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Abstract

This paper considers a new subsidy scheme for supporting the purchase of target products, in which the subsidy payment is inversely related to the product price. The scheme makes the demand faced by producers more elastic, thereby reducing their power to raise prices and increasing subsidy pass-through to consumers. Relative to the commonly-used specific or *ad valorem* subsidy, it induces larger sales with the same government budget (up to 50% more sales than the specific subsidy according to simulations based on the U.S. electric vehicle market) and allows the policymaker to flexibly adjust the incidence on producers.

Keywords: subsidy; efficiency; incidence; Bertrand competition; supermodular games

JEL Classification: D43, H21, H22, L13, Q58

1 Introduction

There are numerous subsidy programs across countries that aim to promote the use of target goods. These programs partially offset the purchase or production cost through financial incentives, such as grants, rebates, and tax credits/deductions, offered to their consumers or producers. Target goods may relate to, for example, "green" products (e.g., electric cars and solar PV panels), childcare and education (e.g., nursery), healthcare (e.g., treatment, insurance, and pharmaceuticals), and housing (e.g., purchase and rental). Typically, the subsidy payment per unit of a target good is either independent of or proportional to its price ("specific" or "*ad valorem*").[1] Sometimes it is given by a mixture of the two schemes (e.g., an *ad valorem* subsidy with a cap as in reference pricing of pharmaceuticals). This paper considers a

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[1] An income tax credit is in effect a specific subsidy of the amount of the credit, and an income tax deduction is an *ad valorem* subsidy at the marginal income tax rate.

new subsidy form to be used in such programs, which embodies a mechanism to reduce the producers' capacity to raise prices and thus significantly enhances the efficiency of these programs in increasing the sales of subsidized products: simulations based on an actual electric car subsidy in the U.S. indicate that switching from the current specific form to the proposed form, holding the total government spending on the program constant, would increase the market sales by up to 50%.

In many cases, the target good of a subsidy program is produced under imperfect competition, and/or associated with positive externalities or merit-good elements, so its supply without a subsidy falls short of the socially efficient level. Also, it is often desirable from a distributional perspective to help low- and middle-income households acquire such goods. For these reasons, government subsidy programs intend to lower the effective price that consumers pay out-of-pocket for the good, and thus encourage its usage.

Under imperfect competition, the design of a tax or subsidy policy (for example, specific or *ad valorem*) has welfare implications.[2] A number of previous studies use theoretical models of imperfect competition in a closed-economy context, and analyze the relative efficiency of different policy designs.[3] On taxes, Suites and Musgrave (1953), Delipalla and Keen (1992), Skeath and Trandel (1994), Anderson, de Palma and Kreider (2001*a*), and Hamilton (2009), to name a handful, compare specific with *ad valorem* taxes. Myles (1996), Hamilton (1999), and Carbonnier (2014) examine more general tax designs that contain specific and *ad valorem* taxes as special cases, and their frameworks relate to the subsidy policy proposed here in that the key channel is a policy-induced increase in the elasticity of demand faced by producers. On subsidies, there are a limited number of papers on the issue: Valido et al. (2014) and Liang, Wang and Chou (2017) contrast specific with *ad valorem* subsidies. This paper considers another form of subsidies and compare it with the commonly-used specific and *ad valorem* forms.

This paper has been motivated by a subsidy program in Japan that has a distinct feature not observed in usual specific or *ad valorem* subsidy schemes. In this Japanese subsidy (rebate) program on the purchase of residential solar photovoltaic (PV) systems, the rebate amount (per unit quantity) to a buyer is *decreasing* in the pre-rebate, unit price of the system, thereby giving sellers and buyers an incentive to trade at lower (pre-rebate) prices. Specifically, Table 1 shows that as the transaction price of a solar PV system per kW of capacity (inclusive of installation and other related costs) is lowered, the buyer (household) becomes eligible for a higher subsidy per kW. For example, in 2012 a household received no rebate if the (pre-rebate, per-kW) price of the purchased system was above ¥550,000; a rebate of ¥30,000 per kW if it was between ¥475,001–¥550,000; and ¥35,000 if it was equal to or below ¥475,000.[4]

[2] In contrast, under perfect competition where the firms are price-takers with no market power, the difference in the design of tax/subsidy policies is insignificant.

[3] There are papers that investigate the difference between the specific and *ad valorem* forms of import tariffs or export subsidies in an open-economy setting where domestic and foreign producers are treated differently (e.g., Brander and Spencer, 1984; Collie, 2006). This paper focuses on a closed-economy setting where producers are treated equally regardless of nationality, and thus export subsidies are out of its scope.

[4] A similar policy design was adopted by a U.K. subsidy scheme for electric and hybrid vehicles in 2016: a threshold at the vehicle price of £60,000.

Table 1: Residential Solar PV Installation Subsidy in Japan

Fiscal year	Subsidy (¥/kW)	Condition on pre-rebate price p_{pre} (¥/kW)
2009	0	if $700,000 < p_{\text{pre}}$
	70,000	if $p_{\text{pre}} \leq 700,000$
2010	0	if $650,000 < p_{\text{pre}}$
	70,000	if $p_{\text{pre}} \leq 650,000$
2011	0	if $600,000 < p_{\text{pre}}$
	48,000	if $p_{\text{pre}} \leq 600,000$
2012	0	if $550,000 < p_{\text{pre}}$
	30,000	if $475,000 < p_{\text{pre}} \leq 550,000$
	35,000	if $p_{\text{pre}} \leq 475,000$
2013	0	if $500,000 < p_{\text{pre}}$
	15,000	if $410,000 < p_{\text{pre}} \leq 500,000$
	20,000	if $p_{\text{pre}} \leq 410,000$

The table shows rebates (per kW of capacity) offered to households for installing residential solar PV systems. The amount of a rebate is conditional on the (pre-rebate) transaction price (¥/kW) of a system.
Source: Japan Photovoltaic Energy Association

The demand for a solar PV system jumps up as its price goes down below a threshold, incentivizing sellers to take advantage of this structure. Thus, the policymakers expected that the scheme would work as a mechanism to lower not only the consumer prices (i.e., post-rebate, out-of-pocket prices for households) but also the producer prices (i.e., pre-rebate, transaction prices received by sellers), leading to more than full pass-through of the subsidy to consumers and further accelerating the diffusion of solar PV systems that have positive externalities. Note that introducing a specific or *ad valorem* subsidy on a good normally reduces the consumer price and increases the producer price at the same time, resulting in less than full pass-through of the subsidy to consumers.

Transaction data suggest that the subsidy design indeed worked well in lowering (pre-rebate) prices. Figure 1 shows the (pre-rebate) price distribution of household solar PV systems that were installed during fiscal year 2012 (April 2012–March 2013). Solar PV system prices bunch in the bins just below the threshold prices (¥475,000 and ¥550,000), indicating that sellers have price-setting power, take account of the subsidy rule, and make transactions at lower prices than they would without such a scheme. As the threshold prices were reduced significantly year by year, the subsidy design, together with the declining production costs, helped lower the (pre-rebate) prices and further accelerate the diffusion of the technology until the national subsidy program was phased out in 2014 following a rapid expansion of residential solar PV.

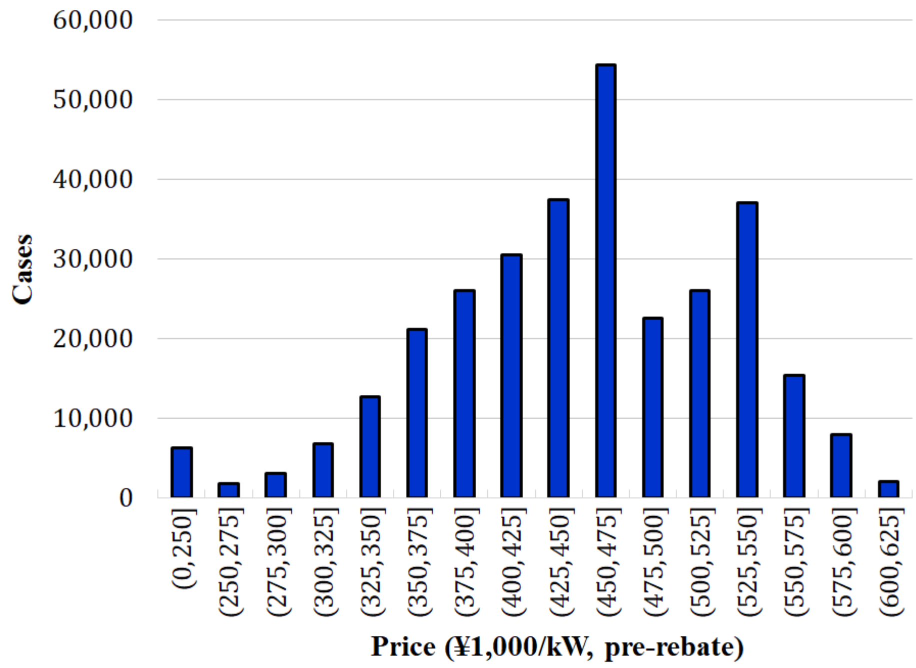

Figure 1: Distribution of Residential Solar PV System Prices (2012)
Source: RTS Cooperation and the Ministry of Economy, Trade and Industry

Despite the thought-provoking observation, no previous research exists, to my knowledge, that uses an economics framework to analyze the effect of making subsidy payment conditional on and inversely related to the price of a target product. This paper proposes and evaluates such a new subsidy scheme. More specifically, the subsidy considered is offered for the purchase or production of a good on the condition that its price is less than a government-set threshold, and as the price goes down, the subsidy per unit of the good increases in proportion to the difference between the threshold and price.[5] That is, the government sets two policy parameters: the threshold price level, and the rate at which subsidy payment increases with price reduction ($\overline{p}_i$ and r, respectively, in the model below). Based on a model of imperfect competition (Bertrand oligopoly with product differentiation) and the theory of supermodular games, I contrast this inversely price-related (IPR) subsidy with the benchmark case of no subsidy and widely-used specific and *ad valorem* subsidies in terms of various equilibrium characteristics (e.g., output, price, profits, and government expenditure).

From the government's perspective, the advantages of the proposed design consist in efficiency and flexibility. First, the IPR design is more efficient than the specific or *ad valorem* design in the sense that the former requires less government spending than the latter for inducing the same output level through subsidization. Equivalently, with a given budget, the IPR form can generate a higher output than the specific or *ad valorem* form. The relative efficiency of the IPR form follows because it makes the demand curve *faced by the producers* more elastic. In effect, the scheme partially compensates the producers for cutting prices, so a $1 reduction in the consumer price can be achieved by a smaller reduction in the producer

[5] With the subsidy schedule in Table 1, subsidy payment and consequently demand is discontinuous at the threshold prices. For tractability, the paper considers a subsidy design that is continuous in the price of the good.

price. This means that they face more elastic demand than under no subsidy, and under the specific or *ad valorem* scheme. Elastic demand erodes the producers' power to raise prices, thus making it easier for the policymaker to induce lower prices (higher outputs).

Second, the other side of the coin is that in inducing a given output level, the IPR scheme allows the policymaker to pursue an additional goal of adjusting the incidence on the producers. Given a target output to be induced, the policymaker can choose the two policy parameters to make a firm's equilibrium profits higher or lower than the case of no subsidy. In other words, besides targeting an output level, the policymaker can in effect choose whether the scheme works as subsidization or implicit taxation on the producers and to what extent, depending on the policymaker's objectives and market situations. For example, while increasing the supply and consumption of subsidized goods, the IPR form can also be used to financially support the producers of an emerging industry (such as electric carmakers) or instead to lower the economic rents due to imperfect competition. This flexibility does not hold for specific or *ad valorem* subsidies because they have just one policy parameter, and the policymaker's choice of it determines the equilibrium output and profits at once.

Simulations based on actual market data reveal substantial impacts of the IPR scheme. I construct a market using the data from the 2017 U.S. electric vehicle market where buyers were eligible for a specific subsidy of $7,500. This constructed market is then used to simulate the impact of replacing the original specific subsidy of $7,500 with an IPR subsidy in such a way that the market sales or the total government budget on the subsidy program remains constant. The results suggest that in inducing the same market sales, the IPR form can reduce the subsidy payment per unit by up to $4,600–$5,100 (61–68%). Put another way, the IPR form can induce up to 48–50% more sales with the same government budget ($382 million) as in the original specific form. As to the incidence on the producers, in spending the budget of $382 million, the government can adjust the parameters of the IPR scheme to flexibly vary producer surplus between $227–251 million higher and $10–16 million lower than in the case of no government intervention.

The IPR form shares with the widely-used *ad valorem* tax the issue of the disincentive for product quality improvement. In both cases, quality improvement is made more costly because an increase in the pre-subsidy/tax price due to higher quality reduces (increases) subsidy (tax) payment. I show that this disincentive under the IPR form can be corrected in a simple way: making the price threshold for subsidy eligibility increasing in quality, and thus rewarding a higher-quality product with a larger subsidy payment. In practice, this result implies that the IPR form works better when information is available about product attributes (e.g., energy-efficient durable goods and pharmaceuticals).

The rest of the paper is organized as follows. Section 2 considers Bertrand competition under various subsidy policies and derives Nash equilibrium(a). Section 3 analyzes the government's choice of policy variables, and compares equilibrium outcomes of different policies. Section 4 evaluates the impacts of the IPR scheme through simulations based on an actual U.S. subsidy program on electric vehicles. Section 5 extends the analysis by discussing how the IPR scheme can be adjusted to remove the disincentive for quality improvement. Section 6 concludes.

2 Theoretical Model

2.1 Subsidy Policies and Nash Equilibria

Consider a market with n (≥ 2) firms, where each firm i produces a differentiated product with a constant marginal cost $c_i (> 0)$. The demand for firm i's product, q_i, is given by $q_i = Q_i(p_i, \mathbf{p}_{-i})$, where $Q_i : \mathbb{R}_+ \times \mathbb{R}_+^{n-1} \to \mathbb{R}_+$ is a continuous function, p_i is the price of firm i's product, and $\mathbf{p}_{-i}$ is a vector of the prices of all the other $n-1$ firms' products.[6]

For each i, the demand function Q_i has the following properties. It is decreasing in p_i and strictly so where $Q_i > 0$. It is also increasing in p_j for any $j \neq i$ and strictly so where $Q_i > 0$ and $Q_j > 0$ (i.e., products are gross substitutes to one another). Also, I make the following common assumption in the literature of supermodular games, a powerful toolbox for analyzing strategic complementarity as featured in Bertrand models with product differentiation (e.g., Milgrom and Roberts, 1990; Vives, 2005; Amir, 2005): given $p_i \geq p_i'$ and $\mathbf{p}_{-i} \geq \mathbf{p}'_{-i}$ (i.e., $p_j \geq p_j'$ for all $j \neq i$),

$$Q_i(p_i, \mathbf{p}_{-i}) Q_i(p_i', \mathbf{p}'_{-i}) \geq Q_i(p_i, \mathbf{p}'_{-i}) Q_i(p_i', \mathbf{p}_{-i}), \tag{2}$$

which means that $\log Q_i$ displays increasing differences in p_i and $\mathbf{p}_{-i}$ (where $Q_i > 0$):

$$\log Q_i(p_i, \mathbf{p}_{-i}) - \log Q_i(p_i', \mathbf{p}_{-i}) \geq \log Q_i(p_i, \mathbf{p}'_{-i}) - \log Q_i(p_i', \mathbf{p}'_{-i}). \tag{3}$$

In the case that Q_i is twice continuously differentiable, (3) is equivalent to the condition that for each j ($\neq i$),

$$\frac{\partial^2 \log Q_i(p_i, \mathbf{p}_{-i})}{\partial p_i \partial p_j} = \frac{1}{Q_i(p_i, \mathbf{p}_{-i})^2} [Q_i(p_i, \mathbf{p}_{-i}) \frac{\partial^2 Q_i(p_i, \mathbf{p}_{-i})}{\partial p_i \partial p_j} - \frac{\partial Q_i(p_i, \mathbf{p}_{-i})}{\partial p_i} \frac{\partial Q_i(p_i, \mathbf{p}_{-i})}{\partial p_j}] \geq 0. \tag{4}$$

An intuitive and exact economic interpretation of (4) is that the own price-elasticity of demand ($-\frac{p_i}{Q_i}\frac{\partial Q_i}{\partial p_i}$) is decreasing in the price of another product (i.e., the demand for product i becomes less own-price elastic as p_j goes up). This condition is satisfied by a large class of demand functions, including, among others, linear, logit, CES, and translog demand functions (Milgrom and Roberts, 1990). Note that no assumption is made on the concavity or convexity

[6] The demand function $q_i = Q_i(p_i, \mathbf{p}_{-i})$ can be regarded as resulting from the following optimization problem of a representative consumer with quasi-linear utility $U(x, q_1, \cdots, q_n) = x + u(q_1, \cdots, q_n)$ (see Vives (1999, Ch. 3) for details):

$$\max_{x, q_1, \cdots, q_n} x + u(q_1, \cdots, q_n) \text{ s.t. } x + \sum_i p_i q_i \leq I, \tag{1}$$

where x is the numéraire good (the composite of all goods other than the n products) and I is income. An interior solution is characterized by $\frac{\partial u(q_1, \cdots, q_n)}{\partial q_i} = p_i \ \forall i$, so that the inverse demand function for product i is expressed as $p_i(q_1, \cdots, q_n) = \frac{\partial u(q_1, \cdots, q_n)}{\partial q_i} \ \forall i$. Inverting the system of inverse demand functions gives the demand function for each product i as $q_i = Q_i(p_1, \cdots, p_n)$. Lastly, with quali-linear utility, the assumption of a representative consumer is not restrictive.

of Q_i or $\log Q_i$.

[Policy A: No Subsidy]

We first look into the baseline case in which no subsidy is provided. Consider Bertrand competition by $n(\geq 2)$ firms: given $\mathbf{p}_{-i}$, firm i maximizes its profits

$$\pi_{iA}(p_i, \mathbf{p}_{-i}) = (p_i - c_i)Q_i(p_i, \mathbf{p}_{-i}) \tag{5}$$

by setting its price p_i in the interval $[c_i, p^{max}]$, where p^{max} is sufficiently large to choke the demand for any of the n products irrespective of the prices of the other $n-1$ products. Let ψ_{iA} be the correspondence from $\prod_{j\neq i}[c_j, p^{max}]$ to $[c_i, p^{max}]$ that gives firm i's best response(s) to $\mathbf{p}_{-i}$ under Policy A.

