Price Competition with Satisficing Consumers∗

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Abstract

The ‘satisficing’ heuristic by Simon (1955) has recently attracted attention both theoretically and experimentally. In this paper I study a price-competition model in which the consumer is satisficing and firms can influence his aspiration price via marketing. Unlike existing models, whether a price comparison is made depends on both pricing and marketing strategies. I fully characterize the unique symmetric equilibrium by investigating the implications of satisficing on various aspects of market competition. The proposed model can help explain well-documented economic phenomena, such as the positive correlation between marketing and prices observed in some markets.

JEL codes: C79, D03, D43.

Keywords: Aspiration Price, Bounded Rationality, Price Competition, Satisficing, Search.

∗This version: August 2017. I would like to thank the Editor of this journal, two anonymous referees, Ed Hopkins, Hans Hvide, Kohei Kawamura, Ran Spiegler, the seminar audience at universities of Aberdeen, East Anglia, and Trento, and the participants to the 2015 OLIGO workshop (Madrid) and the 2015 Econometric Society World Congress (Montreal) for their comments. Financial support from the Aberdeen Principal’s Excellence Fund and the Scottish Institute for Research in Economics is gratefully acknowledged. Any error is my own responsibility.

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1 Introduction

According to Herbert Simon (1955),

in most global models of rational choice, all alternatives are evaluated before a choice is made. In actual human decision-making, alternatives are often examined sequentially. We may, or may not, know the mechanism that determines the order of procedure. When alternatives are examined sequentially, we may regard the first satisfactory alternative that is evaluated as such as the one actually selected.

The ‘satisficing’ heuristic assumes that the decision-maker discovers and analyses alternatives sequentially and, rather than performing a complicated calculation to derive the optimal stopping rule, follows a simple procedure: if you are satisfied with the current alternative stop, if not keep searching. Once you stop, choose the best discovered alternative (Simon, 1955).\(^1\)

I propose a simple market model in which two profit-maximizing firms producing a homogeneous good compete on prices for a satisficing consumer and influence his aspiration price (the price regarded as satisfactory) via marketing.\(^2\) This research question is relevant for several reasons. First, despite the importance of the satisficing theory, little consideration has been devoted to its implications within an industrial organization setting. This study complements the growing behavioural industrial organization literature (Spiegler, 2011). Second, there is experimental evidence supporting the hypothesis that decision-makers behave according to the satisficing heuristic (Caplin, Dean and Martin, 2011; Reutskaja et al., 2011). In addition,\(^1\)

\(^1\)The version of the satisficing heuristic studied here is called best-satisficing in the sense that if the decision-maker does not find any satisfactory product, then he buys the best unsatisfactory one among those discovered. Alternatively, last-satisficing implies that if no satisfactory product is identified, the decision-maker buys the last discovered product (e.g. see Rubinstein and Salant (2006)). A third possibility is that if there is no satisfactory product, the consumer chooses nothing (he postpones). In a market context, best-satisficing is a plausible assumption in the circumstances in which the consumer urgently needs a product.

\(^2\)The assumption firms are able to manipulate the primitives of the consumer’s choice procedure is standard in the behavioural industrial organization literature (e.g. see Eliaz and Spiegler (2011)).
theorists that have recently studied the choice-theoretic foundations of the satisficing heuristic have provided a deeper understanding of its behavioural implications (Rubinstein and Salant, 2006; Caplin and Dean, 2011; Papi, 2012). Third, investigating the interplay between marketing and prices in real-world markets has recently attracted attention (Bertrand et al., 2010; Agarwal and Ambrose, 2011; Gurun, Matvos and Seru, 2016).

In the proposed market model there are two firms that, along with a price, choose between a high or a low ‘marketing signal’. If both firms choose a high (resp., low) marketing signal, then the consumer’s aspiration price is assumed to increase (resp., decrease), where an increase in the aspiration price makes the consumer more ‘satisficing’ and reduces his willingness to search (and vice versa). On the other hand, if firms choose different marketing signals, then the consumer makes a price comparison if and only if the price that the default firm charges exceeds a firm-specific intermediate aspiration price. Firms therefore face a trade-off between, on the one hand, increasing the aspiration price as much as possible in order to reduce search and charge the highest possible price and, on the other hand, reducing the aspiration price to induce a price comparison and undercut the opponent. The main innovation of the model is that, unlike in the recent models of price-frame competition (Carlin, 2009; Piccione and Spiegler, 2012; Chiovenau and Zhou, 2013; Gu and Wenzel, 2014), whether the consumer makes a price comparison depends upon both marketing and pricing strategies.

In this framework marketing is interpreted to be persuasive in the sense that it is aimed at affecting the consumer’s aspiration price. For example, a firm can increase the consumer’s aspiration price by advertising dominated products along with the target product that it intends to sell. If the consumer - along with the target product (say, a bicycle with certain features) - is shown a second product which is dominated by the target product (e.g. a bicycle of lower quality), then the consumer’s desirability for the target product increases. This phenomenon - known as attraction effect - is well-documented

\[\text{3For instance, a satisficing decision-maker’s choice behaviour is irrational from a revealed preference’s viewpoint if and only if either the ordering according to which he examines the alternatives or his aspiration level are not fixed.}\]

\[\text{4See Bagwell (2007) for a review of the economic literature on advertising.}\]
in the marketing literature (Ratneshwar, Shocker and Stewart, 1987). Hence, one could interpret the high (resp., low) marketing signal as ‘advertising (resp., not advertising) a dominated product’. A second example is given by firms making announcements about product availability (e.g. ‘Hurry up, we are running out of stock’). Recent marketing research has shown that ‘advertisement with a scarcity stimulus can increase perception and purchase intentions for the advertised product’ (Nichols, 2017).

In section 3 I fully characterise the unique symmetric equilibrium by showing that firms randomise over both prices and marketing signals in equilibrium. Unlike in price-frame competition models, the pricing strategy consists of a mixture distribution that contains a gap in the support located just above an intermediate aspiration price. Moreover, the presence of satisficing consumers allows firms to earn extra profits above the max-min level and, unlike most standard models of search, the proposed framework displays equilibrium search.

The paper has two main findings. First, the model predicts that conditional on sending out a low (resp., high) signal, firms charge on average a relatively low (resp., high) price. That is, there is a positive correlation between marketing and prices in equilibrium. This finding can help explain some recent empirical evidence on the relationships between advertising and prices. E.g. Gurun, Matvos and Seru (2016) study the US mortgage market and find that lenders that advertise more intensively are able to sell more expensive mortgages. Gurun, Matvos and Seru (2016) argue that their empirical results reject traditional models of informative advertising and, instead, are more consistent with a persuasive view of advertising, where - like in the proposed model - the heterogeneity in the degree of consumer sophistication plays a crucial role.\(^5\) Other studies that document a relationship between advertising and prices include Agarwal and Ambrose (2011) and Bertrand

\(^5\)In models of price-frame competition in which one frame is unambiguously more complex than the other, firms charge a higher (resp., lower) price conditional on choosing a complex (resp., simple) frame (Chiovenau and Zhou, 2013). However, in those models marketing is not interpreted to be persuasive, but consists of framing decisions (e.g. choosing a simple vs a complex fee structure).
et al. (2010).  

