

## ELASTODYNAMIC CONTACT PROBLEM FOR TWO PENNY-SHAPED CRACKS

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**Summary** The contact interaction of the cracks' edges is considered for the case of two penny-shaped cracks located in the plane under normally incident harmonic tension-compression wave. The components of the stress-strain state are examined. The dependence of the mode I stress intensity factor on the wave number and mutual arrangement of cracks is studied. The solution is compared with the results obtained without allowance for the contact interaction.

Let a three-dimensional linearly elastic homogeneous isotropic space  $\mathbb{R}^3$  contain two penny-shaped cracks located in the coordinate plane  $x_1Ox_2$ . The surfaces of cracks are defined by

$$\Omega_1 = \{x_1^2 + x_2^2 \leq a^2, x_3 = 0\}, \quad \Omega_2 = \{(x_1 - c)^2 + x_2^2 \leq b^2, x_3 = 0\}, \quad a + b < c.$$

A time-harmonic tension-compression wave with frequency  $\omega = 2\pi/T$  and amplitude  $\Phi_0$  propagates normally to the cracks' surfaces. In the process of deformation the adjoining cracks' edges move (mutual displacements are exemplified by the displacement discontinuity vector  $[\mathbf{u}(\mathbf{x}, t)]$ ) and interact with formation of the time-dependent contact domains. The mentioned contact interaction implies the transformation of the stress-strain state and corresponding transformation of the stress intensity factors distribution.

The load vector on the cracks' edges has the form

$$\mathbf{p}(\mathbf{x}, t) = \mathbf{p}^*(\mathbf{x}, t) + \mathbf{q}(\mathbf{x}, t), \quad \mathbf{x} \in \Omega_1 \cup \Omega_2, \quad t \in [0; T],$$

where  $\mathbf{p}^*(\mathbf{x}, t)$  is the load caused by the incident wave,  $\mathbf{q}(\mathbf{x}, t)$  is the contact forces vector originating on the cracks' surfaces.

In the absence of the tangential forces and displacements it is sufficient to determine only normal components of all mentioned vectors in order to solve the problem under consideration. According to [1], the following unilateral Signorini constraints must hold for the normal components of the displacement discontinuity and contact forces vectors

$$[u_3(\mathbf{x}, t)] \geq 0, \quad q_3(\mathbf{x}, t) \geq 0, \quad [u_3(\mathbf{x}, t)]q_3(\mathbf{x}, t) = 0, \quad \mathbf{x} \in \Omega_1 \cup \Omega_2, \quad t \in [0; T]. \quad (1)$$

Expand the normal components of the load vector and the displacement discontinuity vector into a Fourier series

$$p_3(\mathbf{x}, t) = Re \left\{ \sum_{k=-\infty}^{+\infty} p_3^k(\mathbf{x}) e^{i\omega_k t} \right\}, \quad [u_3(\mathbf{x}, t)] = Re \left\{ \sum_{k=-\infty}^{+\infty} [u_3^k(\mathbf{x})] e^{i\omega_k t} \right\},$$

where  $\omega_k = 2\pi k/T$  and

$$p_3^k(\mathbf{x}) = \frac{\omega}{2\pi} \int_0^T p_3(\mathbf{x}, t) e^{-i\omega_k t} dt, \quad [u_3^k(\mathbf{x})] = \frac{\omega}{2\pi} \int_0^T [u_3(\mathbf{x}, t)] e^{-i\omega_k t} dt. \quad (2)$$

The Fourier coefficients (2) are related by the following boundary integral system [1]:

$$p_3^k(\mathbf{x}) = - \int_{\Omega_1 \cup \Omega_2} F_{33}(\mathbf{x}, \mathbf{y}, \omega_k) [u_3^k(\mathbf{y})] d\Omega, \quad \mathbf{x} \in \Omega_1 \cup \Omega_2, \quad k = \overline{-\infty, +\infty}. \quad (3)$$