[Policy B: Specific Subsidy]

Suppose that the government offers consumers a specific subsidy of z per unit of the good purchased, where $z > 0$ and $z < c_i$ for all i. The subsidy is provided as a rebate or tax credit, for example. Importantly, it makes no difference whether the direct recipients of the subsidy are consumers or producers (physical neutrality; see, e.g., Weyl and Fabinger, 2013). The paper mainly works in terms of consumption subsidies (offered directly to consumers), but its results are valid for production subsidies (offered directly to producers) as well.

We interpret p_i as the consumer price, or the the effective price that consumers pay out of pocket after accounting for the subsidy. Demand depends on this effective price. The producer price p_i^p (i.e., the price received by a firm) equals $p_i + z$. Thus, firm i sets the price $p_i \in [c_i - z, p^{max}]$ to maximize its profits

$$\pi_{iB}(p_i, \mathbf{p}_{-i}) = (p_i + z - c_i)Q_i(p_i, \mathbf{p}_{-i}). \tag{6}$$

Let ψ_{iB} be the correspondence from $\prod_{j\neq i}[c_j - z, p^{max}]$ to $[c_i - z, p^{max}]$ that gives firm i's best response(s) to $\mathbf{p}_{-i}$ under Policy B.

[Policy C: *Ad Valorem* Subsidy]

Suppose that the government offers consumers an *ad valorem* subsidy of vp_i per unit of the good purchased ($v > 0$). As under Policy B, the consumer price is p_i, and the producer price is $(1+v)p_i$. Thus, firm i sets the price $p_i \in [\frac{c_i}{1+v}, p^{max}]$ to maximize its profits

$$\pi_{iC}(p_i, \mathbf{p}_{-i}) = [(1+v)p_i - c_i]Q_i(p_i, \mathbf{p}_{-i}). \tag{7}$$

Let ψ_{iC} be the correspondence from $\prod_{j\neq i}[\frac{c_j}{1+v}, p^{max}]$ to $[\frac{c_i}{1+v}, p^{max}]$ that gives firm i's best response(s) to $\mathbf{p}_{-i}$ under Policy C.

The theory of supermodular games is known to be useful for analyzing Bertrand competition (e.g., Milgrom and Roberts, 1990; Vives, 2005; Amir, 2005). (2) implies that for $X \in \{A, B, C\}$,

$\log \pi_{iX}$ satisfies increasing differences in p_i and $\mathbf{p}_{-i}$,[7] making each Bertrand competition a log-supermodular game (see, e.g., Milgrom and Roberts, 1990). Therefore, under each of Policies A–C, there exists at least one (pure-strategy) Nash equilibrium $\mathbf{p}^*$ such that $\mathbf{p}^* = \Psi_X(\mathbf{p}^*)$, where $\Psi_X(\mathbf{p}) \equiv \psi_{1X}(\mathbf{p}_{-1}) \times \cdots \times \psi_{nX}(\mathbf{p}_{-n})$ for $X \in \{A, B, C\}$. Moreover, in the case of multiple Nash equilibria, there exists a (pure-strategy) Nash equilibrium, where every firm's price is higher than its price at any other Nash equilibrium, that Pareto-dominates other Nash equilibria and thus is most plausible.

[Policy D: IPR Subsidy]

The government conditionally offers consumers a subsidy that is inversely related to the price of the target product. No subsidy is provided if the consumer price is greater than or equal to a government-set threshold $\overline{p}_i$ (i.e., if $p_i \geq \overline{p}_i$). If the price is below $\overline{p}_i$, the subsidy per unit of the good increases linearly as the price *decreases*. Specifically, if $p_i < \overline{p}_i$, a subsidy of $r[\overline{p}_i - p_i]$ is provided per unit of the good (i.e., the producer price $p_i^p = p_i + r[\overline{p}_i - p_i]$), where $0 < r < 1$ and $r\overline{p}_i < c_i < \overline{p}_i$ $\forall i$.[8,9] The threshold $\overline{p}_i$ may vary across i (Section 5 considers an extension where $\overline{p}_i$ depends on product i's quality).

Noting that profits equal zero when $p_i = \frac{c_i - r\overline{p}_i}{1-r} (< c_i)$, let $\pi_{iD_0} : \prod_j [\frac{c_j - r\overline{p}_j}{1-r}, p^{max}] \to \mathbb{R}_+$ be the function defined by

$$\pi_{iD_0}(p_i, \mathbf{p}_{-i}) = [p_i + r(\overline{p}_i - p_i) - c_i] Q_i(p_i, \mathbf{p}_{-i}). \tag{10}$$

Let ψ_{iD_0} be the correspondence from $\prod_{j \neq i} [\frac{c_j - r\overline{p}_j}{1-r}, p^{max}]$ to $[\frac{c_i - r\overline{p}_i}{1-r}, p^{max}]$ that gives the maximizer(s) of (10) with respect to $p_i \in [\frac{c_i - r\overline{p}_i}{1-r}, p^{max}]$, conditional on $\mathbf{p}_{-i}$. Define $\Psi_{D_0}(\mathbf{p}) \equiv \psi_{1D_0}(\mathbf{p}_{-1}) \times \cdots \times \psi_{nD_0}(\mathbf{p}_{-n})$. As under Policies A–C, the theory of supermodular games implies that there exists at least one fixed point $\mathbf{p}^*$ such that $\mathbf{p}^* = \Psi_{D_0}(\mathbf{p}^*)$.

[7] Given $p_i \geq p_i'$ and $\mathbf{p}_{-i} \geq \mathbf{p}_{-i}'$, it follows from (2) that

$$\begin{aligned} \pi_{iA}(p_i, \mathbf{p}_{-i}) \pi_{iA}(p_i', \mathbf{p}_{-i}') &= (p_i - c_i)(p_i' - c_i) Q_i(p_i, \mathbf{p}_{-i}) Q_i(p_i', \mathbf{p}_{-i}') \\ &\geq (p_i - c_i)(p_i' - c_i) Q_i(p_i, \mathbf{p}_{-i}') Q_i(p_i', \mathbf{p}_{-i}) \\ &= \pi_{iA}(p_i, \mathbf{p}_{-i}') \pi_{iA}(p_i', \mathbf{p}_{-i}). \end{aligned} \tag{8}$$

Thus, $\log \pi_{iA}$ satisfies increasing differences in p_i and $\mathbf{p}_{-i}$ (where $\pi_{iA} > 0$):

$$\log \pi_{iA}(p_i, \mathbf{p}_{-i}) - \log \pi_{iA}(p_i, \mathbf{p}_{-i}') \geq \log \pi_{iA}(p_i', \mathbf{p}_{-i}) - \log \pi_{iA}(p_i', \mathbf{p}_{-i}'). \tag{9}$$

It is shown analogously that $\log \pi_{iX}$ satisfies increasing differences in p_i and $\mathbf{p}_{-i}$ for each $X \in \{A, B, C, D_0\}$, where the case D_0 is to be defined next.

[8] These assumptions set the range on the generosity of the subsidy. The assumption $r < 1$ ensures $dp_i^p / dp_i = 1 - r > 0$, so that the subsidy is not so generous that the producer price can be raised by lowering the consumer price p_i. The assumption $c_i < \overline{p}_i$ means that the subsidy is generous enough to give positive profits for marginal cost pricing ($p_i = c_i$), while $r\overline{p}_i < c_i$ means that it is not so generous that even the price of zero does not result in losses.

[9] Subsidy payment $r[\overline{p}_i - p_i] (= p_i^p - p_i)$ is defined in terms of the consumer price p_i. Alternatively, it can be expressed with the producer price p_i^p as $r^p[\overline{p}_i - p_i^p]$, where the parameter r^p differs from r, while $\overline{p}_i$ is, by construction of the policy, identical to the one in the consumer price-based definition above. Equating the values from the two definitions gives $p_i^p - p_i = r[\overline{p}_i - p_i] = r^p[\overline{p}_i - p_i^p]$. Rearranging this, we obtain $r^p = r/(1-r)$. Since the function $g : (0, 1) \to (0, \infty)$ with $g(r) = r/(1-r)$ is bijective (one-to-one and onto), it does not matter whether the subsidy is defined in terms of the consumer or producer price.

Depending on the relative magnitude of p_i and $\overline{p}_i$, firm i's profits under Policy D, denoted by $\pi_{iD}(p_i, \mathbf{p}_{-i})$, equals either $\pi_{iD_0}(p_i, \mathbf{p}_{-i})$ or $\pi_{iA}(p_i, \mathbf{p}_{-i})$ (its profits under Policy A):

$$\pi_{iD}(p_i, \mathbf{p}_{-i}) = \begin{cases} \pi_{iA}(p_i, \mathbf{p}_{-i}) & \text{if} \quad p_i \geq \overline{p}_i, \\ \pi_{iD_0}(p_i, \mathbf{p}_{-i}) & \text{if} \quad p_i \leq \overline{p}_i, \end{cases} \tag{11}$$

where by definition $\pi_{iA}(p_i, \mathbf{p}_{-i}) = \pi_{iD_0}(p_i, \mathbf{p}_{-i})$ if $p_i = \overline{p}_i$, so $\pi_{iD}(p_i, \mathbf{p}_{-i})$ is continuous.

A few comments follow about the setup of the policy. Since $\pi_{D_0}(p_i, \mathbf{p}_{-i}) = [(1-r)p_i + r\overline{p}_i - c_i]Q_i(p_i, \mathbf{p}_{-i})$, Policy D is viewed as a combination of an *ad valorem* tax with the rate of r (> 0) and a specific subsidy of $r\overline{p}_i$, with a non-negativity constraint that subsidy payment equals $\max\{r(\overline{p}_i - p_i), 0\}$. The constraint reflects the subsidy eligibility condition and keeps the product, which is typically associated with social benefits such as positive externalities, from being taxed even if it is priced high. Viewed as a dual scheme with *ad valorem* and specific elements, Policy D is related with the model of Myles (1996) (or the more generalized model of Hamilton (1999)) that discusses such a scheme within the context of commodity taxation (i.e., for the case of $r(\overline{p}_i - p_i) < 0$ or $\overline{p}_i < p_i$) and under a homogeneous-product Cournot framework with identical firms (as opposed to this paper's differentiated-product Bertrand framework with heterogeneous firms).[10]

As this paper considers subsidization, the non-negativity constraint is essential: it does not make sense that the government imposes a special tax on, for example, solar PV systems when it aims to encourage their diffusion. Thus, each firm, which strategically interacts with other firms, individually selects its subsidy calculation rule ($r(\overline{p}_i - p_i)$ or 0) by setting its price lower or higher than $\overline{p}_i$. In other words, the policymaker in my model can only induce each firm to opt in to the scheme by making r and $\overline{p}_i$ sufficiently attractive for the firm (where the threshold values of r and $\overline{p}_i$ for firm i's opt-in/out are endogenously determined through market interaction among the firms). This setting can result in multiple Nash equilibria in which each firm chooses to opt in to or out of the scheme and earn positive profits. This is in contrast with the analysis on taxation and Ramsey pricing by Myles (1996), where the rule $r(\overline{p}_i - p_i)$ (< 0) is imposed on all firms, leading to a unique corner (limit) solution (with $r = 1$) that achieves Ramsey pricing by essentially enforcing marginal cost pricing and zero profits on the firms.

Setting the non-negativity constraint aside, note also that the dual scheme works fundamentally differently from an economic policy perspective depending on whether it calculates tax payment, $-r(\overline{p}_i - p_i)$ (> 0 for $p_i > \overline{p}_i$), or subsidy payment, $r(\overline{p}_i - p_i)$ (> 0 for $p_i < \overline{p}_i$). As a tax scheme, it exhibits the standard property shared by almost all tax or subsidy schemes that tax or subsidy payment is non-decreasing in the price p_i. As a subsidy scheme, on the other hand, it has a unique feature that subsidy payment is strictly decreasing in p_i, as in the motivating example of the solar PV subsidy described in Section 1.

We now proceed to a detailed analysis of the theoretical model with Policy D added. First,

[10] The product $r\overline{p}_i$ in this paper is treated in Myles (1996) and Hamilton (1999) as a single parameter representing the specific tax element.

based on Topkis's (1978) monotonicity theorem, the following lemma confirms the intuition that subsidy payment lowers the optimal (consumer) price set by firm i in response to $\mathbf{p}_{-i}$. In what follows, we focus on the non-trivial cases in which producing a positive quantity is firm i's best response (i.e., $Q_i(p^b_{iX}, \mathbf{p}_{-i}) > 0$, where $p^b_{iX} \in \psi_{iX}(\mathbf{p}_{-i})$ for $X \in \{A, B, C, D_0\}$).

Lemma 1. *Given* $\mathbf{p}_{-i} \in \prod_{j \neq i}[c_j, p^{max}]$, $p^b_{iA} \geq p^b_{iB}$, $p^b_{iA} \geq p^b_{iC}$, *and* $p^b_{iA} \geq p^b_{iD_0}$, *where* $p^b_{iA} \in \psi_{iA}(\mathbf{p}_{-i})$, $p^b_{iB} \in \psi_{iB}(\mathbf{p}_{-i})$, $p^b_{iC} \in \psi_{iC}(\mathbf{p}_{-i})$, *and* $p^b_{iD_0} \in \psi_{iD_0}(\mathbf{p}_{-i})$.

Proof. See the Appendix. ∎

Next, Lemma 2 below shows that the property of increasing differences holds under Policy D as in the previous cases.

Lemma 2. *The function* $\log \pi_{iD}$ *satisfies increasing differences in* p_i *and* $\mathbf{p}_{-i}$, *where* π_{iD} *is defined in* (11).

Proof. See the Appendix. ∎

As is the case with Policies A–C, Lemma 2 and the theory of supermodular games imply that there exists at least one (pure-strategy) Nash equilibrium under Policy D.

With the existence a Nash equilibrium ensured under Policy D, I now derive firm i's best response correspondence ψ_{iD} from $\prod_{j \neq i}[\frac{c_j - r\overline{p}_j}{1-r}, p^{max}]$ to $[\frac{c_i - r\overline{p}_i}{1-r}, p^{max}]$, and then consider an illustrative example of a duopoly with linear demand. With $\Psi_D(\mathbf{p}) \equiv \psi_{1D}(\mathbf{p}_{-1}) \times \cdots \times \psi_{nD}(\mathbf{p}_{-n})$, a Nash equilibrium $\mathbf{p}^*$ under Policy D is determined by the equation $\mathbf{p}^* = \Psi_D(\mathbf{p}^*)$. Given policy parameters r and $\overline{p}_i$, let $G_i(\mathbf{p}_{-i})$ be the difference between $\max_{p_i \in [c_i, p^{max}]} \pi_{iA}(p_i, \mathbf{p}_{-i})$ and $\max_{p_i \in [\frac{c_i - r\overline{p}_i}{1-r}, p^{max}]} \pi_{iD_0}(p_i, \mathbf{p}_{-i})$, where maximization is unconditional on the (in)eligibility conditions given in (11). That is, with $p^b_{iA} \in \psi_{iA}(\mathbf{p}_{-i})$ and $p^b_{iD_0} \in \psi_{iD_0}(\mathbf{p}_{-i})$,

$$\begin{aligned} G_i(\mathbf{p}_{-i}) &\equiv \pi_{iA}(p^b_{iA}, \mathbf{p}_{-i}) - \pi_{iD_0}(p^b_{iD_0}, \mathbf{p}_{-i}) \\ &= [p^b_{iA} - c_i] \cdot Q_i(p^b_{iA}, \mathbf{p}_{-i}) - [(1-r)p^b_{iD_0} + r\overline{p}_i - c_i] \cdot Q_i(p^b_{iD_0}, \mathbf{p}_{-i}). \end{aligned} \tag{12}$$

The following lemma states that a firm's choice of opting in or out is governed simply by the sign of $G_i(\mathbf{p}_{-i})$, or the relative benefits of opting in and out. This means that the (in)eligibility conditions in (11) can be ignored in deriving the optimal responses because they are implied by the sign on $G_i(\mathbf{p}_{-i})$ (as shown in Lemma 5 in the Appendix).