Second, in standard search models an increase in the consumer’s propensity to search (i.e., a decrease in the search cost) leads to a decrease in the consumer’s reservation price and, as a result, affects the equilibrium price distribution accordingly (Stahl, 1989). In this model, on the contrary, an actual increase in the consumer’s equilibrium search may not have any effect on the equilibrium price and marketing strategies. The reason is that, unlike in standard search frameworks, in this model the consumer disregards the equilibrium price distribution when computing his search strategy. This novel result can help explain why - despite there have been advancements in the search technology that have facilitated search, such as the arrival of the internet - price dispersion has endured (Brynjolfsson and Smith, 2000; Baye, Morgan and Scholten, 2004): the internet has induced consumers to search more, but firms have only partially reacted by changing their pricing strategy accordingly, because a fraction of consumers do not search optimally.

The remainder of the paper is organized as follows. Subsection 1.1 reviews the related literature; Section 2 introduces the formal model; Section 3 presents the equilibrium analysis; Section 4 proposes the comparative statics analysis; Section 5 concludes. The proofs of lemma 1 and proposition 1 are relegated in the appendix. Further details and the proofs of the corollaries can be found in the supplementary material, which is available from the author upon request.

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6See also Della Vigna and Gentzkow (2010) for a review of the literature investigating how persuasion affects consumer choice.

7This is true if the search cost is not too high. Moreover, note that in standard search models like Stahl (1989) there is no equilibrium search.

8I explain the intuition behind this result in the comparative statics section. I also discuss the relationship between satisficing and standard models of search in the literature review section.

9The literature has offered several explanations of this phenomenon, such as supply-side bounded rationality (Baye and Morgan, 2003), loyalty to a brand name (Chen and Hitt, 2003), obfuscation (Ellison and Ellison, 2009). See also Pan, Ratchford and Shankar (2004) for a review on this subject.
1.1 Related Literature

This paper is related to the behavioural industrial organization literature on price-frame competition.\textsuperscript{10} Varian (1980) assumes that firms simultaneously compete on price for a consumer that is shopper and uninformed with positive probability.\textsuperscript{11} Both Carlin (2009) and Chiovenau and Zhou (2013) extend Varian (1980) by assuming that the fraction of shoppers and uninformed consumers is endogenously determined. In Carlin (2009)’s model along with a price firms choose its complexity and it is assumed that more complex price structures reduce the fraction of shoppers.\textsuperscript{12} On the other hand, Chiovenau and Zhou (2013) assume that firms can manipulate the extent to which consumers are informed by framing their products in a certain way and, unlike in Carlin (2009), in their model how a firm’s frame influences the extent to which a price comparison is made depends upon the competitors’ frame decisions. Finally, Piccione and Spiegler (2012) focus their attention on the two-firm case by considering a more general framework than Chiovenau and Zhou (2013).\textsuperscript{13} Unlike all these studies, my model assumes that whether a price comparison is made or not, depends on both pricing and marketing strategies.

The proposed model can also be viewed as a standard search model with two special features that have been investigated separately in the literature. First, like in the literature on obfuscation (Ellison and Wolitzky, 2012), firms are able to manipulate the perceived search costs of the consumers via obfuscation strategies.\textsuperscript{14} Second, the consumer does not consider the equilibrium price distribution when computing his search strategy (see e.g. Parakhonyak (2014)) and, instead, merely compares the perceived search cost with

\textsuperscript{10}Recent surveys of the behavioural industrial organization literature include Ellison (2006), Armstrong (2008), and Spiegler (2011).
\textsuperscript{11}Actually Varian (1980) is considered to be a model of standard industrial organization. However, I discuss it here for expositional purposes.
\textsuperscript{12}See also Gu and Wenzel (2014) that consider a two-stage game in which firms first choose the obfuscation levels and then the prices. In their model the obfuscation strategies influence the fraction of sophisticated (i.e., shoppers) and naive (i.e., uninformed) consumers in the population.
\textsuperscript{13}See Spiegler (2014) for an extension.
\textsuperscript{14}See also Armstrong and Zhou (2016) that assumes that firms can deter search by charging higher prices to returning customers.
the price charged by the default firm.\textsuperscript{15} The latter is consistent with the bounded rationality tradition, according to which, while the fully rational consumer has the cognitive capacity to figure out the equilibrium price distribution and calculate the optimal stopping rule, the boundedly rational one does not (or is not willing to) and, as a result, uses simple rules of thumb to make decisions.\textsuperscript{16}

An interesting work is Chen and Zhang (2011) that studies a price-competition model with three types of consumers: shopper, uninformed, and - what they call - global searcher. A global searcher is a fully rational consumer that searches optimally à la Stahl (1989). Under certain conditions on their parameters, which I will discuss below, they too find that the support of the price distribution contains a gap and there may be equilibrium search. However, their model is different from mine in two respects. First, their consumers are fully rational. Second, the consumer types are exogenous.

Finally, a recent study by de Cippel, Eliaz and Rozen (2014) also has the feature that the consumer’s decision of whether to explore the competitor depends on whether the default charges a price below some threshold. However, their model is different from mine, as they assume that fully rational consumers have to decide how to allocate a limited budget of attention to multiple markets.

\section{The Model}

I assume that there are two profit-maximizing firms that produce a homogeneous good at zero costs. Each firm simultaneously chooses a price in the interval $[0, 1]$ and decides whether to send out either a low ($s = 0$) or

\textsuperscript{15}Parakhonyak (2014) argues that it is unrealistic to assume that the consumer knows (or is able to calculate) in advance the equilibrium price distribution. In his model a fraction of consumers are shoppers and a fraction of them disregard the equilibrium price distribution and derive a stopping rule that satisfies certain consistency requirements. Unlike Parakhonyak (2014), my consumers follow a simple rule of thumb.

\textsuperscript{16}This second aspect captures the main difference between the satisficing and the standard approach: unlike in standard search models, in satisficing models the stopping rule is not necessarily optimal. Empirically, one would distinguish between the two approaches by checking whether the consumer searches more or less with respect to what would be the optimal amount of search.
a high \((s = 1)\) marketing signal. Formally, a firm strategy is denoted by \((p, s) \in [0, 1] \times \{0, 1\}\). Marketing is interpreted as persuasive in the sense that it is aimed at affecting the consumer’s aspiration price. As discussed in the introduction, examples include advertising dominated products and making announcements about product availability.

