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Procedia Structural Integrity 28 (2020) 1621–1628

Structural Integrity Procedia

www.elsevier.com/locate/procedia

1st Virtual European Conference on Fracture

Contact problems for cracks under impact loading

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Abstract

This paper concerns a fracture mechanics problem for elastic cracked materials under transient dynamic loading. The nonlinear contact problem for a linear crack under oblique Heaviside compression pulse is solved by the boundary integral equations method in the frequency domain, and the components of the solution are presented by the Fourier exponential series. The contact forces are calculated and the solution is analysed accounting for the friction. The dynamic stress intensity factors are computed at leading and trailing crack's tips and compared with those obtained neglecting the crack closure and friction.

Keywords: Crack; impact loading; contact; friction; boundary integral equations; stress intensity factors.

1. Introduction

It is well recognized in the literature that the crack closure effects and the friction between the crack faces must be taken into account when considering cracked engineering materials under dynamic loading, because the stress and displacement distribution in the vicinity of cracks changes not only quantitatively, but also qualitatively, see, e.g., Guz et al. (2003), Menshykov et al. (2008). However, the numerical solution of such problems is very complicated as the contact problems are nonlinear due to the nature of the contact and divergent integrals of different order and type should be regularized and computed.

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10.1016/j.prostr.2020.10.133

Nomenclature

a	half-length of the crack
c_1, c_2	velocities of the longitudinal and transversal waves
$\mathbf{p}(\mathbf{x},t)$	traction vector
$\mathbf{q}(\mathbf{x},t)$	vector of contact forces
$[\mathbf{u}(\mathbf{x},t)]$	vector of the displacement discontinuity
E	Yoing's elastic modulus
$H_{\beta}^{(1)}$	Hankel function of the first kind
α	angle of the loading
λ, μ	elastic Lame constants
ρ	material density
ν	Poisson's ratio
ω	frequency of the wave
Ω	middle surface of the crack

Linear crack under normal harmonic tension-compression loading was considered by Menshykov et al. (2005), where the crack faces contact interaction was taken into account and the problem was solved using Galerkin method. The investigation on how the stress intensity factor depends on the wave number was carried out, using piecewise-linear continuous elements in the method. Linear crack under oblique time-harmonic loading was studied by Menshykov et al. (2008), where crack's closure was considered with allowance of the friction. The problem was solved using boundary integral equation method, and incorporating into analysis friction effects governed by Coulomb law. The analysis of results obtained for various angles of incidence and different friction coefficients was carried out, and the results were compared with the ones obtained neglecting the cracks' closure.

The influence of the friction coefficient on K_{II} was also studied by Giner et al. (2011), where the fatigue contact problem was solved using X-FEM. In the paper authors considered two different approaches to set the faces' contact for the crack under cyclic loading, and the accuracy of K_{II} was assessed by various techniques.

Three-dimensional problem of the elliptical crack in homogeneous body under normally incident tension-compression wave was solved by Guz et al. (2003), and the opening mode of the stress intensity factor was studied. Rectilinear crack in homogeneous material under three different contact conditions was also studied by Ostrik (2019), who considered smooth, sliding slip and adhesion between the crack faces. Improved boundary integral equation method was used by Mykhas'kiv (2019) for analysis of time-harmonic longitudinal elastic wave penetration through a double-periodic array of penny-shaped cracks.

The problems of interface crack with contact faces under harmonic loading were considered by Menshykov et al. (2007), Men'shikov et al (2007), Menshykova et al. (2009) and Guz et al. (2009). To solve the problem the boundary integral equations method was used, and a system of boundary integral equations that allows evaluating the displacement and stress fields for an interfacial crack under harmonic loading and the expressions for the integral kernels were obtained. The influence of the frequency on displacements and tractions at the crack under normal tension-compression wave was studied by Menshykov et al. (2007), and under normal shear wave by Guz et al. (2009).

The effects of wave number, material properties, and the crack interfaces distance on the dynamic stress intensity factor were investigated by Mykhaskiv and Stankevych (2019) for the problem of torsion harmonic loading of penny-shaped crack in layered composite. The extension of the boundary integral equation method was used by Golub and Doroshenko (2019) to solve the problem of the elastic wave scattering by a doubly periodic array of planar delaminations of arbitrary shape. The solutions for two types of cracks, rectangular and elliptic, were presented in the paper.

Note that the special attention should be paid to the case of non-harmonic loading. In particular, the problems of impact loading were considered by many of researchers. Wuensche et al. (2009) analyzed two-dimensional crack under transient dynamic loading, comparing two hypersingular time-domain boundary element methods. A combination of the classical displacement boundary integral equations and the hypersingular traction boundary integral equations was used to solve the problem and the analysis of dynamic stress intensity factors was presented. 3D time-domain formulation of boundary element method was implemented to obtain the solution of impact loading of finite elastic cracked members problem by Agrawal and Kishore (2001), and Agrawal (2002). The computation of the critical intersection angle for straight and curved cracks was performed and the influence of free surface on the distribution of stress intensity factor along the crack-front was investigated.

