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Abstract: To better understand the fractal characteristics of coal fracture network and find the relation between 2D and 3D fractal dimensions, we utilized the improved box counting method to calculate 2D and 3D fractal dimensions based on high resolution CT images of 4 coal samples (Ro from 2.915% to 4.69%). Based on the calculated 2D and 3D fractal dimension and porosity, the size of representative element volume (REV), the relationship between Df2 and Df3 and the relationship between porosity and fractal dimension are investigated extensively. It is noticed that the fractal dimension-based REV of coal is smaller than the porosity-based REV. As the complement of previous theoretical studies, it is proved that porosity has an exponential relationship with fractal dimension. By deducing formulas based on fractal theory, a new way to get the lower self-similar region size from relation between porosity and fractal dimension is provided. Evidently, the relation between 2D and 3D fractal dimension of coal could be expressed as Df3=CDf2+ φ, and the slope of the line, C, depends on the average 2D fractal dimension of the sample. Finally, the reference of the relation between Df2 and C of high rank coal is provided as C= -0.75Df2 +2.75.

Research Data Related to this Submission

There are no linked research data sets for this submission. The following reason is given:
Data will be made available on request
Imaged based fractal characterization of micro-fracture structure in coal

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Abstract

To better understand the fractal characteristics of coal fracture network and find the relation between 2D and 3D fractal dimensions, the improved box counting method was utilized to calculate 2D and 3D fractal dimensions based on high resolution CT images of 4 coal samples ($R_o$ from 2.915\% to 4.69\%). Based on the calculated 2D and 3D fractal dimension, the size of representative element volume (REV), the relationship between $D_{f2}$ and $D_{f3}$ and the relationship between porosity and fractal dimension were investigated extensively. As the complement of previous theoretical studies, the exponential relationship between porosity and fractal dimension was proved. By deducing formulas based on fractal theory, a novel way to get the lower self-similar region size from the relation between porosity and fractal dimension was provided. Evidently, the relation between 2D and 3D fractal dimension of coal could be expressed as $D_{f3} = CD_{f2} + 1$, and the slope of the line, C, depends on the average 2D fractal dimension of the sample.

Key words

Fractal dimension; Coal; Micro-CT images; Box-counting method, REV

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1. Introduction

The fractal theory was proposed initially by Mandelbrot [1] to explore the complexity of the natural world, and it was widely applied to characterize complexed pore structure. Avnir, et al. [2] reported that the surface of most materials, at molecular scale, are fractal; with the application of Scanning Electron Microscopy (SEM), Katz and Thompson [3] illustrated that the pore structure of sandstones is typical self-similar and thus could be characterized by fractal theory. Since then, fractal characterization of porous media have been studied extensively over the past three decades [4-9].

According to the fractal theory proposed by Mandelbrot, the pores size distribution of a self-similar pore structure satisfies the cumulative distribution function as given in Eq. (1).

\[ N(L) \propto L^{-D} \quad (1) \]