I assume that there is one consumer assigned to firm 1 and firm 2 with equal probability.\(^{17}\) That is, I assume two equally likely states of the world: one in which the consumer’s default is firm 1 and one in which it is firm 2.\(^{18}\) His maximum willingness to pay for the product is normalized to 1. If each firm sends out a low signal, then the consumer is a *shopper*, in the sense that he inspects both firms regardless of the marking signals (i.e., his aspiration price is always 0); if each firm sends out a high signal, the consumer is *uninformed* in the sense that he stops searching at the default firm regardless of the priced charged (i.e., his aspiration price is 1)\(^{19}\); if firms send out different signals, then the consumer is a *conditional shopper* in the sense that he stops searching at the default firm if and only if the price that the default charges is smaller than or equal to an intermediate aspiration price that depends upon the identity of the firm that sends out the high signal: if the default firm sends out a high signal and the competitor a low signal, then the aspiration price is equal to \(\alpha \in (0, 1)\). On the contrary, if the default firm sends out a low signal and the competitor a high signal, then the aspiration price is equal to \(\beta\), where \(\alpha \geq \beta > 0\). The assumption that \(\alpha \geq \beta\) is meant to capture the idea that the default firm’s marketing has a stronger influence on the consumer relative to the competitor’s.\(^{20}\) The consumer is assumed to

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\(^{17}\) An equivalent interpretation is that there is a unit mass of consumers, half of which are assigned to firm 1 and half to firm 2.

\(^{18}\) This assumption is borrowed from Piccione and Spiegler (2012).

\(^{19}\) The terminology ‘shopper’ and ‘uninformed’ is standard in the literature and here it is used accordingly. To see that ‘shopper’ and ‘uninformed’ are special cases of satisficing, let \(L \equiv \langle p_1, p_2, \ldots, p_n \rangle\) be a list of \(n\) prices and \(\bar{p}\) be the consumer’s aspiration price. According to the definition provided above, the satisficing consumer explores the list \(L\) of prices sequentially and stops searching as soon as he finds a \(p\) that is smaller than or equal to \(\bar{p}\). Notice that if \(\bar{p}\) is sufficiently low (resp., high), then the consumer always explores the whole list \(L\) (resp., only the first element of the list \(L\)), which corresponds to the search behaviour of a shopper (resp., an uninformed). E.g. see Papi (2012).

\(^{20}\) Notice that \(\alpha\) and \(\beta\) capture an aspect of the consumer’s bounded rationality and, therefore, are not necessarily related with a measure of consumer satisfaction.
purchase from the cheapest firm discovered and ties are randomly broken.

Denote by \( a_i(s_1, s_2) \) the consumer’s aspiration price when the default firm is firm \( i \) as defined in the previous paragraph. Firm \( i \)'s profit function is defined as follows.

\[
\pi_i((p_i, s_i), (p_j, s_j)) \equiv \begin{cases} 
  p_i & \text{if } p_i < p_j \text{ and } p_j > a_j(s_1, s_2) \\
  \frac{p_i}{2} & \text{if } p_i = p_j \text{ or } p_k \leq a_k(s_1, s_2) \text{ for all } k \in \{1, 2\} \\
  0 & \text{otherwise}
\end{cases}
\]

Firm \( i \) gets all market shares \( (p_i) \) with probability one whenever it is cheaper than firm \( j \) \( (p_i < p_j) \) and the consumer assigned to firm \( j \) finds its price unsatisfactory \( (p_j > a_j(s_1, s_2)) \). The latter implies that in the state of the world in which the consumer is assigned to firm \( j \), he realizes that the default firm charges an unsatisfactory price, inspects firm \( i \), and finds out that it is cheaper. On the other hand, firms obtain all market shares with equal probability \( (\frac{p_i}{2}) \) whenever they either charge the same price \( (p_i = p_j) \) or a satisfactory price \( (p_k \leq a_k(s_1, s_2) \text{ for all } k \in \{1, 2\}) \). If both firms charge a satisfactory price, then there is no search and the consumer sticks to the default firm in all states of the world. Finally, in all the other cases, firm \( i \) gets zero market shares (0).

### 3 Equilibrium Analysis

I begin with a preliminary observation.

**Lemma 1.** There is no equilibrium in which firms choose marketing signals deterministically.

Lemma 1 implies that firms choose both marketing signals with strictly positive probability in equilibrium. This means that the consumer is shopper, uninformed, and conditional shopper with positive probability. Hence, standard arguments imply that there is no equilibrium in which firms choose the price deterministically. Therefore, in equilibrium firms randomize over both marketing and price.

Throughout I denote a symmetric mixed-strategy by \( \sigma \equiv (\lambda(s), (F^s)_{s \in \{0,1\}}) \), where \( \lambda(s) \) is the probability that the marketing signal \( s \) is sent out and \( F^s \)
is the cdf pricing strategy conditional on the firm sending out signal $s$. For compactness, I slightly abuse notation by writing $\lambda$ (resp., $1 - \lambda$) instead of $\lambda(0)$ (resp., $\lambda(1)$).

As an illustration assume that a firm $i$ chooses the pure strategy $(p, s) = (p, 1)$ with $\beta < p < \alpha$ against the other firm choosing some mixed strategy $\sigma$.\footnote{For the sake of the illustration I assume that $F^0$ is atomless. This property turns out to hold in equilibrium.} Then, firm $i$'s profits are

$$\pi((p, 1), \sigma) = p \left( \lambda \left( \frac{1}{2} + \frac{1 - F^0(p)}{2} \right) + \frac{1 - \lambda}{2} \right) \quad (1)$$

The interpretation is that when firm $j$ sends out a low signal, which occurs with probability $\lambda$, the consumer is a conditional shopper and his aspiration price is equal to $\alpha$ (resp., $\beta$) whenever his default firm is firm $i$ (resp., $j$). Since the price that firm $i$ charges is lower than $\alpha$, then in the state of the world in which the default firm is firm $i$ (which occurs with probability $1/2$), he stops searching at firm $i$ and buys from it, as its price is satisfactory. On the contrary, in the state of the world in which the default firm is firm $j$ - given that the price charged by $i$ is unsatisfactory - he purchases from firm $i$ if and only if the price that firm $j$ charges is greater than that charged by $i$, which occurs with probability $1 - F^0(p)$. On the other hand, when firm $j$ sends out a high signal, which occurs with probability $1 - \lambda$, the consumer is uninformed and his aspiration price is equal to 1. In this case the consumer shops randomly. So overall firm $i$'s profits are given by the price $p$ it charges times the probability that the consumer buys firm $i$'s product.

**Lemma 2.** The max-min payoff of the game that models the market under consideration is $\frac{\alpha^2}{2}$.

Assume that a firm $i$ chooses $(p, s) = (\alpha, 1)$. Independently of how the other firm responds, the consumer’s aspiration price is either $\alpha$ when firm $i$ is the default firm (if firm $j$ chooses $s = 0$) or 1 (if firm $j$ chooses $s = 1$). Hence, by charging at most $\alpha$, firm $i$ ensures that in the state of the world in which the consumer is assigned to it, he will stop searching at firm $i$ and buy its product. This implies that firm $i$ makes at least $\frac{\alpha^2}{2}$ profits. Firm $i$ can possibly make higher profits in the case firm $j$ charges an unsatisfactory price with
some probability, but this will not happen under the max-min assumption. It follows that the unique max-minimizer is \((\alpha, 1)\) and the corresponding max-min payoff is \(\frac{\alpha}{2}\).