Lemma 3. *The maximized profits under Policy D are as follows: for* $p^b_{iA} \in \psi_{iA}(\mathbf{p}_{-i})$ *and* $p^b_{iD_0} \in \psi_{iD_0}(\mathbf{p}_{-i})$,

$$\max_{p_i \in [\frac{c_i - r\overline{p}_i}{1-r}, p^{max}]} \pi_{iD}(p_i, \mathbf{p}_{-i}) = \begin{cases} \pi_{iA}(p^b_{iA}, \mathbf{p}_{-i}) & \text{if } \; G_i(\mathbf{p}_{-i}) \geq 0, \\ \pi_{iD_0}(p^b_{iD_0}, \mathbf{p}_{-i}) & \text{if } \; G_i(\mathbf{p}_{-i}) \leq 0. \end{cases} \tag{13}$$

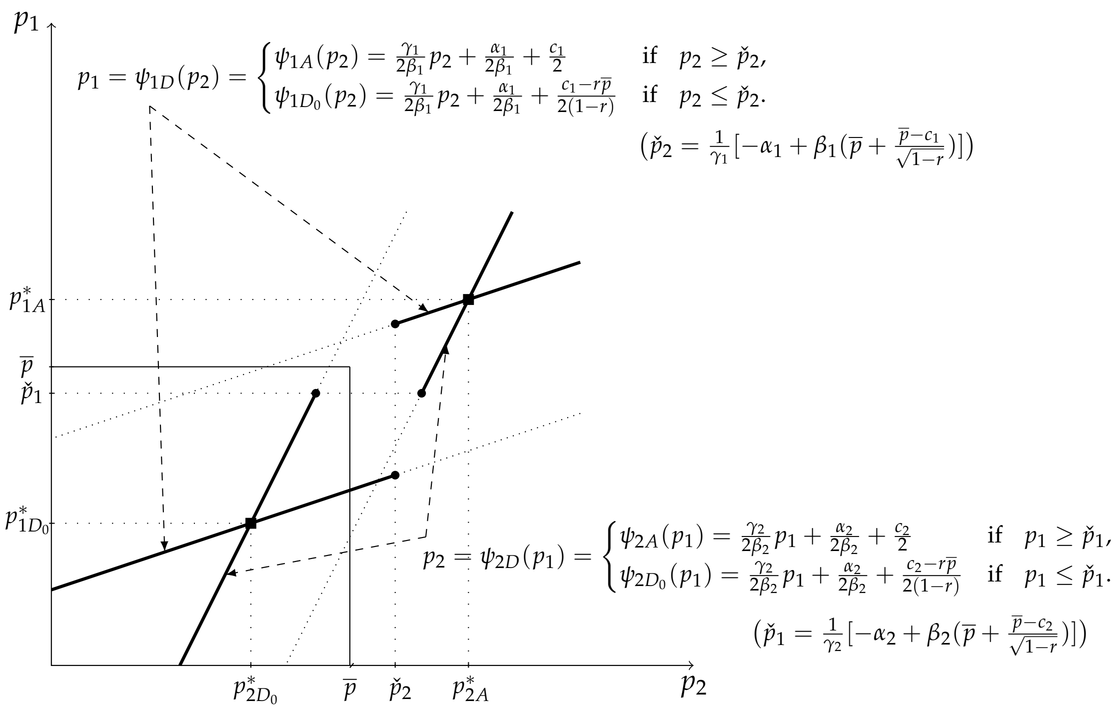

Figure 2: Duopoly under Linear Demand and Policy D

Equivalently, the best response correspondence under Policy D is

$$\psi_{iD}(\mathbf{p}_{-i}) = \begin{cases} \psi_{iA}(\mathbf{p}_{-i}) & \text{if} \quad G_i(\mathbf{p}_{-i}) > 0, \\ \psi_{iD_0}(\mathbf{p}_{-i}) & \text{if} \quad G_i(\mathbf{p}_{-i}) < 0, \\ \psi_{iA}(\mathbf{p}_{-i}) \cup \psi_{iD_0}(\mathbf{p}_{-i}) & \text{if} \quad G_i(\mathbf{p}_{-i}) = 0. \end{cases} \tag{14}$$

Proof. See the Appendix. ■

As an illustration, let us look at a duopoly case ($n = 2$) with linear demand functions:

$$\begin{aligned} q_1 = Q_1(p_1, p_2) = \alpha_1 - \beta_1 p_1 + \gamma_1 p_2, \\ q_2 = Q_2(p_2, p_1) = \alpha_2 - \beta_2 p_2 + \gamma_2 p_1, \end{aligned} \tag{15}$$

where α_i, β_i, and γ_i, $i = 1, 2$, are all positive. It is straightforward to confirm that this demand system satisfies the conditions stated at the beginning of Section 2.1, including the property of increasing differences. Given r and $\overline{p}_1 = \overline{p}_2 = \overline{p}$, Figure 2 depicts each firm's best response correspondence, ψ_{1D} or ψ_{2D}, as well as two Nash equilibria. In this case, $G_i(p_j) < 0$ ($= 0$, > 0) if $p_j < \check{p}_j$ ($= \check{p}_j$, $> \check{p}_j$, respectively), where $\check{p}_j = \frac{1}{\gamma_i}[-\alpha_i + \beta_i(\overline{p} + \frac{\overline{p}-c_i}{\sqrt{1-r}})]$. Thus, as the other firm's price p_j increases, firm i's best response ψ_{iD} jumps up at $p_j = \check{p}_j$ to switch from $p_i = \psi_{iD_0}(p_j) = \frac{\gamma_i}{2\beta_i}p_j + \frac{\alpha_i}{2\beta_i} + \frac{c_i - r\overline{p}}{2(1-r)}$ to $p_i = \psi_{iA}(p_j) = \frac{\gamma_i}{2\beta_i}p_j + \frac{\alpha_i}{2\beta_i} + \frac{c_i}{2}$. Intuitively, given strategic complementarity of Bertrand competition, when firm j sets a sufficiently high price, firm i should also set a high price ($> \overline{p}$) even if it means becoming ineligible for the subsidy. A more generous subsidy scheme (i.e., a larger r or $\overline{p}$) raises $\check{p}_j$, extending the range of p_j in which firm i responds with $\psi_{iD_0}(\cdot)$ (i.e., by opting in to the scheme).

The Nash equilibria under Policy D are where $p_1 = \psi_{1D}(p_2)$ and $p_2 = \psi_{2D}(p_1)$ intersect. In Figure 2, there are two Nash equilibria: the intersection of $p_1 = \psi_{1A}(p_2)$ and $p_2 = \psi_{2A}(p_1)$, (p^*_{1A}, p^*_{2A}), and the intersection of $p_1 = \psi_{1D_0}(p_2)$ and $p_2 = \psi_{2D_0}(p_1)$, $(p^*_{1D_0}, p^*_{2D_0})$. Depending on the values of demand, cost, and policy parameters, which determine where each firm's best response correspondence jumps, these two points may not lie on the best response (solid line) of at least one firm and cannot be a Nash equilibrium. Similarly, depending on the parameter values, there could exist a Nash equilibrium where one firm opts in and the other opts out (the intersection of $p_1 = \psi_{1A}(p_2)$ and $p_2 = \psi_{2D_0}(p_1)$ or that of $p_1 = \psi_{1D_0}(p_2)$ and $p_2 = \psi_{2A}(p_1)$). As discussed above, given a set of parameters, the theory of supermodular games ensures that at least one of these four points is an intersection of $p_1 = \psi_{1D}(p_2)$ and $p_2 = \psi_{2D}(p_1)$, and thus is a Nash equilibrium.

2.2 Nash Equilibria in Symmetric Games

To explore the paper's primary goal of comparing different subsidy schemes in the next section, I focus on the case of identical firms which have a common marginal cost c and face a symmetrically differentiated demand system (as in Weyl and Fabinger (2013) and Anderson, de Palma and Kreider (2001*b*)). The firms are also subject to a common eligibility threshold $\overline{p}$. This makes the game symmetric (i.e., unaffected by permutations of the firms), so the subscript i is dropped from the functions Q_i, π_{iX}, ψ_{iX} (for $X \in \{A, B, C, D_0, D\}$), and G_i. I analyze Nash equilibria under each subsidy policy. The inequality in (3) is now assumed to hold strictly when $p_i > p'_i$ and $\mathbf{p}_{-i} > \mathbf{p}'_{-i}$ (i.e., $p_j \geq p'_j$ for all $j \neq i$ and $\mathbf{p}_{-i} \neq \mathbf{p}'_{-i}$),[11] which implies that all Nash equilibria are symmetric (Vives, 1999, Ch. 2).[12] Thus, it suffices to look into firm i's best response when all the other firms set a common price p_0 and the resulting fixed points where $p_0 \in \psi_X(p_0, p_0, \cdots, p_0)$. For brevity of notation, let $\tilde{Q}(p_i, p_0) \equiv Q(p_i, p_0, \cdots, p_0)$ and analogously define $\tilde{\pi}_X(p_i, p_0)$, $\tilde{\psi}_X(p_0)$, and $\tilde{G}(p_0)$. As discussed in Section 2.1, for each $X \in \{A, B, C, D_0, D\}$, the set of Nash equilibria (fixed points) such that $p \in \tilde{\psi}_X(p)$ is non-empty, which is denoted by F_X (for example, $F_A \equiv \{p | p \in \tilde{\psi}_A(p)\}$). If there are multiple Nash equilibria in F_X, $p^*_X \equiv \max\{p | p \in F_X\}$ constitutes the strictly Pareto-best (in terms of π_X) Nash equilibrium (fixed point) in F_X (see, e.g., Milgrom and Roberts, 1990).[13]

The next lemma characterizes Nash equilibria under the IPR scheme (F_D) for the symmetric case:

Lemma 4. *$F_D = \{p | p \in F_A$ and $\tilde{G}(p) \geq 0\} \cup \{p | p \in F_{D_0}$ and $\tilde{G}(p) \leq 0\}$. Moreover, if there exist p_A and p_{D_0} such that $p_A \in F_D \cap F_A$ and $p_{D_0} \in F_D \cap F_{D_0}$, then $p_{D_0} \leq \overline{p} \leq p_A$ (so $p_{D_0} < p_A$ unless $p_A = p_{D_0} = \overline{p}$), and $\tilde{\pi}_D(p_{D_0}, p_{D_0}) \leq \tilde{\pi}_D(p_A, p_A)$ (with equality if and only if $p_A = p_{D_0} = \overline{p}$).*

[11] That is, $\log Q$ satisfies strictly increasing differences in p_i and $\mathbf{p}_{-i}$.

[12] It is proved by contradiction that $\Psi_X(\mathbf{p}) \equiv \psi_X(\mathbf{p}_{-1}) \times \cdots \times \psi_X(\mathbf{p}_{-n})$ (for $X \in \{A, B, C, D_0, D\}$) has no asymmetric fixed point under this setting. If one exists, there are (at least) two firms (denoted by 1 and 2) such that $p_1 \neq p_2$ at the fixed point. Without loss of generality, assume $p_1 < p_2$. Note that $p_1 \in \psi_X(p_2, p_3, \cdots, p_n)$ and $p_2 \in \psi_X(p_1, p_3, \cdots, p_n)$. Since (3) with a strict inequality implies that every selection of ψ_X is increasing in each argument (see, e.g., Vives, 2005, Lemma 1), it follows from $p_1 < p_2$, $p_1 \in \psi_X(p_2, p_3, \cdots, p_n)$, and $p_2 \in \psi_X(p_1, p_3, \cdots, p_n)$ that $p_1 \geq p_2$, which is a clear contradiction.

[13] Given two fixed points p and p' in F_X such that $p > p'$, $\tilde{\pi}_X(p, p) \geq \tilde{\pi}_X(p', p) > \tilde{\pi}_X(p', p')$, where the last inequality holds because (1) $p > p'$ and (2) Q_i and consequently π_{iX} are strictly increasing in p_j for any $j \neq i$ (from the assumptions stated at the beginning of Section 2.1 and above Lemma 1).

Proof. See the Appendix. ■

That is, F_D consists of a subset of F_A where opting out is the best response despite the possibility of receiving the IPR subsidy, and a subset of F_{D_0} where opting in is the best response to the IPR subsidy. If both opt-in and opt-out Nash equilibria exist,[14] the price and profits are always higher at an opt-out equilibrium than at an opt-in equilibrium, except for an unlikely case of $p_A = p_{D_0} = \overline{p}$.

3 Comparing the Outcomes of Different Policies

I have so far looked into firm behavior and market equilibria under exogenous subsidy policies. I now consider a government's choice of the structure and parameter(s) of its subsidy policy, where the subsidy aims to increase social and consumer surplus by lowering the (consumer) price of a good and thus stimulating its sales/consumption.

Policies A–D are compared in the following setting. I consider identical firms and a symmetrically differentiated demand system and focus on the (strictly) Pareto-best fixed points p_A^*, p_B^*, p_C^*, and $p_{D_0}^*$ (as discussed in Section 2.2), each of which is the the most plausible outcome within F_A, F_B, F_C, and F_{D_0}, respectively. Suppose that the government sets Policy B, C, or D (z, v, or r and $\overline{p}$) with a view to inducing each firm to reduce its (consumer) price from the no-subsidy level (p_A^*) to a common target level $\hat{p}$ (i.e., $\hat{p} = p_B^* = p_C^* = p_{D_0}^*$).[15] Equivalently, it aims to raise its output from the no-subsidy level, $q_A^* = \tilde{Q}(p_A^*, p_A^*) = Q(p_A^*, \cdots, p_A^*)$, to a common target level, $\tilde{Q}(\hat{p}, \hat{p}) = Q(\hat{p}, \cdots, \hat{p})$. This section compares the effectiveness of alternative subsidy forms in terms of the government spending needed to achieve the common target (see Anderson, de Palma and Kreider (2001*b*) for a similar approach to comparing specific and *ad valorem* taxes). The following argument assumes interior solutions for profit maximization and a twice continuously differentiable demand function. Define $\tilde{Q}_1(p_i, p_0) \equiv \frac{\partial \tilde{Q}(p_i, p_0)}{\partial p_i} = \frac{\partial Q(p_i, p_0, \cdots, p_0)}{\partial p_i}$. Notationally, the dependence of the functions $\tilde{\pi}_X$ (or π_X), where $X \in \{B, C, D_0, D\}$, and $\tilde{G}$ (or G) on policy parameters (z, v, or r and $\overline{p}$) has so far been suppressed, but it is made explicit in the following for clarity (e.g., $\tilde{\pi}_{D_0}(p_i, p_0; r, \overline{p})$ and $\tilde{G}(p_0; r, \overline{p})$).

[Policy B]

The first order condition (FOC) for maximizing $\tilde{\pi}_B(p_i, p_0; z) = (p_i + z - c)\tilde{Q}(p_i, p_0)$ that is satisfied at the symmetric Nash equilibrium $p_i = p = p_B^*$ is

$$\tilde{Q}(p_B^*, p_B^*) + (p_B^* + z - c)\tilde{Q}_1(p_B^*, p_B^*) = 0.^{16} \tag{16}$$

[14] As seen below, it is often the case that only one of these two types can be Nash equilibria.

[15] If no externalities are associated with the consumption/production of the good, social surplus (= consumer surplus + producer surplus − government expenditure) is maximized when $\hat{p} = c$ (social optimum). With positive externalities, which are often the very reason for subsidization but not considered explicitly in this paper, the socially optimal $\hat{p}$ becomes lower than c. Note that the following analysis is *not* about setting $\hat{p}$ optimally and thus it holds more generally for $\hat{p}$ such that $\hat{p} < p_A^*$.

[16] Additionally, the second order condition that $\frac{\tilde{Q}(p,p)\tilde{Q}_{11}(p,p)}{[\tilde{Q}_1(p,p)]^2} \leq 2$ needs to hold at $p = p_B^*$, which limits the

With the government target $\hat{p}$ $(< p_A^*)$ given, solving (16) for z and letting $p_B^* = \hat{p}$ shows that

$$z = -(\hat{p} - c) - \frac{\tilde{Q}(\hat{p}, \hat{p})}{\tilde{Q}_1(\hat{p}, \hat{p})} \equiv \sigma_B(\hat{p}) \tag{17}$$

induces $\hat{p}$ as the Pareto-best Nash equilibrium $p = p_B^*$ under Policy B.

[Policy C]

Similarly, the FOC for maximizing $\tilde{\pi}_C(p_i, p_0; v) = [(1+v)p_i - c]\tilde{Q}(p_i, p_0)$ that is satisfied at the symmetric Nash equilibrium $p_i = p = p_C^*$ is

$$(1+v)\tilde{Q}(p_C^*, p_C^*) + [(1+v)p_C^* - c]\tilde{Q}_1(p_C^*, p_C^*) = 0. \tag{18}$$

Thus, solving (18) for v and letting $p_C^* = \hat{p}$ shows that $v = \frac{c\tilde{Q}_1(\hat{p},\hat{p})}{\tilde{Q}(\hat{p},\hat{p}) + \hat{p}\tilde{Q}_1(\hat{p},\hat{p})} - 1$ induces $\hat{p}$ as the Pareto-best Nash equilibrium $p = p_C^*$ under Policy C, and the subsidy payment per unit ($v\hat{p}$) equals

$$\frac{\hat{p}c\tilde{Q}_1(\hat{p}, \hat{p})}{\tilde{Q}(\hat{p}, \hat{p}) + \hat{p}\tilde{Q}_1(\hat{p}, \hat{p})} - \hat{p} \equiv \sigma_C(\hat{p}). \tag{19}$$

[Policy D]

The FOC for maximizing $\tilde{\pi}_{D_0}(p_i, p_0; r, \overline{p}) = [(1-r)p_i + r\overline{p} - c]\tilde{Q}(p_i, p_0)$ that is satisfied at the symmetric Nash equilibrium $p_i = p = p_{D_0}^*$ is

$$(1-r)\tilde{Q}(p_{D_0}^*, p_{D_0}^*) + [(1-r)p_{D_0}^* + r\overline{p} - c]\tilde{Q}_1(p_{D_0}^*, p_{D_0}^*) = 0. \tag{20}$$

Thus, setting r and $\overline{p}$ in such a way to meet the FOC

$$(1-r)\tilde{Q}(\hat{p}, \hat{p}) + [(1-r)\hat{p} + r\overline{p} - c]\tilde{Q}_1(\hat{p}, \hat{p}) = 0 \tag{21}$$

realizes $\hat{p}$ as the Pareto-best fixed point $p_{D_0}^*$ within F_{D_0}. From (21), $\overline{p}$ is given as a function of r conditional on $\hat{p}$ (denoted by $\overline{p}(r; \hat{p})$), and $\frac{d\overline{p}(r;\hat{p})}{dr} < 0$.[17] Hereafter, $\overline{p}$ is eliminated by using (21), which means that when we consider below the effect of changing r conditional on $\hat{p}$, we also implicitly change $\overline{p}$ to satisfy (21), and we observe the total effect of the changes in both r and $\overline{p}$.

Below I discuss three propositions on Nash equilibria under this policy and how they compare to those under previous policies. Figure 3 then summarizes the main findings from these propositions.

First, we investigate the conditions on r (and $\overline{p}$ through (21)) under which $\hat{p} = p_{D_0}^* \in F_{D_0}$ is

convexity of the demand curve at the optimum and corresponds to the usual condition under Cournot competition that a firm's marginal revenue curve should not be upward sloping at an optimum. Analogous conditions should hold under Policies C and D to be discussed below.

[17] $\overline{p} = [c - \frac{\tilde{Q}(\hat{p},\hat{p})}{\tilde{Q}_1(\hat{p},\hat{p})} - \hat{p}]/r + \frac{\tilde{Q}(\hat{p},\hat{p})}{\tilde{Q}_1(\hat{p},\hat{p})} + \hat{p}$, and $\frac{d\overline{p}}{dr} = [\frac{\tilde{Q}(\hat{p},\hat{p})}{\tilde{Q}_1(\hat{p},\hat{p})} + \hat{p} - c]/r^2 < 0$, where the last inequality follows because (21) and the assumption $\overline{p} > c$ imply $\frac{\tilde{Q}(\hat{p},\hat{p})}{\tilde{Q}_1(\hat{p},\hat{p})} + \hat{p} = \frac{c - r\overline{p}}{1-r} < c$.

also in F_D (i.e., a Nash equilibrium under Policy D). Let r_2 be determined as follows: given the government target $\hat{p}$ and $p_j = p_A^*$ for all $j(\neq i)$, r_2 makes firm i indifferent between opting in and out, that is, between $\tilde{\psi}_{D_0}(p_A^*)$ and $p_A^* \in \tilde{\psi}_A(p_A^*)$. Mathematically, $\tilde{\pi}_A(p_A^*, p_A^*) = \tilde{\pi}_{D_0}(p_{D_0}^b(p_A^*), p_A^*; r_2, \overline{p}(r_2; \hat{p}))$, where $p_{D_0}^b(p_A^*) \in \tilde{\psi}_{D_0}(p_A^*)$,[18] or $\tilde{G}(p_A^*; r_2, \overline{p}(r_2; \hat{p})) = 0$. Similarly, given the government target $\hat{p}$ and $p_j = \hat{p}$ for all $j(\neq i)$, r_3 makes firm i indifferent between opting in and out, that is, between $\hat{p} \in \tilde{\psi}_{D_0}(\hat{p})$ and $\tilde{\psi}_A(\hat{p})$. Mathematically, $\tilde{\pi}_A(p_A^b(\hat{p}), \hat{p}) = \tilde{\pi}_{D_0}(\hat{p}, \hat{p}; r_3, \overline{p}(r_3; \hat{p}))$ or $\tilde{G}(\hat{p}; r_3, \overline{p}(r_3; \hat{p})) = 0$. The following proposition gives the range of r (and $\overline{p}$ by (21)) in which $p = \hat{p}$ or $p = p_A^*$ constitutes a Nash equilibrium under Policy D.