here \( N(L) \) is cumulative pore with size larger than \( L \), and \( D \) is the fractal dimension. In the porous media research community, the fractal dimension is normally determined by experimental measurements, such as mercury intrusion curve [10], Small-angle Neutron Scattering measurement [11,12] and Nuclear Magnetic Resonance measurements [13]. Besides these indirect measurements, fractal dimension of pore space and size normally estimated with the well-known box-counting method from two-dimension rock images [14,15]. However, fractal dimensions measured through different experiments are different. For example, fractal dimensions estimated from \( N_2 \) adsorption experiment represent the fractal dimension of surface and volumetric roughness, the fractal dimension measured through mercury intrusion is volumetric fractal dimension, while fractal dimension inferred from NMR data represents pore size distribution fractal. The fractal dimension calculated by box counting method represent the fractal dimension of pore volume/area size and distribution in space, therefore, the fractal dimension calculated through box counting method is a valuable parameter to study seepage properties of porous media [16-18].
The box counting method based fractal dimension of porous media could be improved with the development of advanced and high-resolution imaging techniques (e.g., micro X-ray computerized tomography (micro-CT), Focused Ion Beam Scanning Electron Microscopy (FIB-SEM)), as the 3D high resolution images makes it possible to characterize rock sample with high resolution [19-22]. Over the last few decades, several different versions of algorithms based on box counting method, such as box rotate method [23,24], box flex method [25] and 2 other versions documented by La Pointe [26], have been proposed to calculate the fractal dimension based on porous images. However, some estimated fractal dimension of 2D and 3D sample image are larger than 3 or 2, respectively, which is unreasonable [27]. According to Cai et al. [28], this is because the influence of the minimum scale/cell size. For the improvement of the algorithm, previously, researchers took different approaches to eliminate boundary effect, which is a key factor to reduce the accuracy of the box counting method algorithm [23-27], the most common way is to resize the initial image by adding blank pixels to make the side length of image be $2^n$, $n$ is different integers according to different initial image size, then the side lengths of boxes are set as $2^m$, where $1 \leq m < n$, as a result, all of the initial image data can be covered by small boxes, however, this method changes the original data set by adding additional data. Small boxes used in this work can cover the original data set perfectly without any data loss or data implement by utilizing common divisors of the length and width as box sizes to avoid boundary effect. While as appeared in the literature, there is very limited research about the determination of 3D volume fractal dimension based on 3D high resolution coal sample images. Especially, the relation between 3D fractal dimension and 2D fractal dimension, and the relation between fractal dimension and porosity have not been discussed detailly, which are very important for simplifying the fractal permeability models [16-18].

For the study of the relation between 2D and 3D fractal dimension, the common accepted practice of obtaining the volume fractal dimension in 3D is using $D_f^3 = D_f^2 + 1$ [1]. This is inferred from Euclidean dimension, in which, dimension of 3D object, $D_3$, is 3, while dimension of 2D slices that make up 3D object, $D_2$, is 2, then $D_3 = D_2 + 1$, however, this approach has not been proved effective for fractal networks. Some researchers deduced the relation mathematically as $D_f^3 = 3.5 - S_{D_f^2}$, where $S_{D_f^2}$ is the self-similar parameter of $D_f^2$ [29,30], however it is reported to be a
particular self-similar model and only for three particular projections on the Cartesian coordinate planes [31]. The common practice relation was found incomprehensive to describe the relation of coal samples, and the mathematic practice has already been reported to be inaccurate.

Recently, the influence factors on different types of fractal dimension have been studied extensively, some researchers utilized low temperature nitrogen gas adsorption curve and mercury intrusion curve to calculate fractal dimension of coal samples and analyze the influence factors on fractal dimension. For example, Yao et al. [5] found that surface fractal dimension of coal samples ($R_o$ from 1.47% to 4.21%) has positive correlation with coal rank, Yao et al. [19] reported that volumetric roughness fractal dimension of coal samples ($R_o$ from 0.79% to 4.24%) is affected by composition, such as ash, moisture, carbon, Fu et al. [32] reported the volumetric fractal dimension of low rank coals ($R_o < 0.7\%$) has positive correlation with moisture content. Moreover, the CT technique was also widely used to calculate the fractal dimension of the coal samples. For example, Liu and Nie [33] utilized box counting method to calculate fractal dimension, and paid attention to influenced volatile matter content particularly, and found fractal dimension has a U-shaped curve relationship with $V_{daf}$. Shi et al. [34] found that the 2D fractal dimension of coal ($R_o$ from 0.59% to 2.25%) has positive correlation with coal rank, Zhou et al. [35] used Sierpinski-like model to calculate the fractal dimension and the results showed that fractal dimension increases as the pressure on the coal sample increases. In our study, the relation between fractal dimension and porosity deduced theoretically and the results that calculated directly from samples were compared, the results show that these results fit well with each other. Then pore area/volume fractal dimension of high rank coal ($R_o$ from 2.915% to 4.69%) was found to be mainly influenced by porosity, the higher porosity, the bigger fractal dimension.