Lemma 2 has got strong implications on the equilibrium analysis. In particular it entails that the minimum level of profits that a firm can guarantee in equilibrium is positive and equal to \(\frac{\alpha}{2}\).

The equilibrium pricing and marketing strategies turn out to be different depending on whether \(\alpha\) is greater or smaller than a threshold. I examine both cases in proposition 1.

**Proposition 1.** There exists a unique symmetric equilibrium \(\sigma\) such that

- if \(\alpha < \frac{2}{3}\),
  \[
  \lambda = \frac{2 - \alpha}{2 + 3\alpha}
  \]
  \[
  F^0(p) = \begin{cases}
  0 & \text{if } p < \frac{\alpha}{1 + \alpha} \\
  \frac{2(1 + \alpha)}{2 - \alpha} - \frac{2\alpha}{(2 - \alpha)p} & \text{if } p \in \left[\frac{\alpha}{1 + \alpha}, \alpha\right) \\
  \frac{2\alpha}{2 - \alpha} & \text{if } p \in \left[\alpha, \frac{2\alpha}{2 - \alpha}\right] \\
  \frac{2\alpha}{2 - \alpha} - \frac{2\alpha}{(2 - \alpha)p} & \text{if } p \in \left[\frac{2\alpha}{2 - \alpha}, 1\right) \\
  1 & \text{otherwise}
  \end{cases}
  \]
  \(F^1\) consists of two atoms of equal size located at \(\alpha\) and 1.

- and if \(\alpha \geq \frac{2}{3}\),
  \[
  \lambda = 1 - \alpha
  \]
  \[
  F^0(p) = \begin{cases}
  0 & \text{if } p < \frac{3 - 2\alpha}{3 - 2\alpha} \\
  \frac{3 - 2\alpha}{2(1 - \alpha)} - \frac{\alpha}{2(1 - \alpha)p} & \text{if } p \in \left[\frac{3 - 2\alpha}{3 - 2\alpha}, \alpha\right) \\
  1 & \text{otherwise}
  \end{cases}
  \]
  \(F^1\) consists of two atoms located at \(\alpha\) and 1 of size \(\frac{2\alpha - 1}{\alpha}\) and \(\frac{1 - \alpha}{\alpha}\), respectively.

Proposition 1 possesses multiple noteworthy properties. First, the equilibrium marketing and pricing strategies are independent of \(\beta\). This implies that the extent to which the competitor’s influence on the consumer’s aspiration price is close to the default firm’s is irrelevant. So no matter how
influential the competitor is, as long as $\beta \leq \alpha$ the size of $\beta$ does not have any effect on the equilibrium. This result turns out to have novel implications on the relationship between the consumer’s search behaviour and the firms’ equilibrium strategies that I will discuss in the comparative statics section.

Second, the equilibrium probability that firms send out a high signal is increasing in $\alpha$. That is, a higher conditional shopper’s aspiration price induces firms to enhance marketing.

Third, figure 1 graphically represents the support of $F^0$ and $F^1$ under the assumption that $\alpha < \frac{2}{3}$. Conditional on $s = 0$, firms randomize over the price according to an atomless cdf. This follows from the fact that, conditional on $s = 0$, the consumer is a shopper with positive probability in every realization. The support of $F^0$ consists of the union of two disjoint intervals located just below the aspiration prices $\alpha$ and 1. As $\alpha$ increases, $\frac{2\alpha}{2-\alpha}$ converges to 1 and becomes equal to it as $\alpha$ meets the threshold $\frac{2}{3}$. As soon as that occurs, the right portion of the support of $F^0$ disappears, which corresponds to the second case of proposition 1. On the other hand, conditional on sending out a high signal, firms charge two prices - $\alpha$ and 1 - with positive probability. The intuition behind this result is three-fold. First, conditional on sending a high signal, it does not make sense for a firm

![Figure 1: The support of $F^0$ and $F^1$ when $\alpha < \frac{2}{3}$](image-url)
to charge a price strictly below $\alpha$, because in the worst case (when firm $j$ chooses $s = 0$) the consumer’s aspiration price is $\alpha$ in the state of the world in which the consumer is assigned to $i$. Second, as it will become clear below, increasing the price from $\alpha$ to a price above $\alpha$ conditional on sending out a high signal reduces market power, because the prices above $\alpha$ are unsatisfactory when firms send out different signals. Third, conditional on sending a high signal, the market shares that a firm obtains when the other firm sends out a high signal as well are independent of the price that firm $i$ charges. The reason is that when both firms send out a high signal the consumer is uninformed implying that he sticks to the default regardless of the price they charge. This induces firms to charge the highest possible price.

Fourth, in terms of the relationship between $F^0$ and $F^1$, it turns out that $F^1 \text{ first-order stochastically dominates } F^0$ if and only if $\alpha \geq \frac{2}{5}$.$^{22}$ The latter indicates that there is a strong correlation between prices and marketing in equilibrium. I will further develop this discussion in the comparative statics section.

Fifth, there exists a price interval just above $\alpha$ to which both $F^0$ and $F^1$ assign zero mass. The reason is that by increasing the price from $\alpha$ to a price above $\alpha$ there is a discontinuity in profits, as firms suddenly lose market power. To gain an intuition of why this is the case I now compute a firm $i$’s expected profits of choosing $(p', 1)$ with $p' > \alpha$ against some mixed-strategy $\sigma$.

$$\pi((p', 1), \sigma) = p' \left( \lambda (1 - F^0(p')) + \frac{(1 - \lambda)}{2} \right)$$

(2)

By comparing the above expected profits (eq.(2)) with the expected profits of choosing the strategy $(p, 1)$ with $\beta < p < \alpha$ (see eq. (1)), it can be seen that there is a difference in the realization in which firm $j$ sends out a low signal. In particular as long as firm $i$ charges a price at most equal to $\alpha$, it guarantees that in the state of the world in which the consumer is assigned to it, the conditional shopper stops searching at firm $i$, as its price

$^{22}$When $\alpha \geq \frac{2}{5}$, then necessarily $F^1 \text{ first-order stochastically dominates } F^0$. When $\alpha < \frac{2}{5}$, $F^1 \text{ first-order stochastically dominates } F^0 \text{ if and only if } F^0(\alpha) = \frac{2\alpha}{2 - \alpha} > \frac{1}{2} = F^1(\alpha)$. That is, $\alpha \geq \frac{2}{5}$.
is satisfactory, regardless of the price charged by firm \( j \). On the contrary, as soon as firm \( i \) increases its price above \( \alpha \), it makes its price unsatisfactory implying that in the state of the world in which the consumer is assigned to firm \( i \), the conditional shopper will inspect firm \( j \). For a sufficiently small \( \epsilon > 0 \), this discontinuity in profits implies that an increase in the price from \( \alpha \) to \( \alpha + \epsilon \) does not offset the reduction in market shares.

One of the few models in the literature whose equilibrium pricing strategy also displays a gap in the support is Chen and Zhang (2011) discussed above. Their model generates a price distribution with a gap only if the shopper’s reservation price exceeds by a relatively large amount the global searcher’s reservation price at a benchmark case - i.e., when the consumer is either a shopper or a global searcher (and never an uninformed consumer).