Proposition 1. *Given $\hat{p}$ ($< p_A^*$), $r_2 \leq r_3$. Moreover,*

1. *if $r < r_2$, then $\hat{p} \in F_D$ and $p_A \notin F_D$ for any $p_A \in F_A$ (i.e., $p_{D_0}^* = \hat{p}$ is realized as a Nash equilibrium under Policy D, but any $p_A \in F_A$, including p_A^*, is not);*
2. *if $r_2 \leq r \leq r_3$, then $\hat{p} \in F_D$ and $p_A^* \in F_D$.*
3. *if $r_3 < r$, then $\hat{p} \notin F_D$ and $p_A^* \in F_D$.*

Proof. See the Appendix. ■

If p_A^* and $p_{D_0}^*$ are respectively a unique fixed point of $\tilde{\psi}_A(\cdot)$ and $\tilde{\psi}_{D_0}(\cdot)$ (as is the case with linear, logit, and CES demand systems, for example), this proposition is simplified to the following more intuitive corollary.

Corollary 1. *Suppose that p_A^* and $p_{D_0}^* = \hat{p}$ are respectively a unique element of F_A and F_{D_0}. Then,*

1. *if $r < r_2$, Policy D induces a unique equilibrium $p = p_{D_0}^* = \hat{p}$;*
2. *if $r_2 \leq r \leq r_3$, Policy D induces two Nash equilibria $p = p_{D_0}^* = \hat{p}$ and $p = p_A^*$.*
3. *if $r_3 < r$, Policy D induces a unique equilibrium $p = p_A^*$.*

By Proposition 1, if $r \leq r_3$, $p = p_{D_0}^* = \hat{p}$ is a Nash equilibrium and the subsidy payment per unit (i.e., $r(\overline{p} - \hat{p})$ with (21) satisfied) in this equilibrium is

$$-(\hat{p} - c) - \frac{(1-r)\tilde{Q}(\hat{p}, \hat{p})}{\tilde{Q}_1(\hat{p}, \hat{p})} \equiv \sigma_{D_0}(r, \hat{p}). \tag{22}$$

Since increasing r (with $\hat{p}$ fixed) makes a more generous subsidy policy, it may sound counter-intuitive that (22) implies $\frac{\partial \sigma_{D_0}(r,\hat{p})}{\partial r} < 0$. Note that, in this section, increasing r simultaneously reduces $\overline{p}$ as discussed after (21), thus resulting in the negative partial derivative.

The next proposition shows the relative efficiency of the different subsidy schemes in achieving a given government target ($\hat{p}$).

[18] In the following, an element of $\tilde{\psi}_X(p_0)$ may be denoted by $p_X^b(p_0)$ to clarify that it is a best response to p_0.

Proposition 2. *Suppose Policies B, C, and D (with $r \leq r_3$) are all designed to attain $\hat{p}$ ($< p_A^*$) and $\tilde{Q}(\hat{p},\hat{p})$ in equilibrium. Then, as to the subsidy payment per unit under these policies, $\sigma_{D_0}(r,\hat{p}) < \sigma_B(\hat{p}) < \sigma_C(\hat{p})$. Specifically, the differences in subsidy payment (or, equivalently, in profits) per unit of a good are as follows:*

$$\begin{aligned} \sigma_B(\hat{p}) - \sigma_{D_0}(r,\hat{p}) &= \frac{-r\tilde{Q}(\hat{p},\hat{p})}{\tilde{Q}_1(\hat{p},\hat{p})} > 0, \\ \sigma_C(\hat{p}) - \sigma_B(\hat{p}) &= \Big(\frac{c\tilde{Q}_1(\hat{p},\hat{p})}{\tilde{Q}(\hat{p},\hat{p}) + \hat{p}\tilde{Q}_1(\hat{p},\hat{p})} - 1\Big)\Big(-\frac{\tilde{Q}(\hat{p},\hat{p})}{\tilde{Q}_1(\hat{p},\hat{p})}\Big) > 0. \end{aligned} \tag{23}$$

Proof. The results follow from (17), (19), and (22). ■

Note that the difference in per-unit (per-firm, aggregate) subsidy payment equals the difference in per-unit (per-firm, aggregate, respectively) profits, because the three policies result in the same price and quantity $(\hat{p}, \tilde{Q}(\hat{p},\hat{p}))$.

Proposition 2 shows that Policy D is the most efficient among the three subsidy schemes in the sense that it achieves a given target $\hat{p}$ and $\tilde{Q}(\hat{p},\hat{p})$ with the least government spending, and the difference increases with r. Additionally, in the typical case of less than full subsidy pass-through under the specific subsidy (i.e., $0 > d(\hat{p} + \sigma_B(\hat{p}))/d\hat{p} = d\frac{-\tilde{Q}(\hat{p},\hat{p})}{\tilde{Q}_1(\hat{p},\hat{p})}/d\hat{p}$, where the equality is due to (17)), the difference $\sigma_B(\hat{p}) - \sigma_{D_0}(r,\hat{p}) = r\frac{-\tilde{Q}(\hat{p},\hat{p})}{\tilde{Q}_1(\hat{p},\hat{p})}$ decreases with $\hat{p}$. That is, as the government aims at a larger reduction in the consumer price via subsidization, Policy D becomes even more efficient than Policy B. Policy D can increase the output less expensively because it makes the (inverse) demand curve *faced by each firm i* flatter (on the q_i-p_i plane) and thus increases the own-price elasticity of demand relative to Policies A–C. Price-elastic demand is a factor to incentivize a firm to lower the (producer) price in equilibrium, making it easier for the government to induce a higher output. Similarly, as pointed out by Liang, Wang and Chou (2017) with a homogeneous-Cournot model of quantity competition, Policy B is more efficient than Policy C. In contrast to Policy D, the *ad valorem* subsidy (Policy C) makes the demand faced by a firm less price-elastic, making it harder and more costly for the government to induce the firm to lower the price and increase the output.

Next, I compare a firm's profits at an opt-in Nash equilibrium $p = p_{D_0}^* = \hat{p}$ under Policy D and the Pareto-best Nash equilibrium $p = p_A^*$ under Policy A (and potentially also under Policy D). With the FOC under each case substituted, $\tilde{\pi}_{D_0}(\hat{p},\hat{p};r,\overline{p}(r;\hat{p})) = -\frac{(1-r)\tilde{Q}(\hat{p},\hat{p})^2}{\tilde{Q}_1(\hat{p},\hat{p})}$ and $\tilde{\pi}_A(p_A^*,p_A^*) = -\frac{\tilde{Q}(p_A^*,p_A^*)^2}{\tilde{Q}_1(p_A^*,p_A^*)}$. Thus,

$$\tilde{\pi}_A(p_A^*,p_A^*) \begin{cases} < \tilde{\pi}_{D_0}(\hat{p},\hat{p};r,\overline{p}(r;\hat{p})) & \text{if} \quad r < r_1, \\ = \tilde{\pi}_{D_0}(\hat{p},\hat{p};r,\overline{p}(r;\hat{p})) & \text{if} \quad r = r_1, \\ > \tilde{\pi}_{D_0}(\hat{p},\hat{p};r,\overline{p}(r;\hat{p})) & \text{if} \quad r > r_1, \end{cases} \tag{24}$$

where $r_1 \equiv 1 - \frac{\tilde{Q}(p_A^*,p_A^*)^2/\tilde{Q}_1(p_A^*,p_A^*)}{\tilde{Q}(\hat{p},\hat{p})^2/\tilde{Q}_1(\hat{p},\hat{p})}$. Using r_1, the next proposition gives a further analysis of Nash equilibria under Policy D.

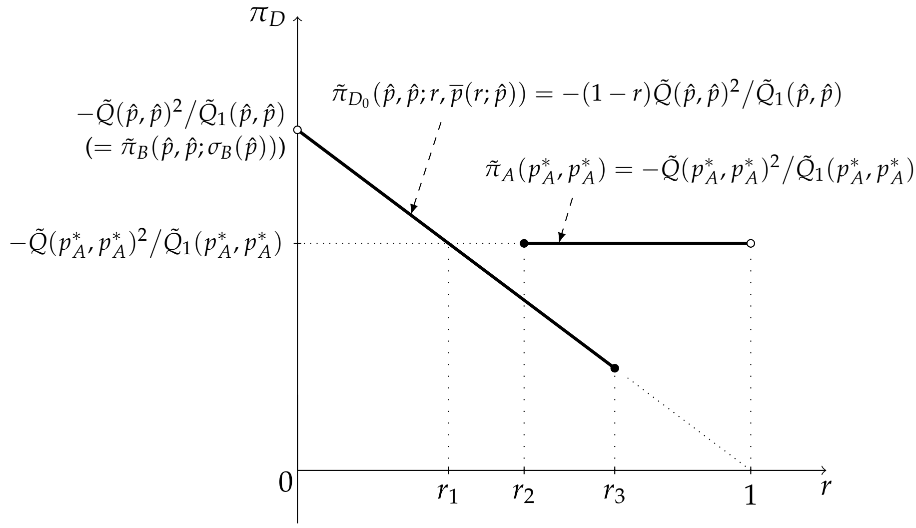

Figure 3: Equilibrium profits and r under Policy D, conditional on $\hat{p}$

Proposition 3. *Given $\hat{p}$ $(< p_A^*)$, $r_1 < r_2$ $(\leq r_3)$. Therefore, by setting r to satisfy $0 < r < r_1$, $r = r_1$, and $r_1 < r \leq r_3$ (and $\overline{p}$ by* (21)*), Policy D can make the profits at the opt-in Nash equilibrium with $p = p_{D_0}^* = \hat{p}$ (that is, $\tilde{\pi}_D(\hat{p},\hat{p};r,\overline{p}(r;\hat{p})) = \tilde{\pi}_{D_0}(\hat{p},\hat{p};r,\overline{p}(r;\hat{p}))$) higher than, equal to, and lower than, respectively, the profits at the Pareto-best Nash equilibrium under Policy A (that is, $\tilde{\pi}_A(p_A^*,p_A^*)$). In particular, when $r_1 < r < r_2$, $p = p_{D_0}^* = \hat{p}$ is the Pareto-best Nash equilibrium under Policy D and $\tilde{\pi}_D(\hat{p},\hat{p};r,\overline{p}(r;\hat{p})) < \tilde{\pi}_A(p_A^*,p_A^*)$.*

Proof. See the Appendix.

■

Figure 3 graphically summarizes the characteristics of the IPR subsidy as stated in Propositions 1–3. Conditional on $\hat{p}$, the figure plots a firm's profits at two potential Nash equilibria under Policy D ($p = p_A^*$ or $p = p_{D_0}^* = \hat{p}$) against r. Under Policy D, the equilibrium p_A^*, where every firm opts out of the IPR scheme by setting $p_i = p_A^*$ and earns $\tilde{\pi}_D(p_A^*,p_A^*;r,\overline{p}(r;\hat{p})) = \tilde{\pi}_A(p_A^*,p_A^*) = -\tilde{Q}(p_A^*,p_A^*)^2/\tilde{Q}_1(p_A^*,p_A^*)$, exists if and only if $r \in [r_2, 1)$. For $r \in (0, r_2)$, no opt-out Nash equilibrium exists (i.e., $p_A \notin F_D$ for any $p_A \in F_A$, including p_A^*). Any pair of policy variables $(r, \overline{p})$ with $r \in (0, r_3]$ and $\overline{p}$ determined by (21) can induce the equilibrium $p = p_{D_0}^* = \hat{p}$, where each firm opts in by setting $p_i = \hat{p}$ and earns $\tilde{\pi}_D(\hat{p},\hat{p};r,\overline{p}(r;\hat{p})) = \tilde{\pi}_{D_0}(\hat{p},\hat{p};r,\overline{p}(r;\hat{p})) = -(1-r)\tilde{Q}(\hat{p},\hat{p})^2/\tilde{Q}_1(\hat{p},\hat{p})$. The opt-in equilibrium $p = p_{D_0}^* = \hat{p}$ always offers lower profits (or, equivalently, is realized with less government spending) than the Policy B equilibrium $p = p_B^* = \hat{p}$, where each firm earns $\tilde{\pi}_B(\hat{p},\hat{p};\sigma_B(\hat{p})) = -\tilde{Q}(\hat{p},\hat{p})^2/\tilde{Q}_1(\hat{p},\hat{p})$. Moreover, for $r \in (0, r_1)$ ($r \in (r_1, r_3]$), the opt-in profits (with $p = p_{D_0}^* = \hat{p}$) are higher (lower, respectively) than $\tilde{\pi}_A(p_A^*,p_A^*)$. In particular, for $r \in (r_1, r_2)$, the firms would be better off if they could collude and jointly opt out of the IPR subsidy to set $p_i = p_A^*$ and earn $\tilde{\pi}_A(p_A^*,p_A^*)$ per firm, but this is not a Nash equilibrium and the prisoner's dilemma ends them up with opting in to earn lower equilibrium profits

$-(1-r)\tilde{Q}(\hat{p},\hat{p})^2/\tilde{Q}_1(\hat{p},\hat{p})$ per firm.[19] For $r \in [r_2, r_3]$, both opt-in ($p = p^*_{D_0} = \hat{p}$) and opt-out ($p = p^*_A$) are Nash equilibria under Policy D, but $p = p^*_A$ is more plausible since it gives each firm higher profits than and thus Pareto-dominates $p = p^*_{D_0} = \hat{p}$. Note that $(r, \pi_D) = (1, 0)$ in Figure 3 corresponds to the corner (limit) solution resulting from the dual tax scheme of Myles (1996), where the enforceability of the tax eliminates the above-mentioned opt-out equilibrium $p = p^*_A$ and leads to the Ramsey pricing outcome of transferring all the economic rents from the firms to the government through taxation.

It is clear from Figure 3 that the IPR subsidy can utilize the two policy variables to flexibly control its incidence on the producers, unlike the specific or *ad valorem* subsidy. In attaining the target outcomes ($\hat{p}$, $\tilde{Q}(\hat{p},\hat{p})$, and associated consumer or social surplus) with the IPR scheme, the policymaker can adjust the benefit or burden of the policy on the producers by changing r in the range of $(0, r_2)$ (and $\overline{p}$ by (21)). If $r \in (0, r_1)$, Policy D increases both consumer and producer surplus relative to the Policy A equilibrium $p = p^*_A$. Thus, it is subsidization for both the consumers and producers. On the other hand, for $r \in (r_1, r_2)$, although Policy D remains to offer the same benefits to the consumers, it works in effect as taxation on the producers because their (Pareto-best) equilibrium profits decline relative to the no subsidy case, and the differential is implicitly transferred to the government as a reduction in government spending for inducing $\hat{p}$. In contrast, there is no such flexibility with the specific or *ad valorem* form, because (17) and (19) mean that setting $\hat{p}$ will uniquely determine the (Pareto-best) equilibrium profits under each of Policies B and C.

With the flexibility provided by the IPR form, the incidence of the policy on producers can be adjusted in line with its objectives and market situations. For example, if a target good requires emerging, innovative technologies (e.g., electric vehicles), producers might have incurred significant fixed costs (e.g., R&D investment). Under these circumstances, the government may want to support the innovative producers by allowing them significantly higher profits than under no subsidy. On the other hand, if prior to government intervention the market for a target good is relatively mature and served by a small number of firms earning large economic rents due to imperfect competition, the government can use the IPR subsidy to induce a larger output and at the same time bring down the oligopolists' profits.

4 Simulating the Impact of the IPR Subsidy

4.1 Simulation Model

To illustrate the impact of the IPR structure in a more empirical setting, this section calibrates the above symmetrically differentiated Bertrand oligopoly model according to the actual data from the U.S. electric vehicle (EV) market, where buyers are eligible for a specific subsidy

[19] For the case of a monopoly ($n = 1$), the following changes should be made to Figure 3. The profit functions π_{D_0} and π_A remain essentially the same as in Figure 3: $\pi_A(p^*_A) = -Q(p^*_A)^2/Q_1(p^*_A)$ and $\pi_{D_0}(\hat{p}) = -(1-r)Q(\hat{p})^2/Q_1(\hat{p})$. The optimal price set by the monopolist, p, equals $\hat{p}$ when $r \in (0, r_1)$; $\hat{p}$ and p^*_A when $r = r_1$; and p^*_A when $r \in (r_1, 1)$. This is because Policy D cannot make the firm worse off than $\pi_A(p^*_A) = -Q(p^*_A)^2/Q_1(p^*_A)$ as it always has the option of disqualifying the product for the IPR subsidy and earns $-Q(p^*_A)^2/Q_1(p^*_A)$.

(tax credit). The calibrated model is then used to simulate what would happen if the specific subsidy form is replaced with other forms.

Following the argument in the previous sections, I particularly consider a hypothetical market with a symmetrically differentiated logit demand system:

$$Q(p_i, \mathbf{p}_{-i}) = \frac{\alpha \exp(\beta p_i)}{1 + \sum_{j=1}^{n} \alpha \exp(\beta p_j)} \quad \forall i, \tag{25}$$

where $\alpha > 0$ and $\beta < 0$, and an outside option is included that can be a substitute for the n goods. This demand system satisfies the conditions given at the beginning of Section 2 and is known to result in a unique, symmetric fixed point p_X^* such that $p_X^* = \tilde{\psi}(p_X^*)$ for each $X \in \{A, B, C, D_0\}$ (see, e.g., Anderson, de Palma and Thisse, 1992, Ch. 7), at which

$$\tilde{Q}(p_X^*, p_X^*) = \frac{\alpha \exp(\beta p_X^*)}{1 + n\alpha \exp(\beta p_X^*)}. \tag{26}$$

To make policy simulations with this demand system realistic, the parameters α, β, and n (as well as c) are set to reflect an actual market environment, as explained below.