In this work, the box counting method was utilized to study the fractal dimension of micro-CT coal images, and then the relationships among 2D fractal dimension, 3D fractal dimension and porosity were discussed. In Section 2, the algorithm of the fractal dimension calculation and the procedure of image processing were introduced. Then in Section 3, the computation results were analyzed and verified, the relationships among 3D fractal dimension, 2D fractal dimension and
porosity were characterized, the influence factors were also discussed. Finally, the coal network was proved self-similar and the REV (representative elementary volume) was determined, furthermore, a novel method of finding the lower self-similar region size was introduced.

2. Materials and methods

2.1 Samples and coal analyses

The coal samples used in this study were collected from Duanshi mine, Sihe mine, Yongan mine and Houcun mine in Qinshui Basin, Shanxi Province, China. The sample identification numbers of them are D3-2, SH3-1, YA3-2 and H3-1, respectively. Fundamental properties, such as, maximum vitrinite reflectance in oil ($R_o$), maceral composition and proximate analysis were measured under the China National Standard GB/T 6948-2008 and GB/T 8899-2013; and the results are shown in Table 1. As shown in this table, the $R_o$ of these samples are 2.92%, 3.05%, 4.69% and 4.06% respectively, which indicates they are anthracite in general.

Table 1  vitrinite reflectance, maceral composition and proximate analysis of the coal samples.

<table>
<thead>
<tr>
<th>Sample NO.</th>
<th>$R_o$ (%)</th>
<th>Coal maceral composition (vol. %)</th>
<th>Proximate analysis (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Vitrinite</td>
<td>Inertinite</td>
</tr>
<tr>
<td>D3-2</td>
<td>2.92</td>
<td>66.10</td>
<td>0.20</td>
</tr>
<tr>
<td>SH3-1</td>
<td>3.05</td>
<td>90.70</td>
<td>1.80</td>
</tr>
<tr>
<td>YA3-2</td>
<td>4.69</td>
<td>76.20</td>
<td>19.00</td>
</tr>
<tr>
<td>H3-1</td>
<td>4.06</td>
<td>63.80</td>
<td>31.70</td>
</tr>
</tbody>
</table>

2.2 X-ray CT scanning

The computed tomography (CT) is a non-destructive technique that can provide quantitative detection of interior 3D structure of rocks, thus it has been used extensively for porous media research [36]. In this study, the sample was analyzed using the GE Phoenix X-ray Nanotom industrial CT instrument. The system consists of X-ray source system, detector system, mechanical turntable system, image processing system. The measured samples were cylindrical
with a diameter of 2 mm and a height of 5 mm. The four samples were scanned with a resolution of 1.1μm.

2.3 Image processing

The rock images obtained from micro-CT scanning, need to be processed with two main steps, first reduce the noise and then do binarization before they could be used as input data to calculate their properties.

2.3.1 Denoise

The raw micro-CT coal images normally contain noises due to the limitation of CT scanning equipment itself. The noises may gloss some essential feature in the image that could significantly affect the accuracy of the subsequent data analyses, and thus noise reduction is prerequisites before any further implementation. In this work, the Java-based open source image processing software, ImageJ, was applied to reduce the noise of the images, the median filter method was utilized, and the radius was set as 2 pixels. Fig.1 shows the procedure of reducing the noise for a slice of coal image. Fig.1(a) presents the slide of initial image, a highlighted yellow square area was selected to be shown as an example of denoising. The selected yellow area was displayed in Fig.1(b), and then it was segmented, where red denotes for the pores and fractures in the coal sample, as shown in this figure, amount of isolated noises were observed. Fig.1(c) shows the results after noise reduction, much less noises (mainly isolated points) were found compared with that in Fig.1(b).
Figure 1. The procedure of reducing the noise in a slice of coal sample image. (a) A slice of coal image; (b) a selected area of the original image before noise reduction, (c) a selected area of image after noise reduction.