4 Comparative Statics

4.1 Average Price and Profits

Corollary 1. In equilibrium

- **Conditional on** \( s=0 \), each firm charges on average
  
  \[
  \frac{2\alpha}{2-\alpha} \left( \ln(1 + \alpha) - \ln \left( \frac{2\alpha}{2-\alpha} \right) \right), \text{ if } \alpha < \frac{2}{3} \\
  \frac{\alpha}{2(\alpha-1)} \left( \ln \left( \frac{1}{3-2\alpha} \right) \right), \text{ otherwise.}
  \]

- **Conditional on** \( s=1 \), each firm charges on average
  
  \[
  2\alpha + \frac{1}{\alpha} \quad \text{if } \alpha < \frac{2}{3} \\
  \alpha + 2, \quad \text{otherwise.}
  \]

- **Firm’s profits are**
  
  \[
  \frac{2\alpha}{2+3\alpha}, \quad \text{if } \alpha < \frac{2}{3} \\
  \frac{\alpha}{2}, \quad \text{otherwise.}
  \]

Figure 2 graphically represents the conditional average prices. Both average prices are increasing in \( \alpha \). A decrease in \( \alpha \) makes the conditional shopper
more and more similar to a shopper leading to an increase in competition and lower prices. Except in the extreme case in which the conditional shopper becomes uninformed (i.e., $\alpha = 1$), there is a positive correlation between marketing and prices in equilibrium in the sense that conditional on sending out a high (resp., low) signal firms charge on average a relatively high (resp., low) price. This result is intuitive, because, unlike a low marketing signal, a high signal increases the consumer’s aspiration price, reduces his willingness to search, and, as a result, makes the market relatively less competitive, which in turn leads firms to charge higher prices. As discussed in the introduction, this finding can help explain the interaction between marketing and prices detected in some real-world markets (Gurun, Matvos and Seru, 2016).

Equilibrium profits are depicted in figure 3. Notice that an increase in $\alpha$ translates into the conditional shopper being more and more ‘satisficing’, which in turn leads to greater profits.

Lemma 2 states that the minimum level of profits that firms can guarantee in this model is equal to $\frac{\alpha}{2}$. It is therefore interesting to examine whether competitive forces drive equilibrium profits to the max-min level or the presence of satisficing consumers allows firms to earn extra profits. Notice that
in equilibrium, when $\alpha < \frac{2}{3}$,

$$\pi((\alpha,1),\sigma) = \alpha \left( \lambda \left( \frac{1}{2} + \frac{1 - F^0(\alpha)}{2} \right) + \frac{(1 - \lambda)}{2} \right)$$

Notice also that $\pi((\alpha,1),\sigma) = \frac{\alpha}{2}$ if and only if $F^0(\alpha) = 1$. That is, firms earn the max-min payoff in equilibrium whenever, conditional on sending out a low signal, they never charge a price greater than $\alpha$. However, in equilibrium firms do charge prices greater than $\alpha$ conditional on sending out a low signal if and only if $\alpha < \frac{2}{3}$. This means that the source of extra profits above the max-min is given by the fact that firms charge with positive probability prices that are never satisfactory to the conditional shopper, conditional on sending out a low signal.

4.2 Consumer Search Behaviour and Switching Rate

**Corollary 2** (Equilibrium Search and Switching Rate). The equilibrium probability that only one firm is inspected is

- $\frac{2\alpha(2+2F^0(\beta)(2-\alpha)+7\alpha)}{(2+3\alpha)^2}$, if $\alpha < \frac{2}{3}$
- $\alpha(3 + F^0(\beta) - \alpha(1 + F^0(\beta))) - 1$, otherwise.
The equilibrium switching rate is

\[ 4 + 3(4 - 5\alpha)\alpha \quad \text{if} \quad \alpha < \frac{2}{3} \]

\[ \frac{3(\alpha - 1)^2}{2} \quad \text{otherwise.} \]

Figure 4: Probability that One Firm Is Inspected when \( F^0(\beta) = 0 \) (Solid Line) and \( F^0(\beta) = 1 \) (Dashed Line)

In terms of search behaviour, corollary 2 contains three main messages (see also figure 4). First, the equilibrium probability that only one firm is inspected increases with the aspiration levels \( \alpha \) and \( \beta \). That is, the more the consumer is ‘satisficing’, the less he searches. Second, unlike most models of consumer search, this model displays equilibrium search. Specifically, the consumer always searches except in the extreme case in which the conditional shopper becomes uninformed (i.e., \( \alpha = 1 \)). Third and most importantly, despite a change in \( \beta \) causes a change in the equilibrium probability that only

---

\(^{24}\) An exception is again the model by Chen and Zhang (2011). Under the conditions specified previously, their model displays a price distribution with a gap. In that case both firms set the price above the global searcher’s reservation price with positive probability in equilibrium. As a result the global searcher may search more than once before purchasing in equilibrium.
one firm is inspected, the resulting change in consumer’s search behaviour does not have any effect on the equilibrium marketing and pricing strategies (see also figure 5 representing the switching rate). The intuition is as follows.

Consider the first case of proposition 1 by assuming $\alpha < \frac{2}{3}$. Suppose that a firm $i$ chooses a pure strategy $(p, s) = (\tilde{p}, 0)$ with $\tilde{p} \in \left[\frac{\alpha}{1+\alpha}, \alpha\right)$ against firm $j$ choosing the equilibrium strategy $\sigma$. Focus the attention on the realization in which firm $j$ sends out a high signal, which occurs with probability $1 - \lambda$, and the state of the world in which the consumer’s default firm is firm $i$. In this case the consumer is a conditional shopper with an aspiration price equal to $\beta$. Assume first that $\beta \geq \tilde{p}$. Firm $i$’s expected profits are:

$$\pi((\tilde{p}, 0), \sigma) = \tilde{p} \left(\lambda(1 - F^0(\tilde{p})) + (1 - \lambda) \left(\frac{1}{2} + \frac{1 - F^1(\alpha)}{2}\right)\right)$$  \hspace{1cm} (3)

That is, the conditional shopper assigned to firm $i$ finds its price satisfactory, stops searching, and purchases from firm $i$ (second addendum of equation 3). Assume now that - ceteris paribus - the conditional shopper’s

\footnote{The same argument goes through if one assumes instead $\alpha \geq \frac{2}{3}$.}
aspiration price $\beta$ decreases below $\tilde{p}$. This means that the conditional shopper assigned to firm $i$ now finds firm $i$’s price unsatisfactory and therefore explores firm $j$. Firm $j$, however, is more expensive than firm $i$ (it charges $\alpha$ and 1 with equal probability), so that the consumer goes back to firm $i$ and purchases from it. Notice that firm $i$’s expected profits in this second case are exactly the same as those of equation 3, despite the fact that the consumer’s search behaviour has changed: while in the first case the consumer has stopped searching at firm $i$, in the second case he has made a price comparison. This result is given by the fact that the consumer is boundedly rational and does not search optimally by disregarding the equilibrium price distribution. In particular in the second case the consumer is unable to realize that if he keeps searching, the price that he will discover is worse with certainty.