I calibrate these parameters to the model year 2017 U.S. market data on small or midsize EVs with 4–5 seats.[20] First, the sales data identify the eight best-selling models in this market, which are produced by eight distinct manufacturers (BMW, Fiat, Ford, General Motors, Kia, Mercedes, Nissan, and Volkswagen) and account for almost all of the sales in this category. The aggregate sales of the eight models are 50,981 and the (sales-weighted) average price is \$34,160 (without options; i.e., low cost/quality) or \$38,799 (with options, if available; i.e., high cost/quality). As EV buyers were eligible for the specific subsidy (tax credit) of \$7,500 in 2017, the (sales-weighted) average consumer price is \$26,600 ($= \$34,160 - \$7,500$) or \$31,299 ($= \$38,799 - \$7,500$). Thus, the parameters of the logit model are set to result in a Nash equilibrium that reflects these observed values ($p_B^* = \$26,660$ or \$31,299 and $q_B^* \equiv \tilde{Q}(p_B^*, p_B^*) = 50,981/n$ with the specific subsidy of $z = \$7,500$). Regarding the outside option, a reasonable choice for the case of the above EV models would be all the other hybrid EVs and plug-in EVs, whose total sales is 500,096. In other words, the above eight models account for about 9% of the hybrid EV and plug-in EV market. As for the supply side, based on UBS Evidence Lab's (2017) engineering estimates of the production cost of Chevrolet Bolt (one of the eight models considered) with and without options, the following simulations set the marginal production cost $c =$ \$27,315 (for the low cost/quality case) or \$29,885 (for the high cost/quality case). Lastly, I set $n = 4.64$ in (26), which is the equivalent number of firms in accordance with the market shares of the eight models.[21] Using these values and (16) with $\hat{p} = p_B^*$ and $z = \$7,500$ pins down the demand parameters α and β in

[20] Vehicle sales and price data are constructed from the information available on the U.S. Department of Energy's websites (`www.fueleconomy.gov` and `afdc.energy.gov/data`) and EV-volumes.com (`www.ev-volumes.com`).

[21] That is, a market with 4.64 equal-sized firms gives the same Herfindahl-Hirschman Index as observed in the data. Using integer values ($n = 4$ or 5) gives very similar results (for example, at most a 0.4% difference in the equilibrium consumer price relative to $n = 4.64$). Non-integer values of n are used in calibrations by, e.g., Bushnell (2007).

(25).

Two sets of parameter values are used to check the robustness of the results against these parameters. One set corresponds to a case in which the production cost and product quality of the n goods are relatively low ($c = \$27,315$, $\alpha = 1.1688$, and $\beta = -0.1491 \times 10^{-3}$), and the other set relatively high ($c = \$29,885$, $\alpha = 0.7902$, and $\beta = -0.1145 \times 10^{-3}$).

4.2 Simulation Results

Based on the demand and supply parameters obtained above, simulations are carried out to derive and compare the Nash equilibria under various policies (Table 2). In simulation #1 of Table 2, the three subsidy schemes (Policies B–D) are all designed to result in the same equilibrium price and sales as targeted by the government ($\hat{p} = \$26,660$ for the low cost/quality case or \$31,299 for the high cost/quality case, and $n\hat{q} \equiv n\tilde{Q}(\hat{p}, \hat{p}) = 50,981$). By construction, the specific scheme (Policy B) with $z = \$7,500$ achieves the target price and sales. With no subsidy, $p_A^* = \$34,071$ and $q_A^* = 18,004$ (for the low cost/quality case) or $p_A^* = \$38,701$ and $q_A^* = 23,070$ (for the high cost/quality case). Thus, relative to the no-subsidy case (Policy A), the specific subsidy lowers the consumer price by about \$7,400 and raises the producer price about \$100 (i.e., subsidy pass-through is less than 100%), thus increasing the sales by 183% (121%; hereafter, the values for the high cost/quality case are shown in parentheses), consumer surplus by \$236 million (\$261 million), and the firms' aggregate profits (producer surplus) by \$227 million (\$251 million).[22] The *ad valorem* subsidy (Policy C) with $v = \frac{z}{c-z} = 0.38$ (0.34) induces the same $(\hat{p}, \hat{q})$ and consumer surplus as Policy B. As indicated by Proposition 2, compared to the above specific subsidy, it needs \$2,591 (\$2,987) or 35% (40%) extra subsidy payment per unit, adding to the producer price by the same amount (\$2,591 (\$2,987)) and producer surplus by \$132 million (\$152 million).

Regarding the IPR subsidy (Policy D), simulation #1 shows the outcomes when r equals r_1 or r_2 (as defined in Section 3) and induces $(\hat{p}, \hat{q})$ at the Nash equilibrium $p = p_{D_0}^* = \hat{p}$. Note that r_1 and r_2 are selected just because they are thresholds, but any $r \in (0, r_2)$ can induce $(\hat{p}, \hat{q})$ as the Pareto-best (indeed, unique in the logit case here) Nash equilibrium under Policy D, with a smaller r leading to larger equilibrium profits and government spending (see Figure 3). By construction, $r = r_1$ results in the same profits as Policy A. I examine r_2 rather than r_3 because for $r \in [r_2, r_3]$, the equilibrium $p = p_{D_0}^* = \hat{p}$ is implausible as it is Pareto-dominated (from the firms' perspective) by the opt-out equilibrium $p = p_A^*$. The value of 0.65 (for the low cost/quality case in simulation #1), for example, indicates that if the subsidy eligibility is met ($p < \overline{p}$), the policy (implicitly) compensates the producer with \$0.65 for every \$1 reduction in the consumer price p.

The result associated with r_1 is interpreted as follows. To induce all the firms to opt in to the subsidy while assuring equilibrium profits at least as large as under Policy A, it must be the case that $r \leq r_1$ and the subsidy payment per vehicle is not less than $r_1(\overline{p}_1 - \hat{p})$, where

[22] For the calculation of consumer surplus in a logit framework, see Anderson, de Palma and Thisse (1992, Ch. 7).

Table 2: Simulation Results

	Simulation #1		Simulation #2	
Cost and quality:	Low	High	Low	High
Policy A (no subsidy)				
p^*_A (consumer price = producer price; $)	34,071	38,701		
nq^*_A (market sales)	18,004	23,070		
Policy B (specific)				
p^*_B (consumer price; $)	26,660 [1]	31,299 [2]		
nq^*_B (market sales)	50,981 [3]	50,981 [3]		
z ($= \sigma_B$; subsidy/unit; $)	7,500	7,500	Same as in	
$p^*_B + \sigma_B$ (producer price; $)	34,160	38,799	simulation #1	
ΔConsumer surplus (vs. Pol. A; mil. $)	236 [4]	261 [5]		
ΔProducer surplus (vs. Pol. A; mil. $)	227	251		
Government outlay ($\sigma_B \times nq^*_B$; mil. $)	382 [6]	382 [6]		
ΔSocial surplus (vs. Pol. A; mil. $)	81 [7]	130 [8]		
Policy C (*ad valorem*)				
p^*_C (consumer price; $)	26,660 [1]	31,299 [2]	27,665	32,558
nq^*_C (market sales)	50,981 [3]	50,981 [3]	44,462	44,694
v	0.38	0.34	0.31	0.26
$v\hat{p}$ ($= \sigma_C$; subsidy/unit; $)	10,091	10,487	8,600	8,555
$p^*_C + \sigma_C$ (producer price; $)	36,751	41,786	36,265	41,113
ΔConsumer surplus (vs. Pol. A; mil. $)	236 [4]	261 [5]	188	201
ΔProducer surplus (vs. Pol. A; mil. $)	359	403	276	298
Government outlay ($\sigma_C \times nq^*_C$; mil. $)	514	535	382 [6]	382 [6]
ΔSocial surplus (vs. Pol. A; mil. $)	81 [7]	130 [8]	82	117
Policy D (IPR) with $r = r_1$				
$p^*_{D_0}$ (consumer price; $)	26,660 [1]	31,299 [2]	23,808	27,498
$nq^*_{D_0}$ (market sales)	50,981 [3]	50,981 [3]	74,345	74,987
r_1	0.65	0.55	0.76	0.70
$\overline{p}_1$ ($)	31,328	35,960	30,547	34,796
$r_1(\overline{p}_1 - \hat{p})$ ($= \sigma_{D_0}$; subsidy/unit; $)	3,041	2,575	5,143	5,099
$p^*_{D_0} + \sigma_{D_0}$ (producer price; $)	29,701	33,874	28,951	32,597
ΔConsumer surplus (vs. Pol. A; mil. $)	236 [4]	261 [5]	413	498
ΔProducer surplus (vs. Pol. A; mil. $)	0 [9]	0 [9]	0 [9]	0 [9]
Government outlay ($\sigma_{D_0} \times nq^*_{D_0}$; mil. $)	155	131	382 [6]	382 [6]
ΔSocial surplus (vs. Pol. A; mil. $)	81 [7]	130 [8]	31	116
Policy D (IPR) with $r = r_2$				
$p^*_{D_0}$ (consumer price; $)	26,660 [1]	31,299 [2]	23,714	27,327
$nq^*_{D_0}$ (market sales)	50,981 [3]	50,981 [3]	75,251	76,263
r_2	0.67	0.57	0.79	0.73
$\overline{p}_2$ ($)	31,037	35,519	30,180	34,221
$r_2(\overline{p}_2 - \hat{p})$ ($= \sigma_{D_0}$; subsidy/unit; $)	2,925	2,410	5,081	5,014
$p^*_{D_0} + \sigma_{D_0}$ (producer price; $)	29,585	33,709	28,795	32,341
ΔConsumer surplus (vs. Pol. A; mil. $)	236 [4]	261 [5]	420	511
ΔProducer surplus (vs. Pol. A; mil. $)	-6	-8	-10	-16
Government outlay ($\sigma_{D_0} \times nq^*_{D_0}$; mil. $)	149	123	382 [6]	382 [6]
ΔSocial surplus (vs. Pol. A; mil. $)	81 [7]	130 [8]	27	113

The entries with a common superscript ([1], [2], $\cdots$, or [9]) are equal by construction.

$\overline{p}_1 \equiv \overline{p}(r_1; \hat{p})$ is as defined in footnote 17. In simulation #1, r_1 is 0.65 (0.55) and $r_1(\overline{p}_1 - \hat{p})$ is \$3,041 (\$2,575). Put differently, under the constraint that each firm is not worse off than at the no-subsidy equilibrium, switching from Policy B to Policy D can cut down the government spending by up to 59% (66%) for attaining the same policy target. The producer price at $r = r_1$ is lower than p_A^* by \$4,370 (\$4,827) in spite of the subsidy ("over-shifting" or over 100% subsidy pass-through).

The result for the case of $r = r_2$ can be interpreted similarly. To induce all the firms to opt in to the subsidy (with equilibrium profits possibly lower than under Policy A), it must be the case that $r < r_2$ and the subsidy payment per vehicle is greater than $r_2(\overline{p}_2 - \hat{p})$. In simulation #1, r_2 is 0.67 (0.57) and $r_2(\overline{p}_2 - \hat{p})$ is \$2,925 (\$2,410). This subsidy payment is slightly below the payment when $r = r_1$, because equilibrium profits can now be lower than at the no-subsidy equilibrium (as is the case for $r \in (r_1, r_2)$). In other words, by switching from Policy B to Policy D in such a way to keep the firms from opting out and still meet the same policy target, the government can cut down the subsidy budget by up to 61% (68%) relative to Policy B. As expected, over-shifting occurs and producer surplus is somewhat lower than under Policy A by \$6 million (\$8 million).

Simulation #2 compares the subsidy schemes from a different but related perspective. Instead of obtaining the subsidy payment under each scheme that is necessary to achieve the target sales level $n\hat{q} = 50,981$ (as simulation #1 did), simulation #2 computes the sales level under each scheme that is realized with the fixed subsidy budget of \$382 million, which is, by construction, what Policy B with $z = \$7,500$ needs to induce $n\hat{q} = 50,981$ (\$382 million $= z \times n\hat{q}$). As Policy C is less efficient than Policy B (Proposition 2), Policy C attains 13% (12%) less sales with this budget than Policy B. On the other hand, Policy D is more efficient than Policy B (Proposition 2). Thus, according to the simulation for $r = r_1 = 0.76$ (0.70), subject to the condition that Policy D results in a unique equilibrium where every firm opts in and makes at least the same profits as in the Policy A equilibrium, Policy D induces up to 46% (47%) larger sales with the given budget than Policy B (74,345 (74,987) vs. 50,981). This means that under Policy D with $r = r_1$, the consumer price, \$23,808 (\$27,498), and subsidy payment per unit, \$5,143 (\$5,099), are lower than under Policy B (\$26,660 (\$31,299) and \$7,500, respectively). Consumer surplus, as a result, increases by \$177 million (\$237 million) relative to the specific subsidy. Lastly, according to the simulation for $r = r_2$, given a weaker constraint that the opt-in equilibrium is a unique equilibrium, Policy D with the given budget and with r_2 marginally less than 0.79 (0.73) can extend the equilibrium sales some more: 48% (50%) larger than Policy B. Thus, the consumer price and subsidy payment per unit are even lower and consumer surplus even higher than in the previous case of $r = r_1 = 0.76$ (0.70). Producer surplus decreases by \$10 million (\$16 million) relative to Policy A and Policy D with $r = r_1$.

Though possibly looking counter-intuitive at first sight, it is not surprising that social surplus in simulation #2 is larger under Policies B and C than under Policy D. Note that social surplus in Table 2 equals consumer surplus + producer surplus − government outlay, so externalities are not explicitly considered (see footnote 15). Social surplus (with externalities ignored) is maximized when the consumer price is made equal to the marginal cost $c = \$27{,}315$ (\$29,885) to eliminate the under-supply due to imperfect competition. A subsidy-induced consumer price lower than c (as in simulation #2, Policy D) is justified by the existence of positive externalities associated with the sale/consumption of the target good (e.g., reduced pollutant emissions, R&D spillovers and so on for the case of EVs). If we compare Policy B and Policy D (simulation #2, $r = r_1$) with taking externalities into account, positive externalities greater than \$2,159 (\$593) per unit of a good make Policy D ($r = r_1$) preferred to Policy B in terms of social surplus (with externalities considered). Note that \$2,159 (\$593) is much smaller than the subsidy per unit of \$5,143 (\$5,099) under Policy D (simulation #2, $r = r_1$).

Overall, the simulations have found the substantial impacts of the IPR form (Policy D) as suggested by the theoretical framework in the previous sections. It saves the government expenditure for achieving a given target $n\hat{q} = 50,981$ by up to 61% (68%), compared to the specific form. Stated differently, with a given subsidy budget (\$382 million), the IPR form can induce up to 48% (50%) more sales than the specific form. At the same time, the government can flexibly affect the policy's incidence on the firms through its choice of policy variables. By letting $r \approx 0$, the IPR subsidy is (almost) equivalent to the specific subsidy (as shown in Figure 3) and can increase the firms' aggregate profits by up to \$227 million (\$251 million) relative to the case of no government intervention. On the other hand, as r approaches r_2, it works as implicit taxation on the firms by making the firms' profits up to \$10 million (\$16 million) lower than under no government intervention (according to simulation #2). The government can flexibly set producer surplus within this range depending on whether and how much it wants to subsidize or tax the producers.

5 Extension: Product Quality

The simple IPR form as discussed above could deter improvements in product quality by making them more costly: an increase in the (pre-subsidy) price of a product due to higher production costs associated with better quality reduces the subsidy payment for the product. Note that we face the same disincentive regularly through the widely-used *ad valorem* taxation because it levies an additional tax payment on a better-quality product with a higher cost and (pre-tax) price (Keen, 1998). In these cases, while a policy-driven increase in the (producer) price elasticity of demand induces a price reduction, thereby allowing the government to more efficiently attain its target, it also motivates a firm to lower product quality as a simple way to cut the cost and price. This issue is obviously shared by the generalized tax forms of Myles (1996), Hamilton (1999), and Carbonnier (2014) that are designed to make the demand faced by a monopolist/oligopolist more elastic, although product quality is out of the scope of these papers. This section considers how the IPR subsidy can be adjusted to deal with the issue of product quality. In short, the disincentive can be curbed by increasing $\overline{p}_i$ with quality.

I supplement the cost and demand functions with quality as follows, while maintaining the basic structure of the model in Section 2.1. Note that identical firms and a symmetrically differentiated demand system are not assumed in this section. While the following model deals with one-dimensional quality for the sake of simplicity, similar results are obtained with multi-dimensional quality, as discussed in Appendix B. The unit production cost of good i depends on its quality $w_i \in \mathbb{R}_+$: $c_i = c_i(w_i)$ with $c_i'(w_i) > 0$ and $c_i''(w_i) > 0$. Demand functions are derived by adding w_i to the setup of Section 2 (footnote 6). Suppose that a unit of good i with quality w_i increases a representative consumer's utility by $f(w_i)$, where $f'(w_i) > 0$ and $f''(w_i) < 0$. The representative consumer's utility due to product quality (aggregated over the n products) is $\sum_{i=1}^n f(w_i)q_i$. By adding this term to $U(x, q_1, \cdots, q_n) = x + u(q_1, \cdots, q_n)$ of footnote 6, her total utility is now expressed as $U(x, \{w_i, q_i\}_{i=1,\cdots,n}) = x + u(q_1, \cdots, q_n) + \sum_{i=1}^n f(w_i)q_i$. Given $\{w_i, p_i\}_{i=1,\cdots,n}$, she solves the following utility maximization problem:

$$\max_{x, q_1, \cdots, q_n} x + u(q_1, \cdots, q_n) + \sum_{i=1}^{n} f(w_i)q_i \text{ s.t. } x + \sum_i p_i q_i \le I. \tag{27}$$

Assuming an interior solution, the first order conditions are $\frac{\partial u(q_1,\cdots,q_n)}{\partial q_i} = p_i - f(w_i)\ \forall i$. This means that the demand for each good depends on $\{p_i - f(w_i)\}_{i=1,\cdots,n}$, i.e., $q_i = Q_i(p_1 - f(w_1), \cdots, p_n - f(w_n))$ for each i (note that $q_i = Q_i(p_1, \cdots, p_n)$ in Section 2). Put differently, $p_i = \frac{\partial u(q_1,\cdots,q_n)}{\partial q_i} + f(w_i)$, so $f(w_i)$ is considered the premium attributable to product i's quality or, in other words, the consumer's willingness to pay (WTP) for quality.

