying vector,  $\mathcal{U}^c$  is the complement of  $\mathcal{U}$ , and  $\mathcal{I}$  is the indication function for a given subscript set. The norm can be taken as the  $L_p$  or  $L_\infty$ . To be representative and without the loss of generality, we study the  $L_2$  norms. As shown in Fig. 1(b), the compound controller  $u_i^S$  switches from  $u_i^L$  to  $u_i^F$  when the system enters the unit sphere.

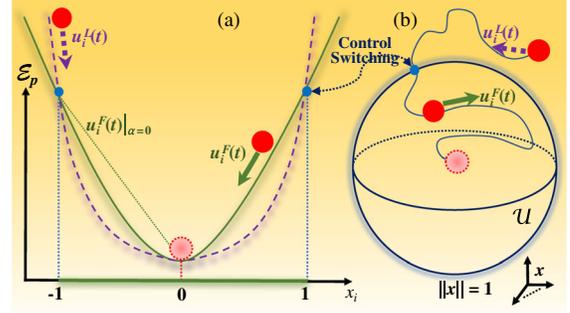


FIG. 1. Physical underpinning of our closed-loop feedback controller. (a) System moving according to the potential function  $\mathcal{E}_p^{L,F}$  (dashed curves) underlying closed-loop feedback controllers  $u_i^{L,F}$ , where  $u_i^L$  specifies a linear feedback controller that acts outside of the unit sphere and  $u_i^F$  denotes a general feedback controller that is activated once the system is inside the unit sphere. (b) Controlled system trajectory in the phase space by  $u_i^S$ , where a control switch occurs when the system crosses the unit sphere  $\|\mathbf{x}\| = 1$ .

We now prove that the controller  $\mathbf{u}^S = [u_i^S]_{1 \leq i \leq M}$  enables finite-time control and provide an estimate of  $T_f^S$ , the time required to achieve control. To be concrete, we set  $M = N$ ,  $b_{ii} = 1$ , and  $b_{im} = 0$  for  $i \neq m$ . As shown in Fig. 1(b), for  $\mathbf{x}(0) \notin \mathcal{U}$ , the control protocol is set as  $u_i^S = u_i^L$ . A direct calculation gives  $d\|\mathbf{x}(t)\|^2/dt \leq -2(k-l-\eta_{\max})\|\mathbf{x}(t)\|^2$ , where  $\eta_{\max}$  is the maximal eigenvalue of the matrix  $\mathbf{H} \equiv [(\mathbf{C} \otimes \mathbf{\Gamma})^T + \mathbf{C} \otimes \mathbf{\Gamma}]/2$  and  $\otimes$  represents the Kronecker product for matrices. Setting  $k > l + \eta_{\max}$  when the networked system is outside of the ball  $\mathcal{U}$ , we get the time instant  $t^*$  such that  $\mathbf{x}(t)|_{t=t^*}$  hits the sphere of  $\mathcal{U}$  with  $t^* \leq [\ln \|\mathbf{x}(0)\|]/\rho$  and  $\rho \triangleq k - l - \eta_{\max} > 0$ . Once the orbit  $\mathbf{x}(t)$  enters  $\mathcal{U}$  after  $t^*$ , because of the dissipation inside  $\mathcal{U}$  (see Supplemental Material [48]), the system will never leave it, so that  $u_i^S$  becomes  $u_i^F$  with the corresponding value  $k$  for  $t > t^*$ , as shown in Fig. 1(b). The dynamical systems theory [48] stipulates that  $d\|\mathbf{x}(t)\|^2/dt \leq -2\rho\|\mathbf{x}(t)\|^{1+\alpha}$  for  $t \geq t^*$  and that  $\mathbf{x}(t) \equiv 0$  for all  $t \geq t^* + 1/\rho(1-\alpha)$ . An analogous analysis applies to the case  $\mathbf{x}(0) \in \mathcal{U}$  with  $u_i^S = u_i^F$ . The upper bound for  $T_f^S$  is then given by

$$T_f^{S, \text{up}} = \begin{cases} \frac{1}{\rho} \left( \ln \|\mathbf{x}(0)\| + \frac{1}{1-\alpha} \right), & \mathbf{x}(0) \notin \mathcal{U}, \\ \|\mathbf{x}(0)\|^{1-\alpha} \frac{1}{\rho(1-\alpha)}, & \mathbf{x}(0) \in \mathcal{U}, \end{cases} \quad (1)$$

with the condition  $\rho > 0$ . We see that, for given values of  $\alpha$  and  $\mathbf{x}(0)$  as well as specific network dynamics with  $l$ ,  $\mathbf{C}$ , and  $\mathbf{\Gamma}$ , the estimation (1) is on the order of  $O(1/k)$ , where  $O(1)$  is a positive and bounded quantity. Accordingly,  $\mathbf{u}^S$  with a larger value of  $k$  can expedite control.