This finding can help explain why - despite there have been advancements in the search technology that have facilitated search, such as the arrival of the internet - price dispersion has persisted (Brynjolfsson and Smith, 2000): the internet has induced consumers to search more actively, but, since a fraction of consumers do not search optimally, firms have only partially adjusted their marketing and pricing strategies accordingly.

5 Concluding Discussions

In this section I acknowledge the main limitations of the model and outline possible extensions. First, the model assumes for simplicity that, for a given marketing profile $(s_i, s_j)$, the consumer’s induced aspiration price is deterministic. Assume instead that $G_i(p; s_i, s_j)$ denotes a continuous cdf with support $[\underline{p}, \overline{p}]$ such that $\underline{p} \leq 0$ and $\overline{p} \geq 1$ measuring the probability with which the consumer’s aspiration price is smaller than or equal to $p$ under the marketing profile $(s_i, s_j)$ when firm $i$ is the default firm. It would be interesting to verify whether the results obtained in this paper extend to this more general setup. A preliminary observation is that in this extended setting mixing over prices is required even when the marketing profile is fixed, as undercutting is always profitable and the max-min payoff is strictly greater
than zero.\textsuperscript{25}

Second, the satisficing heuristic is characterized by two primitives: (i) the ordering according to which the products are examined and (ii) an aspiration price. In this model firms can endogenously manipulate the latter only. A natural extension would be to consider a more general model in which firms can affect also the ordering according to which alternatives are examined and, therefore, which firm is the default firm.

Third, the paper does not derive policy implications of the model. An interesting exercise would be to study the effects of a standard policy intervention of making the market more competitive. The latter would require extending the model to \( n \) firms.

\textbf{A Proof of Lemma 1}

Assume, by contradiction, that there is an equilibrium in which firms choose marketing signals deterministically. I distinguish three cases.

\textbf{Case (i)}: both firms choose \( s=0 \). In this case the consumer is a shopper and standard Bertrand arguments imply that the unique equilibrium candidate is given by both firms charging a zero price, which leads to zero profits. However, each firm \( i \) has an incentive to deviate to \((p, s) = (\alpha, 1)\). The reason is that at the deviation the consumer’s aspiration price is \( \alpha \) (resp., \( \beta \)) in the

\textsuperscript{25}As an illustration, notice that

\[ \pi_i(p, p) = p \left( \frac{G_i(p) + (1 - G_i(p))}{2} + \frac{G_j(p) + 0}{2} \right) \]

and

\[ \pi_i(p - \epsilon, p) = (p - \epsilon) \left( \frac{G_i(p - \epsilon) + (1 - G_i(p - \epsilon))}{2} + \frac{G_j(p) + 0}{2} \right) \]

The latter is greater than the former for a sufficiently small \( \epsilon \). Moreover, charging zero yields zero, but it is profitable to deviate to a positive price, as it yields at least \( p \frac{1 - G_i(p)}{2} > 0 \).
state of the world in which the consumer’s default firm is \( i \) (resp., \( j \)). In the state of the world in which the default is firm \( i \), the consumer finds firm \( i \)’s price satisfactory. This implies that he does not inspect firm \( j \) and buys firm \( i \)’s product. Hence, firm \( i \) makes \( \frac{\alpha}{2} > 0 \) profits, a contradiction.

**Case (ii):** both firms choose \( s=1 \). In this case the consumer is uninformed. Hence, the unique equilibrium candidate is given by firms charging a price equal to 1, which yields \( \frac{1}{2} \) profits. Each firm \( i \) can profitably deviate by reducing the consumer’s aspiration price below 1 (by choosing \( s = 0 \)) and charging a price \( \epsilon > 0 \) smaller than 1. In this way firm \( i \) ensures that the consumer finds the price charged by both firms unsatisfactory and buys firm \( i \)’s product with probability one, as it is cheaper. At the deviation firm \( i \) makes \( 1 - \epsilon \) profits that are greater than \( \frac{1}{2} \) for a sufficiently small \( \epsilon \), a contradiction.

**Case (iii):** firm \( i \) chooses \( s = 0 \) and firm \( j \) \( s = 1 \). Then, in the state of the world in which the default firm is firm \( i \) (resp., \( j \)) the consumer’s aspiration price is \( \beta \) (resp., \( \alpha \)). It can be shown that the unique equilibrium candidate is an asymmetric mixed-strategy profile that depends upon the distance between \( \alpha \) and \( \beta \). I distinguish two sub-cases.

**Sub-case (a):** \( \beta < \frac{\alpha}{2} \). At the equilibrium candidate firm \( i \) randomizes over the price according to the cdf:

\[
F(p) = \begin{cases} 
0 & \text{if } p < \frac{\alpha}{2} \\
2 \left(1 - \frac{\alpha}{2p}\right) & \text{if } p \in \left[\frac{\alpha}{2}, \alpha\right) \\
1 & \text{otherwise}
\end{cases}
\]  

Sub-case (4)

Firm \( j \), on the other hand, with probability \( A = \frac{1}{2} \) charges \( \alpha \) and with probability \( 1 - A \) randomizes over the price according to the cdf of equation 4. Firm \( i \) makes \( \frac{\alpha}{4} \) and firm \( j \) \( \frac{\alpha}{2} \) profits. Firm \( i \) can profitably deviate by choosing \( (p, s) = (1, 1) \). The deviation yields \( \frac{1}{2} > \frac{\alpha}{4} \), a contradiction.

**Sub-case (b):** \( \beta \geq \frac{\alpha}{2} \). At the equilibrium candidate, firm \( i \) with probability \( B = \frac{2\beta - \alpha}{\beta} \) charges \( \beta \) and with probability \( 1 - B \) randomizes over the price according to the cdf:
\[
F(p) = \begin{cases} 
0 & \text{if } p \leq \beta \\
\frac{\alpha(p - \beta)}{p(\alpha - \beta)} & \text{if } p \in (\beta, \alpha) \\
1 & \text{otherwise} 
\end{cases}
\]

(5)

Firm \( j \), instead, with probability \( A = \frac{\beta}{\alpha} \) charges \( \alpha \) and with probability \( 1 - A \) randomizes according to the cdf of equation 5. Firm \( i \) makes \( \frac{\beta}{2} \) and firm \( j \) makes \( \frac{\alpha}{2} \) profits. Firm \( j \) can profitably deviate by choosing \((p, s) = (\beta, 0)\). At the deviation firm \( j \) makes \( \beta > \frac{\beta}{2} \) profits, a contradiction. \( \blacksquare \)

B Proof of proposition 1

Let \( \sigma \) denote a symmetric equilibrium (mixed-) strategy. Denote by \( p_1^L \) and \( p_1^H \) the infimum and the supremum of the support of \( F^* \) with \( s = 0, 1 \), respectively. By standard arguments, \( F^0 \) is atomless and, by lemma 2, \( p_s^L > 0 \) for \( s = 1, 2 \). I prove the statement in a series of steps.