[Policy A]

First, let us consider the determination of Nash equilibrium prices and qualities in the baseline case of no subsidy. Given the qualities and prices of the other products ($\mathbf{w}_{-i}$ and $\mathbf{p}_{-i}$), firm i sets w_i and p_i simultaneously to maximize its profits $\pi_{iA}(w_i, p_i, \mathbf{w}_{-i}, \mathbf{p}_{-i}) = [p_i - c_i(w_i)]Q_i(h_i, \mathbf{h}_{-i})$, where $h_j = p_j - f(w_j)$ and $\mathbf{h}_{-i}$ is a vector of h_j's for all the products other than product i. Assuming an interior solution, the following FOCs are satisfied:

$$\frac{\partial \pi_{iA}}{\partial w_i} = -c_i'(w_i)Q_i(h_i, \mathbf{h}_{-i}) - f'(w_i)[p_i - c_i(w_i)]\frac{\partial Q_i(h_i, \mathbf{h}_{-i})}{\partial h_i} = 0, \tag{28}$$

$$\frac{\partial \pi_{iA}}{\partial p_i} = Q_i(h_i, \mathbf{h}_{-i}) + [p_i - c_i(w_i)]\frac{\partial Q_i(h_i, \mathbf{h}_{-i})}{\partial h_i} = 0. \tag{29}$$

Substituting (29) into (28) shows that the optimal quality w_{iA}^* satisfies

$$f'(w_{iA}^*) = c_i'(w_{iA}^*). \tag{30}$$

The optimal quality w_{iA}^* is uniquely determined because $f'' - c_i'' < 0$. It equates the marginal price (= marginal utility) and marginal cost of quality improvement and maximizes the net value of quality, $f - c_i$. Note, however, that maximizing $f - c_i$ is not needed for the social optimum if product quality is associated with externalities, which is typically the very reason for subsidization. Given $\{w_{iA}^*, c_i(w_{iA}^*)\}_{i=1,\cdots,n}$, Nash equilibrium prices, which satisfy (29) for each i, are determined as in Section 2.1, and the results of Section 2.1 about Policy A apply analogously to the current setting with quality.

[Policies B and C]

Under Policies B and C, profits are respectively given by $\pi_{iB}(w_i, p_i, \mathbf{w}_{-i}, \mathbf{p}_{-i}) = [p_i + z - c_i(w_i)]Q_i(h_i, \mathbf{h}_{-i})$ and $\pi_{iC}(w_i, p_i, \mathbf{w}_{-i}, \mathbf{p}_{-i}) = [(1+v)p_i - c_i(w_i)]Q_i(h_i, \mathbf{h}_{-i})$. Following the same steps as above, it is straightforward to find that the equilibrium qualities (w^*_{iB} and w^*_{iC}) are respectively determined by

$$f'(w^*_{iB}) = c'_i(w^*_{iB}), \tag{31}$$

$$(1+v)f'(w^*_{iC}) = c'_i(w^*_{iC}). \tag{32}$$

Thus, the specific subsidy maintains the same equilibrium quality as no subsidy ($w^*_{iA} = w^*_{iB}$) because it does not distort the effective cost of quality improvement, while the *ad valorem* subsidy lowers the effective cost of quality improvement and thus leads to better quality than with no subsidy ($w^*_{iC} > w^*_{iA}$). Given the equilibrium qualities of all firms, Nash equilibrium prices are determined as in Section 2.1.

[Policy D]

Suppose that each firm's threshold $\overline{p}_i$ is determined by the government as a function of w_i, i.e., $\overline{p}_i = \overline{p}(w_i)$. Assuming a sufficiently generous subsidy scheme, we focus on Nash equilibria where all firms opt in and thus $\pi_{iD} = \pi_{iD_0}$. As above, consider maximizing $\pi_{iD_0}(w_i, p_i, \mathbf{w}_{-i}, \mathbf{p}_{-i}) = [(1-r)p_i + r\overline{p}(w_i) - c_i(w_i)]Q_i(h_i, \mathbf{h}_{-i})$ with respect to w_i and p_i. Assuming an interior solution, the following FOCs are satisfied:

$$\frac{\partial \pi_{iD_0}}{\partial w_i} = [r\overline{p}'(w_i) - c'_i(w_i)]Q_i(h_i, \mathbf{h}_{-i}) - f'(w_i)[(1-r)p_i + r\overline{p}(w_i) - c_i(w_i)]\frac{\partial Q_i(h_i, \mathbf{h}_{-i})}{\partial h_i} = 0. \tag{33}$$

$$\frac{\partial \pi_{iD_0}}{\partial p_i} = (1-r)Q_i(h_i, \mathbf{h}_{-i}) + [(1-r)p_i + r\overline{p}(w_i) - c_i(w_i)]\frac{\partial Q_i(h_i, \mathbf{h}_{-i})}{\partial h_i} = 0. \tag{34}$$

(33) and (34) imply that the optimal quality $w^*_{iD_0}$ satisfies

$$(1-r)f'(w^*_{iD_0}) = c'_i(w^*_{iD_0}) - r\overline{p}'_i(w^*_{iD_0}), \tag{35}$$

where it is assumed for simplicity that $w^*_{iD_0}$ is uniquely determined.[23] (35) means that the policymaker can affect the realized quality by adjusting the gradient $\overline{p}'(\cdot)$. For example, (30) and (35) imply that if $\overline{p}(\cdot)$ is constant, as in previous sections, then the optimal product quality is lower than in the no-subsidy case ($w^*_{iD_0} < w^*_{iA}$). As another example, increasing $\overline{p}_i$ linearly with w_i (i.e., $\overline{p}'_i(w_i) = a$ (> 0) for all w_i) improves $w^*_{iD_0}$ relative to the case of constant $\overline{p}_i$. With $\{w^*_{iD_0}, c_i(w^*_{iD_0})\}_{i=1,\cdots,n}$ determined by (35), Nash equilibrium prices, which satisfy (34) for all i, are determined as discussed in Section 2.1.

Importantly, whereas the *gradient* $\overline{p}'(\cdot)$ appears in (35), the *level* $\overline{p}(\cdot)$ does not. As a result, (35) does not restrict the policymaker's ability to choose the level of $\overline{p}(\cdot)$ and influence the

[23]For example, assume that r and $\overline{p}(\cdot)$ are set in such a way that keeps $(1-r)f + r\overline{p} - c_i$ strictly concave.

equilibrium prices $\{p^*_{iD_0}\}_{i=1,\cdots,n}$ through (34). Hence, augmenting Policy D by making $\overline{p}_i$ dependent on quality does not affect the previous results without product quality.

Following the above argument, let us consider a few examples to which the IPR form can be applied. First, various subsidy programs are in place today across countries to encourage the faster diffusion of green durable goods such as renewable energy systems, low-emission vehicles, energy-efficient home appliances, and energy-saving building renovations (e.g., solar PV and EV subsidies discussed above). Information is often available about energy/environmental and other attributes of these goods, enabling the policymaker to set $\overline{p}_i$ as a function of these attributes (Appendix B shows that the above model can be extended to the case of multi-dimensional attributes).[24] In fact, a number of specific or *ad valorem* subsidy schemes on these products link subsidy payment to product quality (e.g., U.S. federal tax credits for plug-in hybrid vehicles increase with battery capacity). For actual implementation of the scheme, low dimensionality of product attributes would be preferable for keeping the schedule $\overline{p}(\cdot)$ relatively simple and manageable (for example, designing such a schedule would be easier for solar PV systems than for EVs).

Second, actual pharmaceutical drug regulations/subsidies have similarities to the quality-adjusted IPR subsidy discussed above. In many countries, the consumer's out-of-pocket price for a pharmaceutical drug is set below the unregulated level through negotiations between the policymaker and the producer, and in return, the drug is eligible for a government subsidy. The regulated price and subsidy rate depend on such factors as product quality/characteristics (e.g., clinical effectiveness), production costs, and the prices of similar drugs (OECD, 2008; Paris and Belloni, 2013). With theoretical models based on the negotiation process, Johnston and Zeckhauser (1991) and Wright (2004) suggest the possibility that the government can take advantage of its bargaining with pharmaceutical firms and inter-firm strategic interaction to induce lower drug prices (higher consumer surplus) and, at the same time, flexibly adjust producer surplus, two features that are also realized by the IPR scheme.[25]

Besides, many countries adopt reference pricing policies to contain fast-growing public spending on drugs (e.g., Acosta et al., 2014). In these policies, drugs are clustered based on chemical, pharmacological, or therapeutic equivalence criteria, and a reference price is set for each cluster. The reimbursement for each drug linearly increases with its price but is capped at the reference price of the corresponding cluster, providing a downward pressure on the price when it is above the reference price. Note that this design is similar to the quality-adjusted IPR subsidy in that the subsidy rule switches at the threshold price and the threshold depends on product quality, though subsidy payment in reference pricing policies is non-decreasing in the product price, unlike the IPR subsidy. Altogether, these observations about pharmaceutical drug regulations/subsidies show similarities to the IPR form, suggesting that it could

[24] Additionally, economic/engineering/scientific estimates are often available about WTP for product characteristics (by means of, e.g., hedonic regressions or discrete choice models) and positive externalities (e.g., the value of carbon emissions reductions), providing useful benchmarks for setting $\overline{p}_i$ as a function of product attributes.

[25] Wright (2004) points out that in practice the government does not well exploit the second function in the negotiations, as it often simply benchmarks the subsidy level (and thus firm profitability) against foreign markets.

be applied to these cases by, for example, letting $\overline{p}_i$ constant within each cluster and variant across clusters.

6 Conclusion

This paper has been motivated by a unique structure of a government subsidy program in Japan, in which a rebate is paid for the purchase of a target product (residential solar PV system) conditional on the price being below a threshold, and the rebate amount increases as the price is further reduced. Transaction data show that the design was helpful for achieving the government's aim of lowering (post- and) pre-rebate prices and thus boosting the sales. To my knowledge, this type of subsidy policy has not previously been studied in the literature, In this paper, I provide a theoretical foundation for such a policy design and explore its effectiveness. Specifically, I consider a particular scheme (termed the IPR scheme) in which subsidy payment is conditional on the product's price being less than a government-set threshold and increases as the price further goes down, in proportion to the difference between the threshold and price.

With a model of imperfect competition (Bertrand oligopoly with product differentiation), I find that the IPR subsidy has two advantages over the widely-used specific or *ad valorem* subsidy. First, it is more efficient in the sense that it can induce a given level of output (the government's target) with less government expenditure than the other forms (or, equivalently, it can induce a larger sales with a given government budget). Second, while achieving this target, the government can also seek a second goal of affecting the incidence of the policy on the producers: depending on the policy parameters set by the government, firm profits can be increased or decreased relative to the no-subsidy benchmark case.

Calibrations based on the data from the U.S. EV market indicate the substantial magnitude of these advantages. With a given government budget ($382 million), the IPR form can induce up to 48%–50% higher market sales of the target products than the specific form. Additionally, depending on the policymaker's choice of policy variables, the IPR form with the same budget can flexibly adjust producer surplus in the range of $227–$251 million higher and $10–$16 million lower than under no subsidy.

Lastly, an issue with the IPR form is the producer's disincentive to quality improvement, as better quality and associated higher production costs reduces the subsidy payment. By extending the theoretical model and specifically considering product quality, the paper discusses a simple way to curb and correct the disincentive: making the price threshold for subsidy eligibility increase with quality, and thus rewarding a higher-quality product with a larger subsidy payment. Examples of products to which the quality-adjusted IPR scheme is applicable include green durable goods (such as solar PV systems) and pharmaceutical drugs.

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A Proofs

A.1 Proof of Lemma 1

Proof. Given $\mathbf{p}_{-i} \in \prod_{j \neq i}[c_j, p^{max}]$, let K be the interval defined by $K \equiv [0, \inf\{p_i | Q_i(p_i, \mathbf{p}_{-i}) = 0\}]$ and $F_i : K \times \mathbb{R}_{++} \to \mathbb{R}$ be the function defined by $F_i(p_i, x_i) = (p_i - x_i)Q_i(p_i, \mathbf{p}_{-i})$. Then, F_i has strictly increasing differences in (p_i, x_i) on $K \times \mathbb{R}_{++}$ because for $(p_i, x_i) \in K \times \mathbb{R}_{++}$ and $(p'_i, x'_i) \in K \times \mathbb{R}_{++}$ such that $p_i > p'_i$ and $x_i > x'_i$,

$$\begin{aligned}&\left[(p_i - x_i)Q_i(p_i, \mathbf{p}_{-i}) - (p'_i - x_i)Q_i(p'_i, \mathbf{p}_{-i})\right] - \left[(p_i - x'_i)Q_i(p_i, \mathbf{p}_{-i}) - (p'_i - x'_i)Q_i(p'_i, \mathbf{p}_{-i})\right] \\ &= (x_i - x'_i)\left[Q_i(p'_i, \mathbf{p}_{-i}) - Q_i(p_i, \mathbf{p}_{-i})\right] \\ &> 0,\end{aligned} \tag{36}$$

where the last line follows because Q_i is strictly decreasing in i's own price on K. Let $\beta_i(x_i) = \arg\max_{p_i \in [x_i, p^{max}] \cap K} F_i(p_i, x_i)$. The minimum of the constraint set ($[x_i, p^{max}] \cap K = [x_i, \inf\{p_i | Q_i(p_i, \mathbf{p}_{-i}) = 0\}]$) is increasing in x_i, and so is the maximum (trivially). Hence, Topkis's (1978) monotonicity theorem (see Amir, 2005, Theorem 1) implies that if $x_i > x'_i$ and $b_i \in \beta_i(x_i)$ and $b'_i \in \beta_i(x'_i)$, then $b_i \geq b'_i$. Given $\mathbf{p}_{-i} \in \prod_{j \neq i}[c_j, p^{max}]$, note that $\psi_{iA}(\mathbf{p}_{-i}, c_i) = \beta_i(c_i)$, $\psi_{iB}(\mathbf{p}_{-i}, c_i, z) = \beta_i(c_i - z)$, $\psi_{iC}(\mathbf{p}_{-i}, c_i, v) = \beta_i(\frac{c_i}{1+v})$, and $\psi_{iD_0}(\mathbf{p}_{-i}, c_i, r, \overline{p}_i) = \beta_i(\frac{c_i - r\overline{p}_i}{1-r})$ (for clarity purposes, I have made explicit the dependence of the best response correspondences on the parameters c_i, z, v, r, and $\overline{p}_i$). Also, $c_i > c_i - z$, $c_i > \frac{c_i}{1+v}$, and $c_i > \frac{c_i - r\overline{p}_i}{1-r}$. Hence, the statement of the lemma follows. ■

A.2 Proof of Lemma 2

Proof. Given $p_i \geq p'_i$ and $\mathbf{p}_{-i} \geq \mathbf{p}'_{-i}$ (i.e., $p_j \geq p'_j$ for all $j \neq i$), it suffices to show $\Delta \equiv \pi_{iD}(p_i, \mathbf{p}_{-i})\pi_{iD}(p'_i, \mathbf{p}'_{-i}) - \pi_{iD}(p'_i, \mathbf{p}_{-i})\pi_{iD}(p_i, \mathbf{p}'_{-i}) \geq 0$.

If $p_i > p'_i \geq \overline{p}_i$, it follows from (11) and footnote 7 that

$$\Delta = \pi_{iA}(p_i, \mathbf{p}_{-i})\pi_{iA}(p'_i, \mathbf{p}'_{-i}) - \pi_{iA}(p'_i, \mathbf{p}_{-i})\pi_{iA}(p_i, \mathbf{p}'_{-i}) \geq 0. \tag{37}$$

If $\overline{p}_i \geq p_i > p'_i$, it analogously follows from (11) and footnote 7 that

$$\Delta = \pi_{iD_0}(p_i, \mathbf{p}_{-i})\pi_{iD_0}(p'_i, \mathbf{p}'_{-i}) - \pi_{iD_0}(p'_i, \mathbf{p}_{-i})\pi_{iD_0}(p_i, \mathbf{p}'_{-i}) \geq 0. \tag{38}$$

If $p_i > \overline{p}_i > p'_i$, it follows from (11) and (2) that

$$\begin{aligned}\Delta &= \pi_{iA}(p_i, \mathbf{p}_{-i})\pi_{iD_0}(p'_i, \mathbf{p}'_{-i}) - \pi_{iD_0}(p'_i, \mathbf{p}_{-i})\pi_{iA}(p_i, \mathbf{p}'_{-i}) \\ &= (p_i - c_i)Q_i(p_i, \mathbf{p}_{-i})[p'_i + r(\overline{p}_i - p'_i) - c_i]Q_i(p'_i, \mathbf{p}'_{-i}) \\ &\qquad - [p'_i + r(\overline{p}_i - p'_i) - c_i]Q_i(p'_i, \mathbf{p}_{-i})(p_i - c_i)Q_i(p_i, \mathbf{p}'_{-i}) \\ &= (p_i - c_i)[p'_i + r(\overline{p}_i - p'_i) - c_i][Q_i(p_i, \mathbf{p}_{-i})Q_i(p'_i, \mathbf{p}'_{-i}) - Q_i(p'_i, \mathbf{p}_{-i})Q_i(p_i, \mathbf{p}'_{-i})] \\ &\geq 0.\end{aligned} \tag{39}$$

Therefore, if $p_i \geq p'_i$ and $\mathbf{p}_{-i} \geq \mathbf{p}'_{-i}$, then $\Delta \geq 0$ in any case.

■

A.3 Proof of Lemma 3

Proof. First, Lemma 5 below is proved, which is to be used in the proofs of Lemmas 3 and 4.