2.3.2 Binarization

After the coal images treated with denoise, the gray images were segmented, this was accomplished by setting a threshold value first, and then the grayscale of the pixels which were bigger than the threshold were set as 255 or set up as 0 if the grayscale of the pixels were smaller than the threshold. This process is termed as binarization, which is essential before pore and/or fracture extraction from rock images [37,38].

Several methods have been used to determine the threshold for binarization, such as the bimodal method, porosity restriction method and the Digital Terrain Model (DTM). The grayscale histogram of the conventional reservoir rock, for example sandstone, images normally contain two peaks, one relates to the pores and the other relate to the rock skeleton, and thus the bimodal method is widely used for determining the threshold in conventional reservoir rock images [39,40]. While unlike the situation of conventional reservoir rock, the grayscale histogram of coal usually contains one peak, so the bimodal method is not suitable for coal image threshold estimation. Currently, the porosity is normally used as criteria to determine the threshold for coal images [41], and this is normally completed through an iterative process to select the threshold for matching the pre-measured core scale porosity, while this approach could be problematic when there is uncertainty of the pre-measured porosity, for example the coal sample may be deformed during measurement under high pressure [42].
In this work, the DTM method, widely applied in mapping field [43], was used to determine the threshold of coal images. This method was initially adopted by Taud, et al. [44] to estimate the threshold of the micro-CT rock image for porosity calculation. And recently, the DTM method was used successfully for determining the threshold value of coal sample [41]. The algorithm of DTM threshold segmentation method has been described in detail previously [44]. For any CT image, the relationship estimated porosity $\phi(x)$ to its corresponding gray value $x$ can be obtained [41]. The essence of calculating surface porosity of an image is to find the minimum of the function $\phi(x)$, then the threshold is said to be the grey value corresponds to the minimum of the function $\phi(x)$ [44]. For example, choose 4 images equidistantly from all the images of D3-2 and calculate their threshold values, the results are 40, 42, 40, 43, the average is 41, so the threshold value of D3-2 in this work is set as 41. Subsequently, the same method was used to determine the threshold of another 3 samples.

![Porosity curves obtained by DTM threshold segmentation method, the first column is](image-url)
images that are used to analyze, the second column images are the results, while the third column images are the enlarged results of the maximum value of the minimum on the curve of the second column images. x-coordinate is the gray value and the y-coordinate is porosity.

2.4 Fractal dimension calculation based on coal images

2.4.1 Box counting method (BCM)

The box-counting method is the one of the most well accepted method to determine the fractal dimension and it has been used extensively for determining pore surface/area in 2D rock images [14,15]. It contains three main steps: a. divide the binary image by boxes under different length r, b. compute the number N of the boxes whose number of pore pixels is bigger than 1, c. plot logN vs log(1/r) on and use the linear correlation to match the data, the slope of the correlated line denotes the fractal dimension. A flow chart is given in Fig.3 below to demonstrate the procedures of estimate fractal dimension in 2D rock images.

![Flow chart for fractal dimension calculation](chart.png)
Figure 3. The flow chart of estimating fractal dimension from rock images.

Assuming the binary CT images with the size of M×M pixels and the gray value of 0 indicate pores, which is also said to be black pixels. The square boxes (or cubes for 3D) with side length of \( r \) will be used to cover the binary images, \( r \) is chosen from divisors of the length and width of the image, then the original image will be divided into \( M/r \times M/r \) (or \( M/r \times M/r \times M/r \) for 3D) boxes. After that, counting the amounts of black pixels in each box, then calculate \( N(r) \), which is the number of the boxes (or cubes for 3D) whose number of pore pixels is bigger than 1, a set of \( (N(r), r) \) can be obtained by changing \( r \), the data pair \((\text{lg} r, \text{lg} N(r))\) can be fitted and the slop is the fractal dimension according to the fractal dimension law.