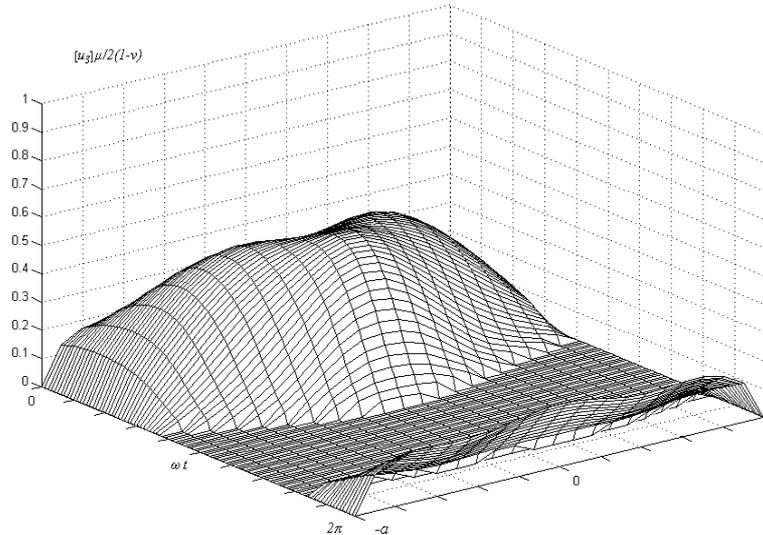
Due to the presence of nonintegrable terms in the integral kernel  $F_{33}(\mathbf{x}, \mathbf{y}, \omega_k)$ , whose rank exceeds the dimension of the integration region, the hypersingular integrals included in the boundary integral systems (3) should be treated only in the sense of the Hadamard finite part [1, 4].

The present paper contains a great deal of numerical results and figures corresponding the distribution of the components of the stress-strain state and the mode I stress intensity factor versus frequency of the incident wave and geometrical parameters of considered problem (mutual arrangement of the cracks and ratio between the radii  $a$  and  $b$ ).

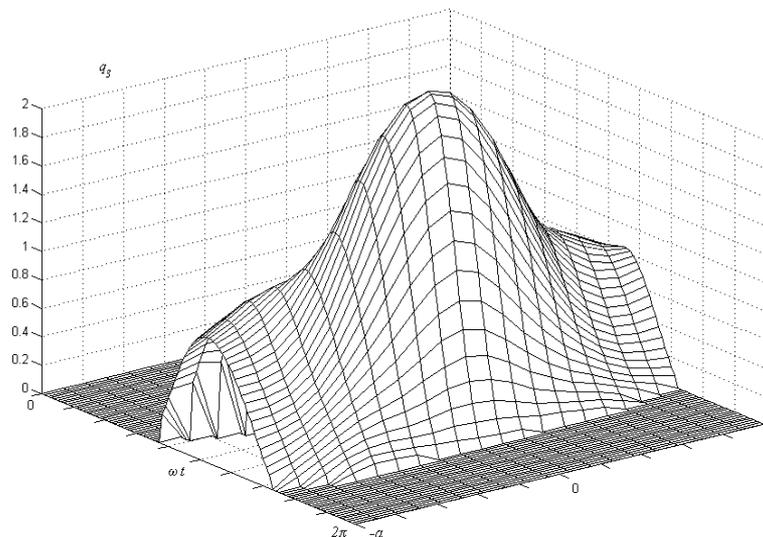
The results obtained with allowance for contact interaction are compared with those neglecting the contact interaction [2, 3]. It is important to observe that the cracks' edges contact interaction changes the solution of the present problem essentially, both quantitatively and qualitatively.

The submitted solution is not symmetric, but with growth of the distance between the cracks, the interference of the cracks considerably decreases and the solution tends to the axisymmetric solution obtained for unique penny-shaped crack [1, 4, 5].

Information on the distribution of the normal components of the displacement discontinuity and contact forces on the central section  $S = \{-a \leq x_1 \leq a, x_2 = 0, x_3 = 0\}$  of the crack  $\Omega_1$  versus time is presented in Figs. 1 and 2. The reduced wave number  $k_2 a = 3.0$ ;  $a/b = 1$ ; distance between cracks' centers  $c = 2.1a$ ; elastic modulus  $E = 200$  GPa, Poisson's ratio  $\nu = 0.25$ , density  $\rho = 7800 \text{ kg/m}^3$ .



**Figure 1.** Normal components of the displacement discontinuity



**Figure 2.** Normal components of the contact forces

The results presented here and in our previous publications indicate the necessity of the allowance for cracks' edges contact interaction in strength analyses of structures by methods of the fracture mechanics.

## References

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