Lemma 5. *For $p^b_{iA} \in \psi_{iA}(\mathbf{p}_{-i})$ and $p^b_{iD_0} \in \psi_{iD_0}(\mathbf{p}_{-i})$, $G_i(\mathbf{p}_{-i}) \geq 0$ implies $p^b_{iA} \geq \overline{p}_i$, and $G_i(\mathbf{p}_{-i}) \leq 0$ implies $p^b_{iD_0} \leq \overline{p}_i$. Equivalently, $p^b_{iA} < \overline{p}_i$ implies $G_i(\mathbf{p}_{-i}) < 0$, and $p^b_{iD_0} > \overline{p}_i$ implies $G_i(\mathbf{p}_{-i}) > 0$.*

Proof. Suppose $p^b_{iA} < \overline{p}_i$. Then,

$$\begin{aligned}
\pi_{iD_0}(p^b_{iD_0}, \mathbf{p}_{-i}) &\geq \pi_{iD_0}(p^b_{iA}, \mathbf{p}_{-i}) \\
&= [(1-r)p^b_{iA} + r\overline{p}_i - c_i] \cdot Q_i(p^b_{iA}, \mathbf{p}_{-i}) \\
&= [p^b_{iA} - c_i] \cdot Q_i(p^b_{iA}, \mathbf{p}_{-i}) + r[\overline{p}_i - p^b_{iA}] \cdot Q_i(p^b_{iA}, \mathbf{p}_{-i}) \\
&= \pi_{iA}(p^b_{iA}, \mathbf{p}_{-i}) + r[\overline{p}_i - p^b_{iA}] \cdot Q_i(p^b_{iA}, \mathbf{p}_{-i}) \\
&> \pi_{iA}(p^b_{iA}, \mathbf{p}_{-i}),
\end{aligned}$$

where the last line follows from $Q_i(p^b_{iA}, \mathbf{p}_{-i}) > 0$. Thus, $G_i(\mathbf{p}_{-i}) < 0$. In other words, if $G_i(\mathbf{p}_{-i}) \geq 0$, then $p^b_{iA} \geq \overline{p}_i$.

Similarly, suppose $p^b_{iD_0} > \overline{p}_i$. Then,

$$\begin{aligned}
\pi_{iA}(p^b_{iA}, \mathbf{p}_{-i}) &\geq \pi_{iA}(p^b_{iD_0}, \mathbf{p}_{-i}) \\
&= [p^b_{iD_0} - c_i] \cdot Q_i(p^b_{iD_0}, \mathbf{p}_{-i}) \\
&> [p^b_{iD_0} - c_i] \cdot Q_i(p^b_{iD_0}, \mathbf{p}_{-i}) + r[\overline{p}_i - p^b_{iD_0}] \cdot Q_i(p^b_{iD_0}, \mathbf{p}_{-i}) \\
&= [(1-r)p^b_{iD_0} + r\overline{p}_i - c_i] \cdot Q_i(p^b_{iD_0}, \mathbf{p}_{-i}) \\
&= \pi_{iD_0}(p^b_{iD_0}, \mathbf{p}_{-i}),
\end{aligned}$$

where the third line follows from $Q_i(p^b_{iD_0}, \mathbf{p}_{-i}) > 0$. Thus, $G_i(\mathbf{p}_{-i}) > 0$. In other words, if $G_i(\mathbf{p}_{-i}) \leq 0$, then $p^b_{iD_0} \leq \overline{p}_i$. ■

By the definition of $\pi_{iD}(p_i, \mathbf{p}_{-i})$ in (11),

$$\max_{p_i \in [\frac{c_i - r\overline{p}_i}{1-r}, p^{max}]} \pi_{iD}(p_i, \mathbf{p}_{-i}) \leq \max_{p_i \in [\frac{c_i - r\overline{p}_i}{1-r}, p^{max}]} \{\max\{\pi_{iA}(p_i, \mathbf{p}_{-i}), \pi_{iD_0}(p_i, \mathbf{p}_{-i})\}\}. \tag{40}$$

Suppose $G_i(\mathbf{p}_{-i}) > 0$. By definition, this means that

$$\begin{aligned}
\pi_{iA}(p^b_{iA}, \mathbf{p}_{-i}) &\equiv \max_{p_i \in [c_i, p^{max}]} \pi_{iA}(p_i, \mathbf{p}_{-i}) = \max_{p_i \in [\frac{c_i - r\overline{p}_i}{1-r}, p^{max}]} \pi_{iA}(p_i, \mathbf{p}_{-i}) \\
&> \pi_{iD_0}(p^b_{iD_0}, \mathbf{p}_{-i}) \equiv \max_{p_i \in [\frac{c_i - r\overline{p}_i}{1-r}, p^{max}]} \pi_{iD_0}(p_i, \mathbf{p}_{-i}),
\end{aligned} \tag{41}$$

so that $\pi_{iA}(p^b_{iA}, \mathbf{p}_{-i}) = \max_{p_i \in [\frac{c_i - r\overline{p}_i}{1-r}, p^{max}]} \{\max\{\pi_{iA}(p_i, \mathbf{p}_{-i}), \pi_{iD_0}(p_i, \mathbf{p}_{-i})\}\}$. Also, Lemma 5 and $G_i(\mathbf{p}_{-i}) > 0$ imply $p^b_{iA} \geq \overline{p}_i$. Thus, it follows from (11) that $\pi_{iD}(p^b_{iA}, \mathbf{p}_{-i}) = \pi_{iA}(p^b_{iA}, \mathbf{p}_{-i})$. Therefore, noting (40), we find $\max_{p_i \in [\frac{c_i - r\overline{p}_i}{1-r}, p^{max}]} \pi_{iD}(p_i, \mathbf{p}_{-i}) = \pi_{iD}(p^b_{iA}, \mathbf{p}_{-i}) = \pi_{iA}(p^b_{iA}, \mathbf{p}_{-i})$ or, equivalently, $\psi_{iD}(\mathbf{p}_{-i}) = \psi_{iA}(\mathbf{p}_{-i})$.

Similarly, suppose $G_i(\mathbf{p}_{-i}) < 0$. By definition, this means that

$$\begin{aligned} \pi_{iA}(p^b_{iA}, \mathbf{p}_{-i}) \equiv \max_{p_i \in [c_i, p^{max}]} \pi_{iA}(p_i, \mathbf{p}_{-i}) &= \max_{p_i \in [\frac{c_i - r\overline{p}_i}{1-r}, p^{max}]} \pi_{iA}(p_i, \mathbf{p}_{-i}) \\ < \pi_{iD_0}(p^b_{iD_0}, \mathbf{p}_{-i}) &\equiv \max_{p_i \in [\frac{c_i - r\overline{p}_i}{1-r}, p^{max}]} \pi_{iD_0}(p_i, \mathbf{p}_{-i}), \end{aligned} \tag{42}$$

so that $\pi_{iD_0}(p^b_{iD_0}, \mathbf{p}_{-i}) = \max_{p_i \in [\frac{c_i - r\overline{p}_i}{1-r}, p^{max}]} \{\max\{\pi_{iA}(p_i, \mathbf{p}_{-i}), \pi_{iD_0}(p_i, \mathbf{p}_{-i})\}\}$. Also, Lemma 5 and $G_i(\mathbf{p}_{-i}) < 0$ imply $p^b_{iD_0} \leq \overline{p}_i$. Then, it follows from (11) that $\pi_{iD}(p^b_{iD_0}, \mathbf{p}_{-i}) = \pi_{iD_0}(p^b_{iD_0}, \mathbf{p}_{-i})$. Therefore, noting (40), we find $\max_{p_i \in [\frac{c_i - r\overline{p}_i}{1-r}, p^{max}]} \pi_{iD}(p_i, \mathbf{p}_{-i}) = \pi_{iD}(p^b_{iD_0}, \mathbf{p}_{-i}) = \pi_{iD_0}(p^b_{iD_0}, \mathbf{p}_{-i})$ or, equivalently, $\psi_{iD}(\mathbf{p}_{-i}) = \psi_{iD_0}(\mathbf{p}_{-i})$.

Lastly, $G_i(\mathbf{p}_{-i}) = 0$ means that

$$\begin{aligned} \pi_{iA}(p^b_{iA}, \mathbf{p}_{-i}) \equiv \max_{p_i \in [c_i, p^{max}]} \pi_{iA}(p_i, \mathbf{p}_{-i}) &= \max_{p_i \in [\frac{c_i - r\overline{p}_i}{1-r}, p^{max}]} \pi_{iA}(p_i, \mathbf{p}_{-i}) \\ = \pi_{iD_0}(p^b_{iD_0}, \mathbf{p}_{-i}) &\equiv \max_{p_i \in [\frac{c_i - r\overline{p}_i}{1-r}, p^{max}]} \pi_{iD_0}(p_i, \mathbf{p}_{-i}), \end{aligned} \tag{43}$$

so that $\pi_{iA}(p^b_{iA}, \mathbf{p}_{-i}) = \pi_{iD_0}(p^b_{iD_0}, \mathbf{p}_{-i}) = \max_{p_i \in [\frac{c_i - r\overline{p}_i}{1-r}, p^{max}]} \{\max\{\pi_{iA}(p_i, \mathbf{p}_{-i}), \pi_{iD_0}(p_i, \mathbf{p}_{-i})\}\}$. Also, Lemma 5 and $G_i(\mathbf{p}_{-i}) = 0$ imply $p^b_{iA} \geq \overline{p}_i$ and $p^b_{iD_0} \leq \overline{p}_i$. Then, it follows from (11) that $\pi_{iD}(p^b_{iA}, \mathbf{p}_{-i}) = \pi_{iA}(p^b_{iA}, \mathbf{p}_{-i})$ and $\pi_{iD}(p^b_{iD_0}, \mathbf{p}_{-i}) = \pi_{iD_0}(p^b_{iD_0}, \mathbf{p}_{-i})$. Therefore, noting (40), we find $\max_{p_i \in [\frac{c_i - r\overline{p}_i}{1-r}, p^{max}]} \pi_{iD}(p_i, \mathbf{p}_{-i}) = \pi_{iD}(p^b_{iA}, \mathbf{p}_{-i}) = \pi_{iD}(p^b_{iD_0}, \mathbf{p}_{-i}) = \pi_{iA}(p^b_{iA}, \mathbf{p}_{-i}) = \pi_{iD_0}(p^b_{iD_0}, \mathbf{p}_{-i})$ or, equivalently, $\psi_{iD}(\mathbf{p}_{-i}) = \psi_{iA}(\mathbf{p}_{-i}) \cup \psi_{iD_0}(\mathbf{p}_{-i})$.

■

A.4 Proof of Lemma 4

Proof. The first result follows immediately from Lemma 3. As for the second result, Lemma 5 and $p_A \in F_D \cap F_A$ imply $\overline{p} \leq p_A$. Similarly, Lemma 5 and $p_{D_0} \in F_D \cap F_{D_0}$ imply $p_{D_0} \leq \overline{p}$. Thus, $p_{D_0} \leq \overline{p} \leq p_A$. The result about $\tilde{\pi}_D$ follows from footnote 13. ■

A.5 Proof of Proposition 1

Proof. When the other firms set a common price p_0, an optimal response $p^b_{D_0} \in \tilde{\psi}_{D_0}(p_0) = \psi_{D_0}(p_0, \cdots, p_0)$ is characterized by the FOC, $(1-r)\tilde{Q}(p^b_{D_0}, p_0) + [(1-r)p^b_{D_0} + r\overline{p} - c]\tilde{Q}_1(p^b_{D_0}, p_0) = 0$. Substituting (21) into the FOC gives

$$\tilde{Q}(p^b_{D_0}, p_0) + [p^b_{D_0} - \hat{p} - \frac{\tilde{Q}(\hat{p}, \hat{p})}{\tilde{Q}_1(\hat{p}, \hat{p})}]\tilde{Q}_1(p^b_{D_0}, p_0) = 0. \tag{44}$$

By applying the envelope theorem to $\tilde{\pi}_{D_0}(p^b_{D_0}, p_0; r, \overline{p}(r;\hat{p})) \equiv \pi_{D_0}(p^b_{D_0}, p_0, \cdots, p_0; r, \overline{p}(r;\hat{p})) = [(1-r)p^b_{D_0} + r\overline{p} - c]\tilde{Q}(p^b_{D_0}, p_0)$ with noting that $\overline{p} = [c - \frac{\tilde{Q}(\hat{p},\hat{p})}{\tilde{Q}_1(\hat{p},\hat{p})} - \hat{p}]/r + \frac{\tilde{Q}(\hat{p},\hat{p})}{\tilde{Q}_1(\hat{p},\hat{p})} + \hat{p}$ by (21), we have

$$\begin{aligned}\frac{d\tilde{\pi}_{D_0}(p^b_{D_0}, p_0; r, \overline{p}(r;\hat{p}))}{dr} &= \big(-p^b_{D_0} + \frac{\tilde{Q}(\hat{p},\hat{p})}{\tilde{Q}_1(\hat{p},\hat{p})} + \hat{p}\big)\tilde{Q}(p^b_{D_0}, p_0) \\ &= \frac{\tilde{Q}(p^b_{D_0}, p_0)^2}{\tilde{Q}_1(p^b_{D_0}, p_0)} \\ &< 0,\end{aligned} \tag{45}$$

where the second line follows from (44). Then,

$$\frac{d\tilde{G}(p_0; r, \overline{p}(r;\hat{p}))}{dr} = -\frac{d\tilde{\pi}_{D_0}(p^b_{D_0}, p_0; r, \overline{p}(r;\hat{p}))}{dr} > 0. \tag{46}$$

Lemma 6. *Given r and $\overline{p}$, if there exists some $\check{p}$ such that $\tilde{G}(\check{p}; r, \overline{p}) = 0$, then*

$$\tilde{G}(p_0; r, \overline{p}) \begin{cases} \leq 0 & \text{for all } p_0 < \check{p}, \\ = 0 & \text{for } p_0 = \check{p}, \\ \geq 0 & \text{for all } p_0 > \check{p}. \end{cases} \tag{47}$$

Proof. By the envelope theorem, $\frac{d\tilde{\pi}_A(p^b_A, p_0)}{dp_0} = [p^b_A - c]\tilde{Q}_2(p^b_A, p_0)$, where $p^b_A \in \tilde{\psi}_A(p_0)$, and $\frac{d\tilde{\pi}_{D_0}(p^b_{D_0}, p_0; r, \overline{p})}{dp_0} = [(1-r)p^b_{D_0} + r\overline{p} - c]\tilde{Q}_2(p^b_{D_0}, p_0)$. Thus,

$$\begin{aligned}\frac{d\tilde{G}(p_0; r, \overline{p})}{dp_0} &= [p^b_A - c]\tilde{Q}_2(p^b_A, p_0) - [(1-r)p^b_{D_0} + r\overline{p} - c]\tilde{Q}_2(p^b_{D_0}, p_0) \\ &= [p^b_A - c]\tilde{Q}(p^b_A, p_0)\frac{\tilde{Q}_2(p^b_A, p_0)}{\tilde{Q}(p^b_A, p_0)} - [(1-r)p^b_{D_0} + r\overline{p} - c]\tilde{Q}(p^b_{D_0}, p_0)\frac{\tilde{Q}_2(p^b_{D_0}, p_0)}{\tilde{Q}(p^b_{D_0}, p_0)} \\ &\geq \Big\{[p^b_A - c]\tilde{Q}(p^b_A, p_0) - [(1-r)p^b_{D_0} + r\overline{p} - c]\tilde{Q}(p^b_{D_0}, p_0)\Big\}\frac{\tilde{Q}_2(p^b_A, p_0)}{\tilde{Q}(p^b_A, p_0)} \\ &= \tilde{G}(p_0; r, \overline{p})\frac{\tilde{Q}_2(p^b_A, p_0)}{\tilde{Q}(p^b_A, p_0)},\end{aligned} \tag{48}$$

where the inequality holds because $\frac{\tilde{Q}_2(p^b_A, p_0)}{\tilde{Q}(p^b_A, p_0)} \geq \frac{\tilde{Q}_2(p^b_{D_0}, p_0)}{\tilde{Q}(p^b_{D_0}, p_0)}$ due to the property of increasing differences of $\log Q$ in $(p_i, \mathbf{p}_{-i})$.[26] Then, it follows from (48) that

$$\frac{d\tilde{G}(p_0; r, \overline{p})}{dp_0} \begin{cases} > 0 & \text{if } \tilde{G}(p_0; r, \overline{p}) > 0, \\ \geq 0 & \text{if } \tilde{G}(p_0; r, \overline{p}) = 0, \end{cases} \tag{50}$$

which implies (47).

■

As $\tilde{G}(\hat{p}; r_3, \overline{p}(r_3; \hat{p})) = 0$ and $\hat{p} < p_A^*$, Lemma 6 implies $\tilde{G}(p_A^*; r_3, \overline{p}(r_3; \hat{p})) \geq 0$. Then, (46) and $\tilde{G}(p_A^*; r_2, \overline{p}(r_2; \hat{p})) = 0$ implies $r_2 \leq r_3$.

By (46) and $\tilde{G}(p_A^*; r_2, \overline{p}(r_2; \hat{p})) = 0$, $\tilde{G}(p_A^*; r, \overline{p}(r; \hat{p})) < 0$ for all $r < r_2$ and $G(p_A^*; r, \overline{p}(r; \hat{p})) > 0$ for all $r > r_2$. Therefore, Lemma 4 implies that $p_A^* \notin F_D$ for all $r < r_2$, and $p_A^* \in F_D$ for all $r \geq r_2$. In addition, as $p_A < p_A^*$ for any other $p_A \in F_A$, $\tilde{G}(p_A; r_2, \overline{p}(r_2; \hat{p})) \leq 0$ (by Lemma 6 and $\tilde{G}(p_A^*; r_2, \overline{p}(r_2; \hat{p})) = 0$), so that $\tilde{G}(p_A; r, \overline{p}(r; \hat{p})) < 0$ for all $r < r_2$ and thus $p_A \notin F_D$ for all $r < r_2$.

Similarly, by (46) and $\tilde{G}(\hat{p}; r_3, \overline{p}(r_3; \hat{p})) = 0$, $\tilde{G}(\hat{p}; r, \overline{p}(r; \hat{p})) < 0$ for all $r < r_3$ and $G(\hat{p}; r, \overline{p}(r; \hat{p})) > 0$ for all $r > r_3$. Therefore, Lemma 4 implies that $\hat{p} \in F_D$ for all $r \leq r_3$, and $\hat{p} \notin F_D$ for all $r > r_3$.

Hence, the three statements of the proposition follow.