For our controller  $\mathbf{u}^S$ , the required energy cost is [7]  $\mathcal{E}_c^S = \int_0^{T_f^S} \sum_{i=1}^N \|u_i^S(t)\|^2 dt$ . A lengthy calculation [48] leads to the following upper bound for the energy cost:

$$\mathcal{E}_c^{S_{\text{up}}} = \begin{cases} k^2 \frac{1}{2\rho} \left(1 - \|\mathbf{x}(0)\|^{-2} + \frac{2\zeta}{1+\alpha}\right), & \mathbf{x}(0) \notin \mathcal{U}, \\ k^2 \frac{\zeta}{\rho(1+\alpha)} \|\mathbf{x}(0)\|^{1+\alpha}, & \mathbf{x}(0) \in \mathcal{U}, \end{cases} \quad (2)$$

where  $\zeta = (Nd)^{1-\alpha}$ . Since  $\rho \sim k$ ,  $\mathcal{E}_c^{S_{\text{up}}}$  is bounded from above by a quantity on the order of  $O(k)$ . This indicates that, for a given network and given values of  $\alpha$  and  $\mathbf{x}(0)$ , increasing  $k$  will raise the energy cost. In addition, for fixed values of  $\alpha$  and  $\mathbf{x}(0)$ , if  $k$  is sufficiently large, increasing  $l$  or  $\eta_{\max}$  will lead to larger upper bounds for both the control time and energy. For example, for an unweighed and undirected network with  $\Gamma$  being an identity matrix, the quantity  $\eta_{\max}$  becomes  $\lambda_{\max}(\mathbf{C})$ , so increasing the maximum eigenvalue would demand more time and energy for the  $\mathbf{u}^S$ -driven control to be successful.

Using  $\partial_{\alpha}(\ln T_f^{S_{\text{up}}}) = \ln \|\mathbf{x}(0)\|^{-1} + 1/(1-\alpha)$ ,  $\|\mathbf{x}(0)\| \leq 1$ , and  $\alpha \in (0, 1)$ , we can prove that  $T_f^{S_{\text{up}}}$  is an increasing function of  $\alpha$ , i.e.,  $\partial_{\alpha}(T_f^{S_{\text{up}}}) > 0$ , implying that control can be expedited by using a smaller value of the steepness exponent  $\alpha$ . In addition, the condition  $\partial_{\alpha}(\mathcal{E}_c^{S_{\text{up}}}) < 0$  implies that smaller values of  $\alpha$  lead to higher energy costs. The dependence of the energy on  $\alpha$  is consistent with the intuitive, potential-landscape-based physical scenario of control. These results reveal a trade-off between the control time and energy cost for our controller  $\mathbf{u}^S$  with respect to variations in  $\alpha$  or  $k$ . For example, consider the index  $\mathcal{J}_{\gamma,\beta}(k) = \gamma[T_f^{S_{\text{up}}}] + \beta[\mathcal{E}_c^{S_{\text{up}}}]$ , where  $\gamma$  and  $\beta$  are adjustable weights determined by the specific system and  $[\cdot]$  is a normalization function. Since  $\mathcal{J}_{\gamma,\beta}(k) \sim O(1/k) + O(k)$ , there must exist a number  $k_c \gtrsim l + \eta_{\max}$  at which the quantity  $\mathcal{J}_{\gamma,\beta}$  reaches its minimum. The optimal control strength is thus given by  $k = k_c$  in the sense that control can be achieved in less time with a lower energy cost in terms of the index  $\mathcal{J}_{\gamma,\beta}$ .

We demonstrate the working of our optimal closed-loop controller  $\mathbf{u}^S$ , its superior performance as compared with the conventional controllers  $\mathbf{u}^L = [u_i^L]_{1 \leq i \leq M}$ , and the corresponding analytic bounds of the control time and energy, using a number of representative real-world complex nonlinear dynamical networks.