The dynamic stress intensity factors for different stress pulses were computed by Menshykova et al. (2016) for cracked homogeneous materials. In the paper the components of solution were presented by the Fourier exponential series, solving the problem by boundary integral equations in frequency domain. Piezoelectric cracked solids under dynamic transient load were analyzed by Garcia-Sanchez et al. (2007) and Zhao et al. (2015). For linear crack of finite length in infinite body the dynamic stress intensity factors were calculated. To solve the problem the regular integrals were calculated numerically and singular and hypersingular integrals were taken analytically. The sawtooth shock pulse problem was considered by Zhang et al. (2020), investigating the factors that are likely to influence the dynamic stress.

The normal impact loading of the linear interface cracks was also considered in Menshykov et al (2020a), where the analysis of the stress intensity factors (opening and transverse shear modes) dependence on the bimaterial properties was carried out. In Menshykov et al (2020b) the problem for the normal transient loading of the linear crack in homogeneous material was solved for the first time taking the friction into account (under some specific assumptions made for the distribution of the normal contact forces in order to test the adapted iterative algorithm initially developed for interface cracks in Menshykova et al (2011). The current study is devoted to oblique impact loading of interface crack in homogeneous material. The actual distributions of the contact forces are computed and used to obtain the solution satisfying contact constraints (unilateral normal contact and the Coulomb friction law).

2. Boundary integral equations

Let us consider a two-dimensional homogeneous, isotropic linearly elastic material under external dynamic loading. The material contains a finite length linear crack without any initial opening and the Heaviside compression pulse propagates (with the velocity of the longitudinal wave) in the oblique direction to the surface of the crack, please see Fig. 1.

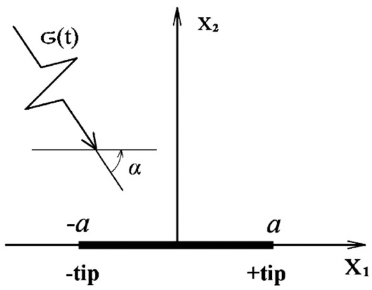


Fig. 1. Linear crack under oblique Heaviside compression pulse.

For an isotropic material the equation of motion and the generalized Hooke's law lead to the linear Lamé equations of elastodynamics for the displacement field with the appropriate initial and boundary conditions. The components of the displacement and tractions could be represented in terms of boundary displacements and tractions using the Somigliana dynamic identity and the appropriate fundamental solutions, see, for example, Menshykov et al (2008) and Menshykova et al (2016).

Furthermore, in order to use the methodology developed by authors in the frequency domain for cracked materials under harmonic loading, see Menshykova et al (2016), and Menshykov et al (2020a, 2020b) for the detailed literature reviews, the external transient dynamic load can be approximated by the Fourier exponential series. In particular, the Heaviside impact pulse $H(t)$ can be approximated by the repeating "steep and long" trapezoidal stress pulse:

$$\sigma(t) = \sigma^* \begin{Bmatrix} \frac{t}{t^*}(H(t) - H(t-t^*)) + (H(t-t^*) - H(t-t^*-t_d)) \\ +\left(2 - \frac{t-t_d}{t^*}\right)(H(t-t^*-t_d) - H(t-2t^*-t_d)) \end{Bmatrix}, \quad (1)$$

where $c_2t^*=0.1$ and $c_2t_d=12$, and the trapezoidal pulse can be approximated by the Fourier series with an appropriate number of the Fourier coefficients. According to Menshykova et al (2016) for the impact (or sharp pulse) loading at least 30 Fourier coefficients should be used to adequately approximate the Heaviside pulse. Some details of the solution convergence analysis with respect to the number of Fourier coefficients are also given in Menshykov et al (2020a) for linear interface cracks with the recommended number of Fourier coefficients being equal to 50. For the consistency sake, in the current study we will use 50 Fourier coefficients to represent the external loading and the components of the solution.