2.4.2 Validation

According to the definition of fractal dimension, the Hausdorff-Besicovitch dimension of Sierpinski Carpet (see Fig.4(a)) and Menger Sponge (see Fig.4(c)) are \( D = \frac{\log 8}{\log 3} = 1.8928 \) and \( D = \frac{\log 20}{\log 3} = 2.7268 \), respectively. As shown in Fig.4(b) and Fig.4(d), excellent linear correlation could be found between \( \log N \) vs \( \log (1/r) \), and the calculated fractal dimension for Sierpinski Carpet and Menger Sponge are 1.8928 and 2.7268, respectively, there are no deviations for when compared with the analytical result, which means our program is theoretically right.
Figure 4. The fractal dimension of Sierpinski Carpet and Menger Sponge. (a) and (c) are the images of Sierpinski carpet and Menger Sponge, respectively; (b) and (d) are fractal dimensions of Sierpinski carpet and Menger Sponge estimated from boxing account method, respectively.

2.4.3 Computation of fractal dimension in 2D and 3D coal images

The BCM, as introduced in section 2.4.1, is used to calculate the fractal dimension directly from the rock images. The size of the 3D analyzed images are $900^3$, $800^3$, $400^3$ and $400^3$ voxels for coal sample D3-2, SH3-1, H3-1 and YA3-2, respectively. To obtain more essential data, the computation domain of 3D images constructed by different cubes initialized from 9 positions (see Fig.5).

Fig.5 gives an example of the approach that is used to generate 3D cubes with different sizes from position B. There are 9 positions, includes eight corner points, A, B, C, D, F, H, I, and the center point, E. As shown in Fig.5, 3 cubes initialized from point B are presented with different color. In this work, we increase the size of the cube with 10 pixels at each coordinate. For sample, D3-2 (with size of $900^3$ voxels), it will generate 90 different cubes with size change from $10^3$ to $900^3$ voxels at each initial position, and thus 810 3D cubes could be constructed that initialized from the 9 points, while 9 among the 810 constructed 3D cubes are identical as the full cube with $900^3$ voxels in each direction, so eventually 802 different 3D cubes could be established. With the same procedure, 632, 352 and 352 3D cubes could be constructed for SH3-1, H3-1 and YA3-2,
respectively. For each of the 3D constructed image, the 3D fractal dimension was calculated using the 3D box counting method, and the relevant 2D fractal dimensions were presented by the averaged 2D fractal dimension estimated from the slices of the 3D image.

![3D Coal Image](image)

*Figure 5. An example of constructing 3D coal image with different size and initial position.*

The 9 initial position contains 8 corner points, A, B, C, D, F, G, H, I, and the center position, E. An example of 3 different 3D images that initialized from position B are highlighted with different color.

### 3. Results and discussion

#### 3.1 REV of coal images

With the box counting method, the averaged $D_{3D}$ of these 4 samples are presented in Fig.6, which shows the averaged $D_{3D}$ of 9 positions as a function of the cubic side length. As shown in this figure, $D_{3D}$ fluctuates at small computation domains and then remains constant if the computation domain size surplus a critical value, and this critical value is referred as REV determined by fractal dimension. As shown in Fig.6, side lengths of fractal dimension-based REV of D3-2, YA3-2, H3-1 and SH3-1 are 290, 120, 120 and 160 pixels, respectively, fractal dimensions of their REVs are 2.55, 2.48, 2.48 and 2.65, respectively. Fig.7 shows the relation...
between average porosity and computation domain size, the average porosities of these 4 samples become stable as the computation domain size larger than certain pixels.

*Figure 6. The relation between computation domain size and average $D_f$..*
238 Figure 7. The relation between computation domain size and average porosity.