Step 1: \( p_0^0 < p_1^0 \leq p_1^1 = 1 \).

Proof. Assume, by contradiction, that \( p_0^0 > p_1^1 \). Suppose first that \( p_0^0 > \beta \). Then, \( \pi((p_0^0, 0), \sigma) = 0 \), which contradicts lemma 2. Assume that \( p_0^0 \leq \beta \). Then, \( \pi((p_0^0, 0), \sigma) = p_0^0 (1 - \lambda) \frac{1}{2} \). However, deviating to \((p, s) = (\alpha, 1)\) yields \( \pi((\alpha, 1), \sigma) = \frac{\alpha}{2} (1 - \lambda) > p_0^0 (1 - \lambda) \frac{1}{2} \), a contradiction. Hence, \( p_0^0 \leq p_1^1 \).

Next, assume, by contradiction, that \( p_1^1 < 1 \). Suppose first that \( p_1^1 > \alpha \). Then, \( \pi((p_1^1, 1), \sigma) = \frac{p_1^1}{2} (1 - \lambda) \), but then it is profitable to deviate to \((p, s) = (1, 1)\). Next, assume that \( p_1^1 < \alpha \). Notice that \( \pi((p_1^1, 1), \sigma) = p_1^1 \frac{1}{2} \), but then it is profitable to deviate to \( \alpha \). It remains to examine the case \( p_1^1 = \alpha \). Suppose first that \( p_0^0 > \beta \). Then, \( \pi((p_0^0, 0), \sigma) = p_0^0 \left( (1 - \lambda) \left( 1 - \frac{F^0(p_0^0)}{2} \right) \right) \). But deviating to \((p, s) = (p_0^0, 1)\) yields \( \pi((p_0^0, 1), \sigma) = \frac{p_0^0}{2} \) and is profitable. A similar argument applies if \( p_0^0 \leq \beta \). Hence, \( p_1^1 = 1 \). \( \square \)

Step 2: \( p_0^0 < p_1^1 \leq \alpha \).

Proof. Suppose, by contradiction, that \( p_1^1 > \alpha \). Notice that \( \pi((p_1^1, 1), \sigma) = p_1^1 \left( \lambda (1 - F^0(p_1^1)) + \frac{1 - \lambda}{2} \right) \). However, deviating to \((p, s) = (p_1^1, 0)\) is profitable, as it yields \( p_1^1 \left( \lambda (1 - F^0(p_1^1)) + (1 - \lambda) \right) \), which leads to a contradiction. Hence, \( p_1^1 \leq \alpha \). 22
Next, assume, by contradiction, that \( p_0^L \geq p_1^L \). Notice that \( \pi((p_1^L, 1), \sigma) = p_L^1 \left( \lambda \left( \frac{1}{2} + \frac{1 - F^0(\beta)}{2} \right) + \frac{(1 - \lambda)}{2} \right) \). Assume first that \( F^1 \) does not have an atom at \( p_1^L \). Then, deviating to \((p, s) = (p_1^L, 0)\) yields \( p_L^1 \left( \lambda + (1 - \lambda) \left( \frac{1}{2} + \frac{1 - F^1(\alpha)}{2} \right) \right) \) and is profitable, as, by step 1, \( F^1(\alpha) < 1 \). Notice that, if \( F^1 \) has an atom at \( p_1^L \), then \((p, s) = (p_1^L - \epsilon, 0)\) constitutes a profitable deviation for a sufficiently small \( \epsilon \). Hence, \( p_0^L < p_1^L \).

**Step 3:** Either \( F^0 \) or \( F^1 \) is strictly increasing over the interval \([p_0^L, \alpha)\).

**Proof.** Suppose, by contradiction, that there exists an interval \([p', p'']\) with \( \alpha > p'' > p' > p_0^L \) over which both \( F^0 \) and \( F^1 \) are flat. By standard arguments, this cannot be the case, as expected profits of \((p'', s)\) are greater than expected profits of \((p', s)\) against \( \sigma \).

It remains to show that there cannot be an interval \([p', p'']\) with \( \alpha > p'' > p' > p_0^L \) over which both \( F^0 \) and \( F^1 \) are strictly increasing. Suppose not. Assume first that \( p'' \leq \beta \). Notice that

\[
\pi((p', 0), \sigma) = p' \left( \lambda(1 - F^0(p')) + (1 - \lambda) \left( \frac{1}{2} + \frac{1 - F^1(\alpha)}{2} \right) \right)
\]

\[
\pi((p', 1), \sigma) = p' \left( \lambda \left( \frac{1}{2} + \frac{1 - F^0(\beta)}{2} \right) + \frac{(1 - \lambda)}{2} \right)
\]

By equating these two and solving for \( F^1(\alpha) \),

\[
F^1(\alpha) = \frac{1 - \lambda - 2\lambda F^0(p') + \lambda F^0(\beta)}{1 - \lambda}
\]

(6)

Next, notice that

\[
\pi((p'', 0), \sigma) = p'' \left( \lambda(1 - F^0(p'')) + (1 - \lambda) \left( \frac{1}{2} + \frac{1 - F^1(\alpha)}{2} \right) \right)
\]

\[
\pi((p'', 1), \sigma) = p'' \left( \lambda \left( \frac{1}{2} + \frac{1 - F^0(\beta)}{2} \right) + \frac{(1 - \lambda)}{2} \right)
\]

By equating these two and solving for \( F^1(\alpha) \), \( F^1(\alpha) = \frac{1 - \lambda - 2\lambda F^0(p'') + \lambda F^0(\beta)}{1 - \lambda} \), which contradicts equation 6.

A similar argument applies if \( p' > \beta \).

**Step 4:** \( F^1 \) has an atom at \( p_1^L = \alpha \).
Proof. I first show that $p_1^L = \alpha$. Assume, by contradiction, that $p_1^L < \alpha$. By step 3, either $F^0$ or $F^1$ is strictly increasing over the interval $[p_1^L, \alpha)$. Assume first that $F^1$ is strictly increasing over the interval $[p_1^L, \alpha)$. Since $\pi((p, 1), \sigma)$ is independent of $p$ in the realization in which the opponent chooses $s = 1$, then $F^0$ must be strictly increasing as well over the same interval. However, this contradicts step 3. Therefore, $F^1$ must have an atom at $p_1^L < \alpha$ and $F^0$ is strictly increasing over the interval $[p_0^L, \alpha)$. However, this is impossible as well, as the presence of the atom makes undercutting profitable conditional on $s = 0$. Hence, $p_1^L = \alpha$.