*Controlling stem cell fate.*—We demonstrate that our closed-loop controller can drive two different cell fates to the critical expression level to enable stem cells to remaster their cell fate for cellular differentiation. Specifically, we consider the following network model for hematopoietic stem cells [56], which describes the interaction between two suppressors during cellular differentiation for neutrophil and macrophage cell fate choices [57,58]:  $\dot{x}_1 = 0.5 - x_1$ ,  $\dot{x}_2 = 5x_1/[(1+x_1)(1+x_3^4)] - x_2$ ,  $\dot{x}_3 = 5x_4/(1+x_4)(1+x_2^4) - x_3$ ,  $\dot{x}_4 = 0.5/(1+x_2^4) - x_4$ ,  $\dot{x}_5 = [x_1x_4/(1+x_1x_4) + 4x_3/(1+x_3)]/(1+x_2^4) - x_5$ , and  $\dot{x}_6 = [x_1x_4/(1+x_1x_4) + 4x_2/(1+x_2)]/(1+x_3^4) - x_6$ , where  $x_{2,3}$  are the expression levels of two lineage-specific counteracting suppressors Gfi-1 and

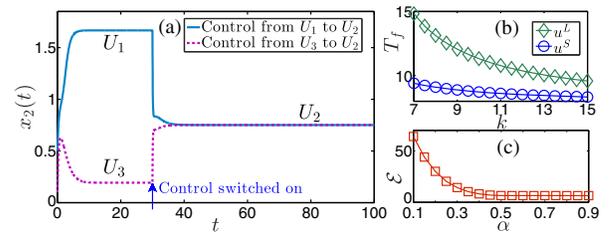


FIG. 2. Controlling a cellular differentiation network model from the steady state  $U_1 = (0.5, 1.66, 0.03, 0.06, 0.02, 2.53)$  or  $U_3 = (0.5, 0.19, 1.66, 0.50, 2.69, 0.10)$  to the steady state  $U_2 = (0.5, 0.75, 1.05, 0.38, 1.69, 0.83)$ . (a) Uncontrolled dynamics [for  $t \in [0, 30]$ ] and controlled dynamics (for  $t \geq 30$ ) for the expression levels of suppressor  $x_2$ , where  $k = 10$  and  $\alpha = 0.5$  when  $\mathbf{u}^S$  is switched on. (b) For  $\alpha = 0.5$ , the control time versus  $k$  for the two controllers  $\mathbf{u}^{S,L}$ . (c) For  $k = 10$ , the control energy versus  $\alpha$  for controller  $\mathbf{u}^S$ .

Egr(1,2), which are activated by their transcription factors  $x_{1,4}$  and simultaneously regulate the downstream genes  $x_{5,6}$ , respectively. As specified in Fig. 2, the system has three steady states:  $U_{1,2,3}$ , where  $U_{1,3}$  correspond to different cell fates and are stable and  $U_2$  represents a critical expression level connecting the two fates and is unstable. Figure 2(a) shows that initially  $x_2$  of the uncontrolled system converges to the stable steady state  $U_1$  or  $U_3$ . From  $t = 30$ , we apply the finite-time controller  $\mathbf{u}^S = \mathbf{u}^F \mathcal{I}_{\mathcal{U}} + \mathbf{u}^L \mathcal{I}_{\mathcal{U}^c}$  with  $\mathcal{U} = \{\|\mathbf{x} - U_2\| < 1\}$ ,  $\mathbf{u}^L = -k(x_2 - U_{22})$ , and  $\mathbf{u}^F = -k \text{sgn}(x_2 - U_{22}) |x_2 - U_{22}|^\alpha$  to  $x_2$ , which is the only variable experimentally accessible [56]. Here,  $U_{22}$  is the second component of  $U_2$ . The controlled system in either of the stable states is driven rapidly to the critical state  $U_2$ , indicating that a finite-time, closed-loop intervention can make the stem cells remaster their cell fate for cellular differentiation. Furthermore, for sufficiently strong control strength  $k$ , the converging time with the controller  $\mathbf{u}^S$  is shorter than that with  $\mathbf{u}^L$ , as shown in Fig. 2(b). Figure 2(c) shows that, for a fixed value of  $k$ , the required control energy decreases with the steepness exponent  $\alpha$ , as predicted by our analysis.