239

240 3.2 Relationship between \( D_{f3} \) and \( D_{f2} \)

241 As addressed in the literatures, the common practice of obtaining the volume fractal
dimension in 3D is using \( D_{f3} = D_{f2} + 1 \) (see Fig.8). As shown in the Fig.8, \( D_{f2} \) and \( D_{f3} \) are 2 and 3
respectively when porosity is 1, but we argue that \( D_{f3} \) calculated from 3D image whose porosity is
not 100\%, could be not be simply correlated with average \( D_{f2} \) by \( D_{f3} = D_{f2} + 1 \) (see Fig.9), which
presents \( D_{f3} \) as a function of \( D_{f2} \) of these four coal samples and it is clear that \( D_{f3} \) is a linear
function of \( D_{f2} \), therefore, Eq. (2) is proposed to describe the relationship between \( D_{f3} \) and average
\( D_{f2} \).

\[
D_{f3} = CD_{f2} + 1 \quad (2)
\]

244 here \( C \) is a parameter characterizes the relation between \( D_{f3} \) and average \( D_{f2} \), when average \( D_{f2} \) is
246 2, \( C \) will be 1 (see Fig.8). \( C \) is close to 1 when average 2D fractal dimension \( D_{f2} \) larger than 1.45
247 (see Fig.9(a)), which means the relation between \( D_{f3} \) and average \( D_{f2} \) of high rank coal samples
can be expressed as $D_3 = D_2 + 1$, however, there will be deviation to characterize the relation using $D_3 = D_2 + 1$ when average 2D fractal dimension is smaller than 1.45 (see Fig. 9(b)), then Eq. (2) is better than the common practice to describe this relation. Evidently, the average $D_2$ of these 4 coal samples from 3 different directions are very close (see Table 2).

![Figure 8](image)

**Figure 8.** The fractal dimensions of 2D and 3D objects with porosity is 1.
Figure 9. The relationship between 2D fractal dimension and 3D fractal dimension. (a) and (b) are the results of average 2D fractal dimension bigger and smaller than 1.45, respectively.

Table 2

Average 2D fractal dimensions from different directions

<table>
<thead>
<tr>
<th></th>
<th>X</th>
<th>Y</th>
<th>Z</th>
</tr>
</thead>
<tbody>
<tr>
<td>SH3-1</td>
<td>1.72</td>
<td>1.72</td>
<td>1.70</td>
</tr>
<tr>
<td>D3-2</td>
<td>1.63</td>
<td>1.62</td>
<td>1.62</td>
</tr>
<tr>
<td>H3-1</td>
<td>1.57</td>
<td>1.58</td>
<td>1.58</td>
</tr>
<tr>
<td>YA3-2</td>
<td>1.56</td>
<td>1.58</td>
<td>1.57</td>
</tr>
</tbody>
</table>
3.3 Relationship between fractal dimension and porosity

Katz and Thompson [3] proposed a correlation between porosity and fractal dimension and it is given as

\[ \phi = a \left( \frac{r_{\text{max}}}{r_{\text{min}}} \right)^{3-D} \]  

(3)

where \( a \) is a constant of order one, \( \phi \) is porosity, \( D \) is fractal dimension, \( r_{\text{max}} \) and \( r_{\text{min}} \) are upper limit and lower limit self-similar region respectively. Yu and Li [45] modified Eq. (3) to allow for more generalized computation domain and it is given as below in Eq. (4),

\[ \phi = a \left( \frac{r_{\text{min}}}{r_{\text{max}}} \right)^{d-D} \]  

(4)

in Eq. (4), \( d \) denotes the Euclidean dimension of the computation domain, and \( d \) equals to 2 in two dimensional spaces, and it equals to 3 in three dimensional spaces.