To prove that $F^1$ has an atom at $\alpha$, notice that:

$$\pi((\alpha, 1), \sigma) = \alpha \left( \lambda \left( \frac{1}{2} + \frac{1 - F^0(\alpha)}{2} \right) + \frac{1 - \lambda}{2} \right)$$

$$\pi((\alpha + \epsilon, 1), \sigma) = (\alpha + \epsilon) \left( \lambda \left( 1 - F^0(\alpha + \epsilon) \right) + \frac{1 - \lambda}{2} \right)$$

Since for a sufficiently small $\epsilon$ the former is greater than the latter, then it must be the case that $F^1$ has an atom at $\alpha$.

\[\square\]

**Step 5:** $F^0$ is flat over the interval $(\alpha, \hat{p})$ for some $\hat{p} \in (\alpha, 1)$.

**Proof.** Notice that

$$\lim_{p \to \alpha} \pi((p, 0), \sigma) = \alpha \left( \lambda(1 - F^0(\alpha)) + (1 - \lambda) \left( \frac{1}{2} + \frac{1 - F^1(\alpha)}{2} \right) \right)$$

$$\pi((\alpha + \epsilon, 0), \sigma) = (\alpha + \epsilon) \left( \lambda(1 - F^0(\alpha + \epsilon)) + (1 - \lambda) \left( 1 - F^1(\alpha + \epsilon) \right) \right)$$

Notice that the former is greater than the latter for a sufficiently small $\epsilon$. Denote by $\hat{p}$ the supremum of the interval $(\alpha, \alpha + \epsilon)$ for which $\lim_{p \to \alpha} \pi((p, 0), \sigma) - \pi((\alpha + \epsilon, 0), \sigma) > 0$.

\[\square\]

**Step 6:** $F^1$ consists of two atoms located at $\alpha$ and 1.

**Proof.** For similar arguments to step 5, $F^1$ must be flat over the interval $(\alpha, \min\{\hat{p}, 1\})$. If $\hat{p} \geq 1$, then the result follows immediately. Assume, instead, that $\hat{p} < 1$. By the same arguments of step 3, either $F^0$ or $F^1$ is strictly increasing over the interval $(\hat{p}, 1)$. If $F^1$ is strictly increasing, then, by the arguments of step 3, $F^0$ must be strictly increasing as well, a contradiction. Hence, $F^0$ is strictly increasing over the interval $(\hat{p}, 1)$, $F^1$ is flat over the same interval and has an atom at 1, and $p_1^H = 1$, which leads to the desired result.

\[\square\]
Step 7: \( p_H^0 \in \{ \alpha, 1 \} \).

Proof. I first show that \( p_H^0 \geq \alpha \). Suppose not. Hence, \( p_H^0 < \alpha \). Then, \( \pi((p_H^0, 0), \sigma) = p_H^0 \left( (1 - \lambda) \left( \frac{1}{2} + \frac{1 - F^1(\alpha)}{2} \right) \right) \), as, by step 4, \( p_L^1 = \alpha \). Assume that a firm deviates to \((p, s) = (\alpha - \epsilon, 0)\) for a sufficiently small \( \epsilon > 0 \). In the limit as \( \epsilon \to 0 \), this deviation yields \( \alpha \left( (1 - \lambda) \left( \frac{1}{2} + \frac{1 - F^1(\alpha)}{2} \right) \right) \) and is profitable if and only if \( p_H^0 < \alpha \), which is true by assumption, a contradiction.

Next, suppose, by contradiction, that \( p_H^0 \in (\alpha, 1) \). By step 6, \( F^1 \) is flat over the interval \((\alpha, 1)\). Note that choosing \((p_H^0, 0)\) yields \( p_H^0 \left( (1 - \lambda) \left( \frac{1}{2} + \frac{1 - F^1(\alpha)}{2} \right) \right) \). However, deviating to \((p, s) = (1 - \epsilon, 0)\) for a sufficiently small \( \epsilon > 0 \) is profitable, as in the limit this strategy yields \( (1 - \lambda)(1 - F^1(\alpha)) \). Hence, it must be that \( p_H^0 \in \{ \alpha, 1 \} \).

In the remaining part of the proof I distinguish two cases.

CASE 1: \( \hat{p} < 1 \).

By the arguments of step 3, \( F^0 \) is strictly increasing over the interval \([\hat{p}, 1)\). Therefore, \( p_H^0 = 1 \). This implies that the support of \( F^0 \) consists of the disjoint interval \([p_L^0, \alpha) \cup [\hat{p}, 1)\) and \( F^1 \) consists of two atoms located at \( \alpha \) and 1. Notice that

\[
\pi((p_L^0, 0), \sigma) = p_L^0 \left( \lambda + (1 - \lambda) \left( \frac{1}{2} + \frac{1 - F^1(\alpha)}{2} \right) \right)
\]

\[
\lim_{p \to -\alpha} \pi((p, 0), \sigma) = \alpha \left( (1 - F^0(\alpha)) + (1 - \lambda) \left( \frac{1}{2} + \frac{1 - F^1(\alpha)}{2} \right) \right)
\]

\[
\pi((\alpha, 1), \sigma) = \alpha \left( \lambda \left( \frac{1}{2} + \frac{1 - F^0(\alpha)}{2} \right) + (1 - \lambda) \right)
\]

\[
\pi((\hat{p}, 0), \sigma) = \hat{p} \left( (1 - F^0(\alpha)) + (1 - \lambda)(1 - F^1(\alpha)) \right)
\]

\[
\lim_{p \to -1} \pi((p, 0), \sigma) = (1 - \lambda)(1 - F^1(\alpha))
\]

\[
\pi((1, 1), \sigma) = \frac{(1 - \lambda)}{2}
\]

By equating the above equations, I find that \( \lambda, p_L^0, \hat{p}, F^0(\alpha), \) and \( F^1(\alpha) \) are defined as in the first case of proposition 1. Notice that \( p_H^0 > \hat{p} \) if and only if \( \alpha < \frac{2}{3} \). Once that these are obtained, I find that \( F^0 \) and \( F^1 \) are defined as in the first case of proposition 1. In the supplementary material (available from the author upon request) I show that there are no profitable deviations.

CASE 2: \( \hat{p} \geq 1 \).
Therefore, $p_0^L = \alpha$. Hence, the support of $F^0$ consists of the interval $[p_0^L, \alpha)$ and $F^1$ consists of two atoms located at $\alpha$ and 1. Notice that

\[
\pi((p_0^L, 0), \sigma) = p_0^L \left( \lambda + (1 - \lambda) \left( \frac{1}{2} + \frac{1 - F^1(\alpha)}{2} \right) \right)
\]

\[
\lim_{p \to \alpha^-} \pi(p, 0), \sigma = \alpha \left( (1 - \lambda) \left( \frac{1}{2} + \frac{1 - F^1(\alpha)}{2} \right) \right)
\]

\[
\pi((\alpha, 1), \sigma) = \alpha \left( \lambda \left( \frac{1}{2} \right) + \frac{(1 - \lambda)}{2} \right)
\]

\[
\pi((1, 1), \sigma) = \frac{(1 - \lambda)}{2}
\]

By equating the above equations, I find that $\lambda$, $p_0^L$, and $F^1(\alpha)$ are defined as in the second case of proposition 1. Once that these are obtained, I find that $F^0$ and $F^1$ are defined as in the second case of proposition 1. In the supplementary material (available from the author upon request) I show that there are no profitable deviations if and only if $\alpha \geq \frac{2}{3}$. ■

References