*Controlling nonlinear ecosystems on food-web networks.*—The nonlinear ecological model is described by  $\dot{x}_i = x_i(1 - x_i/K_i)(x_i/A_i - 1) \triangleq f(x_i)$ , where  $x_i$  is the species abundance,  $f$  characterizes the logistic growth, and the carrying capacity is  $K_i$ . The model includes the Allee effect, where the species is *destined for extinction* if its abundance is lower than a threshold value ( $x_i < A_i$ ) [59–61]. We demonstrate that our control method can successfully restore the system out of extinction to a sustainable state. In particular, for each  $i$ , the model has two stable steady states ( $x_i = 0, K_i$ , corresponding to species extinction and capacity overload, respectively) and one unstable steady state ( $x_i = A_i$ ). To prevent the system from evolving into one of the stable steady states, we choose the control target to be  $x_i = A_i$  for all  $i$  that represents restoration or sustainment of species to a state with moderate abundance.

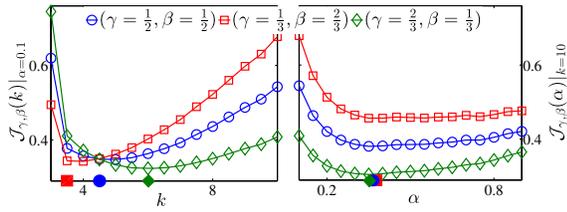


FIG. 3. Dependence of the optimal control strength or steepness exponent on preferential weights. For the Florida food web, optimal locations of the control indices  $\mathcal{J}_{\gamma,\beta}(k)|_{\alpha=0.1}$  and  $\mathcal{J}_{\gamma,\beta}(\alpha)|_{k=10}$  versus the weights, as indicated by the markers along the horizontal axis.

The coupling matrices  $\mathbf{C}$  are constructed from a large number of real food-web networks [48]. For the three controllers  $\mathbf{u}^{S,F,L}$ , we calculate the respective control time  $T_f^{S,F,L}$  required to drive the system into the neighborhood of the target:  $|x_i(t) - A_i| \leq 10^{-4}$ ,  $1 \leq i \leq N$ . The controller  $\mathbf{u}^S$  results in the least control time (see Table S1 in [48] for detailed values from all 22 food-web networks).

To verify our analytic prediction of optimal control through the control indices  $\mathcal{J}_{\gamma,\beta}$ , we use the Florida food web [48] and calculate the indices as a function of  $k$  or  $\alpha$ . Figure 3 shows that the optimal values of  $k_c$  and  $\alpha_c$  depend on the combination of the preferential weights  $(\gamma, \beta)$ , which agree well with the respective analytic results. Simulations further reveal that the optimal value  $k_c$  is more sensitive to the choice of the preferential weights than  $\alpha_c$ , which is reasonable as decreasing the control time tends to make the value of  $k_c$  larger.

*Controlling complex random ecosystems.*—Consider a general ecosystem described by  $\dot{\mathbf{x}} = \mathbf{C}\mathbf{x}$ , where each species  $x_i$  is one-dimensional,  $\mathbf{C} = (c_{ij})_{N \times N}$  describes the random mutual interactions with  $c_{ii} = -r$ , and  $N$  is the population size. Three types of random matrices  $\mathbf{C}$  were studied extensively, which correspond to three typical ecosystems: (a) May’s classic ecosystem [62], where, with probability  $P$ , the off-diagonal elements  $c_{ij}$  are set as mutually independent Gaussian random variables  $\mathcal{N}(0, \sigma_0^2)$  and the probability for the elements to be zero is  $(1 - P)$ ; (b) a mixed ecosystem of competition and mutualism [63], where the off-diagonal elements  $c_{ij}$  and  $c_{ji}$  have the same sign, which are drawn from the distribution  $(\pm|\mathcal{Y}|, \pm|\mathcal{Y}|)$  with probability  $P$  and are zero with probability  $(1 - P)$ ; and (c) the predator-prey (PP) ecosystem [63], where  $c_{ij}$  and  $c_{ji}$  have the opposite signs and are from the distribution  $(\pm|\mathcal{Y}|, \mp|\mathcal{Y}|)$ . As either  $N$  or the variance of  $\mathbf{C}$ ’s elements increases, all three ecosystems eventually become unstable, reflecting the instability of a certain steady state in the original ecosystem from which the linear random system was derived [62,63].