Take the logarithm on both sides of Eq. (4), the correlation can be rewritten as

\[ \ln \phi = -D \ln \left( \frac{r_{\text{min}}}{r_{\text{max}}} \right) + \ln \left( a \left( \frac{r_{\text{min}}}{r_{\text{max}}} \right)^{d} \right) \]  

(5)

this equation could be simplified as:

\[ \ln \phi = eD + f \]  

(6)

where \( e \) and \( f \) are constants, and \( e \) equals to \( -\ln \left( \frac{r_{\text{min}}}{r_{\text{max}}} \right) \), \( f \) equals to \( \ln \left( a \left( \frac{r_{\text{min}}}{r_{\text{max}}} \right)^{d} \right) \), respectively.

The 2D cross section of the four coal sample images are used as input to calculate the \( D_2 \) and their relevant porosity. Fig.10 presents \( \ln \phi \) as function of \( D_2 \), as shown in this figure there is a linear correlation between \( \ln \phi \) and \( D_2 \), which satisfies the theoretical derivation as given in Eq. (6). Here we can calculate 2D \( r_{\text{min}} \) from \( e \) and \( f \), and the 2D \( r_{\text{min}} \) of D3-2, YA3-2, H3-1 and SH3-1 are 5, 3, 4 and 2, respectively.
The relationship between porosity and $D_f$ for the 4 coal samples are also investigated. As shown in Fig. 11, $\ln \phi$ is presented as function of $D_f$. Like the cases in 2D images and expressed theoretically, $\ln \phi$ could be expressed as a linearly function of $D_f$. We can calculate 3D $r_{\text{min}}$ from $e$ and $f$, then the 3D $r_{\text{min}}$ of D3-2, YA3-2, H3-1 and SH3-1 are 76, 98, 181 and 30, respectively. The 3D lower limit size of self-similar region is bigger than 2D lower limit size of self-similar region, and they are both smaller than the observed fractal dimension-based REV.

Here in the coal sample, the main contribution of porosity is the fracture/cleat system, and thus $r_{\text{max}}$ could be considered as the size of the image due to the natural extension of the main fracture/cleat identified in the coal samples.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure10.png}
\caption{The relation between $D_{f2}$ and porosity.}
\end{figure}
4. Summary and conclusion

In this work, we utilized the box counting method to estimate the fractal dimension of 3D objective, $D_{f3}$, and the average 2D fractal dimension, $D_{f2}$, directly from 3D coal samples that imaged with the advanced micro-CT imaging techniques. Based on the calculated 2D and 3D fractal dimension, the size of REV, the relationship between $D_{f2}$ and $D_{f3}$, and the relationship between porosity and fractal dimension are investigated extensively. Based on our work, the detailed conclusion could be summarized as following:

- There exists the fractal dimension-based REV of 3D coal image and the bigger original image size, the bigger REV. Fractal dimensions of REVs of D3-2, YA3-2, H3-1 and SH3-1 are 2.55, 2.48, 2.48 and 2.65, respectively, which are positively correlated with porosity.
• Not like the previous result, the fractal dimension in 3D and 2D coal image could be expressed as $D_{f3} = CD_{f2} + 1$.
  o The slope of line, $C$, increases with the increasement of average 2D fractal dimension, $D_{f2}$, which is around 1 as the $D_{f2}$ larger than 1.45 for high rank coal.
  o It was noticed that average $D_{f2}$ of high rank coal from different directions are very close.

• Consistent with the previous theoretical derivation, we prove that porosity has an exponential relationship with fractal dimension, and it is influenced by coal composition and porosity, the lower self-similar region size can be deduced from the equation of the relation between porosity and fractal dimension.

• The minimum self-similar region, $r_{min}$ in 3D coal samples is larger than that in 2D sample images.

Future work of this study would be extending the proposed approach to characterize other materials, such as sandstone, carbonate, shale, and low rank coal, using the fractal theory. And look at the relationship between permeability and fractal theory. The relation between average 2D fractal dimension and 3D fractal dimension in this work was obtained based on experiments, so future work about this is to deduce the theoretical relation based on mathematic and physic theories and to improve the experimental relation.

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