■

A.6 Proof of Proposition 3

Proof. By the definitions of r_1 and r_2, $\tilde{\pi}_A(p_A^*, p_A^*) = \tilde{\pi}_{D_0}(\hat{p}, \hat{p}; r_1, \overline{p}(r_1; \hat{p})) = \tilde{\pi}_{D_0}(p_{D_0}^b(p_A^*), p_A^*; r_2, \overline{p}(r_2; \hat{p}))$. By the envelope theorem, $\frac{d\tilde{\pi}_{D_0}(p_{D_0}^b(p_0), p_0; r, \overline{p})}{dp_0} = [(1-r)p_{D_0}^b(p_0) + r\overline{p} - c]\tilde{Q}_2(p_{D_0}^b(p_0), p_0) > 0$. Then, $\hat{p} < p_A^*$ means $\tilde{\pi}_{D_0}(\hat{p}, \hat{p}; r_1, \overline{p}(r_1; \hat{p})) < \tilde{\pi}_{D_0}(p_{D_0}^b(p_A^*), p_A^*; r_1, \overline{p}(r_1; \hat{p}))$, which means $\tilde{\pi}_{D_0}(p_{D_0}^b(p_A^*), p_A^*; r_2, \overline{p}(r_2; \hat{p})) < \tilde{\pi}_{D_0}(p_{D_0}^b(p_A^*), p_A^*; r_1, \overline{p}(r_1; \hat{p}))$ by the first sentence of this proof. This implies $r_1 < r_2$ because $\frac{d\tilde{\pi}_{D_0}(p_{D_0}^b(p_0), p_0; r, \overline{p}(r; \hat{p}))}{dr} < 0$ by (45). The remaining statements follow from Proposition 1 and (24).

■

[26]By using (4),

$$\begin{aligned} \frac{\partial \frac{\tilde{Q}_2(p_i, p_0)}{\tilde{Q}(p_i, p_0)}}{\partial p_i} &= \frac{1}{\tilde{Q}^2}[\tilde{Q}\tilde{Q}_{12} - \tilde{Q}_1\tilde{Q}_2] \\ &= \frac{n-1}{Q^2}\left[Q(p_i, p_j, \mathbf{p}_{-ij}) \times \frac{\partial^2 Q(p_i, p_j, \mathbf{p}_{-ij})}{\partial p_i \partial p_j} - \frac{\partial Q(p_i, p_j, \mathbf{p}_{-ij})}{\partial p_i} \times \frac{\partial Q(p_i, p_j, \mathbf{p}_{-ij})}{\partial p_j}\right]\Bigg|_{(p_j, \mathbf{p}_{-ij}) = (p_0, \cdots, p_0)} \\ &\geq 0. \end{aligned} \tag{49}$$

In addition, $p_A^b \geq p_{D_0}^b$ by Lemma 1. Therefore, $\frac{\tilde{Q}_2(p_A^b, p_0)}{\tilde{Q}(p_A^b, p_0)} \geq \frac{\tilde{Q}_2(p_{D_0}^b, p_0)}{\tilde{Q}(p_{D_0}^b, p_0)}$.

B Multi-Dimensional Quality

This appendix extends the model of Section 5 to the case of multi-dimensional quality. Overall, multiple product attributes can be analyzed in analogous steps and leads to similar results.

The unit production cost of good i depends on its K-dimensional attributes $\mathbf{w}_i \in \mathbb{R}_+^K$ and is expressed as $c_i(\mathbf{w}_i)$, where the function $c_i : \mathbb{R}_+^K \to \mathbb{R}_+$ has such properties that $\frac{\partial c_i(\mathbf{w}_i)}{\partial w_i^k} > 0 \; \forall \mathbf{w}_i$ for each attribute $k \in \{1, \cdots K\}$ and $\nabla^2 c_i(\mathbf{w}_i)$ is positive definite for all $\mathbf{w}_i$ (so $c_i(\cdot)$ is strictly convex). As for demand, a unit of good i with quality $\mathbf{w}_i$ increases a representative consumer's utility by $f(\mathbf{w}_i)$, where the function $f : \mathbb{R}_+^K \to \mathbb{R}_+$ has such properties that $\frac{\partial f(\mathbf{w}_i)}{\partial w_i^k} > 0$ for each k and $\nabla^2 f(\mathbf{w}_i)$ is negative definite for all $\mathbf{w}_i$ (so $f(\cdot)$ is strictly concave). As in the single attribute case, the representative consumer's utility due to product quality (aggregated over the n products) is $\sum_{i=1}^n f(\mathbf{w}_i) q_i$, and her total utility is expressed as $U(x, \{\mathbf{w}_i, q_i\}_{i=1,\cdots,n}) = x + u(q_1, \cdots, q_n) + \sum_{i=1}^n f(\mathbf{w}_i) q_i$. Given $\{\mathbf{w}_i, p_i\}_{i=1,\cdots,n}$, she solves the following utility maximization problem:

$$\max_{x, q_1, \cdots, q_n} x + u(q_1, \cdots, q_n) + \sum_{i=1}^n f(\mathbf{w}_i) q_i \;\; \text{s.t.} \;\; x + \sum_i p_i q_i \leq I. \tag{51}$$

Assuming an interior solution, the first order conditions are $\frac{\partial u(q_1, \cdots, q_n)}{\partial q_i} = p_i - f(\mathbf{w}_i) \; \forall i$. This means that the demand for each good depends on $\{p_i - f(\mathbf{w}_i)\}_{i=1,\cdots,n}$, i.e., $q_i = Q_i(p_1 - f(\mathbf{w}_1), \cdots, p_n - f(\mathbf{w}_n))$ for each i. Put differently, $p_i = \frac{\partial u(q_1, \cdots, q_n)}{\partial q_i} + f(\mathbf{w}_i)$, so $f(\mathbf{w}_i)$ is considered the premium attributable to product i's quality or, in other words, the consumer's willingness to pay (WTP) for quality.

[Policy A]

First, let us consider the determination of Nash equilibrium prices and qualities in the baseline case of no subsidy. Given the qualities and prices of the other products, firm i sets $\mathbf{w}_i$ and p_i simultaneously to maximize its profits $\pi_{iA}(\mathbf{w}_i, p_i, \{\mathbf{w}_j, p_j\}_{j \neq i}) = [p_i - c_i(\mathbf{w}_i)] Q_i(h_i, \mathbf{h}_{-i})$, where $h_i = p_i - f(\mathbf{w}_i)$ and $\mathbf{h}_{-i}$ is a vector of h_j's for all the products other than product i. Assuming an interior solution, the following FOCs are satisfied:

$$\frac{\partial \pi_{iA}}{\partial w_i^k} = -\frac{\partial c_i(\mathbf{w}_i)}{\partial w_i^k} Q_i(h_i, \mathbf{h}_{-i}) - [p_i - c_i(\mathbf{w}_i)] \frac{\partial Q_i(h_i, \mathbf{h}_{-i})}{\partial h_i} \frac{\partial f(\mathbf{w}_i)}{\partial w_i^k} = 0 \quad \forall k, \tag{52}$$

$$\frac{\partial \pi_{iA}}{\partial p_i} = Q_i(h_i, \mathbf{h}_{-i}) + [p_i - c_i(\mathbf{w}_i)] \frac{\partial Q_i(h_i, \mathbf{h}_{-i})}{\partial h_i} = 0. \tag{53}$$

Substituting (53) into (52) shows that the optimal quality $\mathbf{w}_{iA}^*$ satisfies

$$\nabla f(\mathbf{w}_{iA}^*) - \nabla c_i(\mathbf{w}_{iA}^*) = \mathbf{0}. \tag{54}$$

The optimal quality $\mathbf{w}_{iA}^*$ is uniquely determined because $f - c_i$ is strictly concave. It equates the marginal price (= marginal utility) and marginal cost of quality improvement for each attribute and maximizes the net value of quality, $f - c_i$. Note, however, that maximizing $f - c_i$

is not needed for the social optimum if some attributes are associated with externalities, which is typically the very reason for subsidization. Given $\{\mathbf{w}_{iA}^*, c_i(\mathbf{w}_{iA}^*)\}_{i=1,\cdots,n}$, Nash equilibrium prices, which satisfy (53) for each i, are determined as in Section 2.1, and the results of Section 2.1 about Policy A apply analogously to the current setting with quality.

[Policies B and C]

Under Policies B and C, profits are respectively given by $\pi_{iB}(\mathbf{w}_i, p_i, \{\mathbf{w}_j, p_j\}_{j\neq i}) = [p_i + z - c_i(\mathbf{w}_i)]Q_i(h_i, \mathbf{h}_{-i})$ and $\pi_{iC}(\mathbf{w}_i, p_i, \{\mathbf{w}_j, p_j\}_{j\neq i}) = [(1+v)p_i - c_i(\mathbf{w}_i)]Q_i(h_i, \mathbf{h}_{-i})$. Following the same steps as above, it is straightforward to find that the equilibrium qualities ($\mathbf{w}_{iB}^*$ and $\mathbf{w}_{iC}^*$) are respectively determined by

$$\nabla f(\mathbf{w}_{iB}^*) - \nabla c_i(\mathbf{w}_{iB}^*) = \mathbf{0}, \tag{55}$$

$$(1+v)\nabla f(\mathbf{w}_{iC}^*) - \nabla c_i(\mathbf{w}_{iC}^*) = \mathbf{0}. \tag{56}$$

Thus, the specific subsidy maintains the same equilibrium quality as no subsidy ($\mathbf{w}_{iA}^* = \mathbf{w}_{iB}^*$) because it does not distort the effective cost of quality improvement, while the *ad valorem* subsidy lowers the effective cost of quality improvement and thus leads to better quality in terms of WTP and the unit production cost (that is, $f(\mathbf{w}_{iA}^*) < f(\mathbf{w}_{iC}^*)$ and $c_i(\mathbf{w}_{iA}^*) < c_i(\mathbf{w}_{iC}^*)$).[27] Given the equilibrium qualities of all firms, Nash equilibrium prices are determined as in Section 2.1.

[Policy D]

Suppose that each firm's threshold $\overline{p}_i$ is determined by the government as a function of $\mathbf{w}_i$, i.e., $\overline{p}_i = \overline{p}(\mathbf{w}_i)$. Assuming a sufficiently generous subsidy scheme, we focus on Nash equilibria where all firms opt in and thus $\pi_{iD} = \pi_{iD_0}$. As above, consider maximizing $\pi_{iD_0}(\mathbf{w}_i, p_i, \{\mathbf{w}_j, p_j\}_{j\neq i}) = [(1-r)p_i + r\overline{p}(\mathbf{w}_i) - c_i(\mathbf{w}_i)]Q_i(h_i, \mathbf{h}_{-i})$ with respect to $\mathbf{w}_i$ and p_i. Assuming an interior solution, the following FOCs are satisfied:

$$\frac{\partial \pi_{iD_0}}{\partial w_i^k} = [r\frac{\partial \overline{p}(\mathbf{w}_i)}{\partial w_i^k} - \frac{\partial c_i(\mathbf{w}_i)}{\partial w_i^k}]Q_i(h_i, \mathbf{h}_{-i}) - [(1-r)p_i + r\overline{p}(\mathbf{w}_i) - c_i(\mathbf{w}_i)]\frac{\partial Q_i(h_i, \mathbf{h}_{-i})}{\partial h_i}\frac{\partial f(\mathbf{w}_i)}{\partial w_i^k} = 0 \quad \forall k, \tag{59}$$

$$\frac{\partial \pi_{iD_0}}{\partial p_i} = (1-r)Q_i(h_i, \mathbf{h}_{-i}) + [(1-r)p_i + r\overline{p}(\mathbf{w}_i) - c_i(\mathbf{w}_i)]\frac{\partial Q_i(h_i, \mathbf{h}_{-i})}{\partial h_i} = 0. \tag{60}$$

[27] From (54) and (56), $\mathbf{w}_{iA}^* \neq \mathbf{w}_{iC}^*$. Since $\mathbf{w}_{iA}^*$ is a unique maximizer of $f(\mathbf{w}_i) - c_i(\mathbf{w}_i)$,

$$f(\mathbf{w}_{iA}^*) - c(\mathbf{w}_{iA}^*) > f(\mathbf{w}_{iC}^*) - c(\mathbf{w}_{iC}^*). \tag{57}$$

Similarly, since $\mathbf{w}_{iC}^*$ is a unique maximizer of $(1+v)f(\mathbf{w}_i) - c(\mathbf{w}_i)$,

$$(1+v)f(\mathbf{w}_{iC}^*) - c_i(\mathbf{w}_{iC}^*) > (1+v)f(\mathbf{w}_{iA}^*) - c_i(\mathbf{w}_{iA}^*). \tag{58}$$

With (57) and (58), $(1+v)f(\mathbf{w}_{iC}^*) - c_i(\mathbf{w}_{iC}^*) > f(\mathbf{w}_{iC}^*) - c_i(\mathbf{w}_{iC}^*) + vf(\mathbf{w}_{iA}^*)$, so $f(\mathbf{w}_{iC}^*) > f(\mathbf{w}_{iA}^*)$. From (57), $c_i(\mathbf{w}_{iC}^*) - c_i(\mathbf{w}_{iA}^*) > f(\mathbf{w}_{iC}^*) - f(\mathbf{w}_{iA}^*) > 0$, so $c_i(\mathbf{w}_{iC}^*) > c_i(\mathbf{w}_{iA}^*)$.

(59) and (60) imply that the optimal quality $\mathbf{w}^*_{iD_0}$ satisfies

$$(1-r)\nabla f(\mathbf{w}^*_{iD_0}) - \nabla c_i(\mathbf{w}^*_{iD_0}) + r\nabla \overline{p}(\mathbf{w}^*_{iD_0}) = \mathbf{0}, \tag{61}$$

where it is assumed for simplicity that $\mathbf{w}^*_{iD_0}$ is uniquely determined.[28] (61) means that the policymaker can affect the realized quality by adjusting the gradient $\nabla\overline{p}(\cdot)$. For example, (54) and (61) imply that if $\overline{p}_i$ is independent of $\mathbf{w}_i$ (i.e., $\nabla\overline{p}(\mathbf{w}_i) = \mathbf{0}\ \forall \mathbf{w}_i$), then the optimal product quality (denoted by $\tilde{\mathbf{w}}^*_{iD_0}$) is lower than in the no-subsidy case in terms of WTP and the unit production cost: $f(\mathbf{w}^*_{iA}) > f(\tilde{\mathbf{w}}^*_{iD_0})$ and $c_i(\mathbf{w}^*_{iA}) > c_i(\tilde{\mathbf{w}}^*_{iD_0})$.[29] On the other hand, setting $\nabla\overline{p}(\mathbf{w}_i) = \nabla f(\mathbf{w}_i)\ \forall \mathbf{w}_i$ makes (54) and (61) equivalent and the equilibrium attributes remain the same with or without the subsidy ($\mathbf{w}^*_{iD_0} = \mathbf{w}^*_{iA}$), although such a rule may be impractical (or if externalities exist, even unnecessary for the social optimum).

As another illustration, consider $\overline{p}(\mathbf{w}_i)$ that is linear in each argument (and non-zero) (i.e., $\nabla\overline{p}(\mathbf{w}_i) = \mathbf{a}\ (\neq \mathbf{0})$ for all $\mathbf{w}_i$). This linear schedule induces the optimal quality $\mathbf{w}^*_{iD_0}$ such that $\overline{p}(\mathbf{w}^*_{iD_0}) > \overline{p}(\tilde{\mathbf{w}}^*_{iD_0})$,[29] so $\mathbf{w}^*_{iD_0}$ lies above the hyperplane through $\tilde{\mathbf{w}}^*_{iD_0}$ that is perpendicular to $\mathbf{a}$ (i.e., $\mathbf{a}\cdot(\mathbf{w}^*_{iD_0} - \tilde{\mathbf{w}}^*_{iD_0}) > 0$).[30] In addition, if the policymaker is interested in improving a particular attribute k (because of, for example, positive externalities associated with it), increasing a_k (the kth element of $\mathbf{a}$) raises its equilibrium quality (i.e., $\partial w^{k*}_{iD_0}/\partial a_k > 0$).[31]

With $\{\mathbf{w}^*_{iD_0}, c_i(\mathbf{w}^*_{iD_0})\}_{i=1,\cdots,n}$ determined by (61), Nash equilibrium prices, which satisfy (60) for all i, are determined as discussed in Section 2.1. Importantly, whereas the *gradient* $\nabla\overline{p}(\cdot)$ appears in (61), the *level* $\overline{p}(\cdot)$ does not. As a result, (61) does not restrict the policymaker's ability to choose the level of $\overline{p}(\cdot)$ and influence the equilibrium prices $\{p^*_{iD_0}\}_{i=1,\cdots,n}$ through (60). Hence, augmenting Policy D by making $\overline{p}_i$ dependent on quality does not affect the previous results without product quality.

[28]For example, assume that r and $\overline{p}(\cdot)$ are set in such a way that keeps $(1-r)f + r\overline{p} - c_i$ strictly concave.

[29]This can be proved as in footnote 27.

[30]More generally, given a (possibly nonlinear) schedule $\overline{p}(\mathbf{w}_i)$ (where $\nabla\overline{p}(\tilde{\mathbf{w}}^*_{iD_0}) \neq \mathbf{0}$), the optimal quality $\mathbf{w}^*_{iD_0}$ satisfies $\overline{p}(\mathbf{w}^*_{iD_0}) > \overline{p}(\tilde{\mathbf{w}}^*_{iD_0})$, $\nabla\overline{p}(\tilde{\mathbf{w}}^*_{iD_0})\cdot(\mathbf{w}^*_{iD_0} - \tilde{\mathbf{w}}^*_{iD_0}) > 0$, and $\nabla\overline{p}(\mathbf{w}^*_{iD_0})\cdot(\mathbf{w}^*_{iD_0} - \tilde{\mathbf{w}}^*_{iD_0}) > 0$.

[31]Conditional on r, the optimal quality $\mathbf{w}^*_{iD_0}$ is determined by the FOC (61) with $\nabla\overline{p}(\mathbf{w}_i) = \mathbf{a}$. By the implicit function theorem, $\mathbf{w}^*_{iD_0}$ is expressed as a function of $\mathbf{a}$ (let $\mathbf{w}^*_{iD_0} = g(\mathbf{a})$), and $\nabla g(a) = -r[(1-r)\nabla^2 f(\mathbf{w}^*_{iD_0}) - \nabla^2 c_i(\mathbf{w}^*_{iD_0})]^{-1}$. As $\nabla^2 f(\mathbf{w}^*_{iD_0})$ is negative definite and $\nabla^2 c_i(\mathbf{w}^*_{iD_0})$ is positive definite, $\nabla g(a)$ is positive definite, so that the diagonal elements of $\nabla g(a)$ are positive (that is, $\partial w^{k*}_{iD_0}/\partial a_k > 0$).