We employ  $\mathbf{u}^S$  to control the ecosystems, which becomes a particular case of our general nonlinear network control

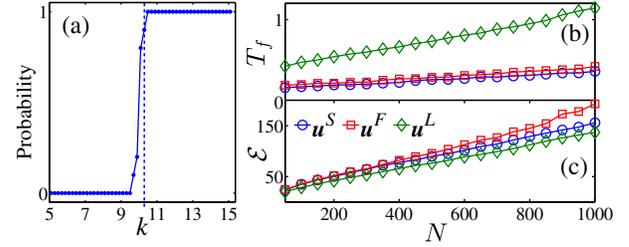


FIG. 4. For May’s classic ecosystems, the probability of successful control versus  $k$  (a), where the vertical dashed line corresponds to  $\eta_{\max}$ ,  $N = 250$ ,  $P = 0.25$ ,  $\sigma_0 = 1$ ,  $r = 1$ , and  $\alpha = 0.6$ , and the required control time cost (b) and energy cost (c), respectively, with the increase of  $N$  for  $k = 1.1\eta_{\max}$ .

framework with  $l = 0$ ,  $\mathbf{\Gamma} = 1$ ,  $b_{ii} = 1$ , and all other  $b_{im} = 0$ . To achieve finite-time control, we estimate the maximal eigenvalue  $\eta_{\max}$  of  $\mathbf{H} = (\mathbf{C}^T + \mathbf{C})/2$  (see Supplemental Material). For May’s classic ecosystem, the well-known semicircle law for random matrices stipulates that  $\mathbf{H}$ ’s eigenvalues are located in  $[-\sqrt{2NP}\sigma_0 - r, \sqrt{2NP}\sigma_0 - r]$  as  $N \rightarrow \infty$  (Supplemental Material). According to Eq. (1), to realize finite-time control requires  $k > \eta_{\max} = \sqrt{2NP}\sigma_0 - r$  (condition A). As shown in Fig. 4(a), successful control is achieved for sufficiently large values of  $k$ . However, from the estimates of the control time and energy [Eqs. (1) and (2), respectively], we see that, for a fixed large value of  $k$ , an increase in either  $N$  or  $\sigma_0$  slows down the control and consumes more energy, eventually violating condition A and causing the control to fail, as shown in Figs. 4(b) and 4(c). While the controller  $\mathbf{u}^S$  requires the least control time among the three available controllers, for a large system size the corresponding energy cost is not necessarily minimum.

For the mixed ecosystem with  $\mathcal{Y} \sim \mathcal{N}(0, \sigma_0^2)$ , from  $\mathbf{H}$ ’s eigenvalue distribution obtained in Ref. [48], we have  $k > \sqrt{2NP(1 + 2/\pi)}\sigma_0 - r$  (condition B) that ensures finite-time control in the probabilistic sense. Similarly for the PP system, we require  $k > \sqrt{2NP(1 - 2/\pi)}\sigma_0 - r$  (condition C). Overall, conditions A–C reveal a *hierarchy* where the PP, May’s classic, and mixed ecosystems require the weakest, intermediate, and strongest control strength  $k$ , respectively. The control time for the three systems can be made finite and identical, because the respective choices of the  $k$  value can result in the same value of  $\rho$  in Eq. (1). In spite of this, the ordering of the control energy for the three types of ecosystems cannot be altered, because  $k$  appears still in Eq. (2) in addition to  $\rho$ .

Akin to the previous example of controlling stem cell fate via only one suppressor, we apply our finite-time controller to different numbers of species in the ecosystem with an undirected scale-free coupling matrix  $\mathbf{C}$ , which reveals a high flexibility of our controller (see [48]).

In summary, we develop a closed-loop control framework for nonlinear dynamical networks to drive the system to a desired unstable steady state in a finite time and with a

predictable energy. Because of the closed-loop nature and high flexibility of the controller, it is suitable for the experimental control of nonlinear networks. We obtain physical and mathematical understandings of the trade-off between the control time and energy. Our closed-loop controller is also effective for realizing synchronization in nonlinear neuronal networks (see [48]). While the issue of optimal energy associated with closed-loop control and single- or two-layer structure has been investigated [64,65], prior to our work a closed-loop control scheme for nonlinear dynamical networks with both optimal time and energy had not been achieved. Our work provides a base for developing a general, physically realizable closed-loop control scheme for complex nonlinear networks with completely unknown steady states.

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