The normal and tangential components of the displacement discontinuity vector, $[\mathbf{u}(\mathbf{x},t)] = \mathbf{u}^{(1)}(\mathbf{x},t) - \mathbf{u}^{(2)}(\mathbf{x},t)$, and the traction vector at the crack surface can be approximated by the following trigonometric Fourier series with respect to the time:

$$p_j(\mathbf{x},t) = \frac{p^0_{j,cos}(\mathbf{x})}{2} + \sum_{k=1}^{+\infty}\left(p^k_{j,\cos}(\mathbf{x})\cos(\omega_k t)\right) + \left(p^k_{j,\sin}(\mathbf{x})\sin(\omega_k t)\right), \quad (2)$$

$$[u_j(\mathbf{x},t)] = \frac{[u^0_{j,\cos}(\mathbf{x})]}{2} + \sum_{k=1}^{+\infty}\left([u^k_{j,\cos}(\mathbf{x})]\cos(\omega_k t)\right) + \left([u^k_{j,\sin}(\mathbf{x})]\sin(\omega_k t)\right), \quad (3)$$

where $\mathbf{x} \in \Omega$, $\omega_k = 2\pi\, k/T$, $j = 1,2$ and

$$p^k_{j,\cos}(\mathbf{x}) = \frac{\omega}{2\pi}\int_0^T p_j(\mathbf{x},t)\cos(\omega_k t)\,dt, \quad p^k_{j,\sin}(\mathbf{x}) = \frac{\omega}{2\pi}\int_0^T p_j(\mathbf{x},t)\sin(\omega_k t)\,dt, \quad (4)$$

$$[u^k_{j,\cos}(\mathbf{x})] = \frac{\omega}{2\pi}\int_0^T [u_j(\mathbf{x},t)]\cos(\omega_k t)\,dt, \quad [u^k_{j,\sin}(\mathbf{x})] = \frac{\omega}{2\pi}\int_0^T [u_j(\mathbf{x},t)]\sin(\omega_k t)\,dt, \quad (5)$$

$$k \in N_0 = 0,1,\ldots,+\infty.$$

Thus, the system of boundary integral equations can be represented as follows:

$$p^k_{j,\cos}(\mathbf{x}) - ip^k_{j,\sin}(\mathbf{x}) = -\sum_{m=1}^{2}\int_\Omega \left(F^{Re}_{mj}(\mathbf{x},\mathbf{y},\omega_k) + iF^{Im}_{mj}(\mathbf{x},\mathbf{y},\omega_k)\right)\left([u^k_{m,\cos}(\mathbf{y})] - i[u^k_{m,\sin}(\mathbf{y})]\right)d\mathbf{y}, \quad (6)$$

$$\mathbf{x} \in \Omega, \quad k \in N_0, \quad j = 1,2,$$

where i is the imaginary unit; and the real and the imaginary parts of the integral kernel $F_{mj}\,(\mathbf{x},\mathbf{y},\omega_k)$ can be obtained from the fundamental displacement

$$U_{ij}\,(\mathbf{x},\mathbf{y},t-\tau) = \frac{1}{2\pi\mu}\left(\psi\delta_{ij} - \chi\frac{(y_i-x_i)}{r}\frac{(y_j-x_j)}{r}\right), \quad (7)$$

applying the following differential operator with respect to $\mathbf{x}$ and $\mathbf{y}$

$$P_{ik}[\bullet,(\mathbf{y})] = \lambda n_i(\mathbf{y})\frac{\partial[\bullet]}{\partial y_k} + \mu\left[\delta_{ik}\frac{\partial[\bullet]}{\partial \mathbf{n}(\mathbf{y})} + n_k(\mathbf{y})\frac{\partial[\bullet]}{\partial y_i}\right]. \quad (8)$$

For a linear crack the integral kernels in (6) can be written as:

$$F_{12}(\mathbf{x},\mathbf{y},\omega_k) = F_{21}(\mathbf{x},\mathbf{y},\omega_k) = 0, \quad (9)$$

$$F_{11}(\mathbf{x},\mathbf{y},\omega_k)=\frac{i\mu}{4}\left\{\begin{array}{c}H_0^{(1)}\left(\frac{\omega_k r}{c_2}\right)-4\frac{\omega_k}{c_2 r}H_1^{(1)}\left(\frac{\omega_k r}{c_2}\right)+\\ 4\frac{\mu}{\lambda+2\mu}\frac{\omega_k}{c_1 r}H_1^{(1)}\left(\frac{\omega_k r}{c_1}\right)+\frac{12}{r^2}\left(H_2^{(1)}\left(\frac{\omega_k r}{c_2}\right)-\frac{\mu}{\lambda+2\mu}H_2^{(1)}\left(\frac{\omega_k r}{c_1}\right)\right)\end{array}\right\}, \tag{10}$$

$$F_{22}(\mathbf{x},\mathbf{y},\omega_k)=\frac{i\mu}{2}\left\{\begin{array}{c}\left[\frac{\lambda^2}{(\lambda+2\mu)^2}\left(\frac{\omega_k}{c_2}\right)^2 H_0^{(1)}\left(\frac{\omega_k r}{c_1}\right)+2\frac{\omega_k}{c_2 r}H_1^{(1)}\left(\frac{\omega_k r}{c_2}\right)+\right.\\ 2\frac{\omega_k}{c_1 r}\frac{\lambda}{\lambda+2\mu}H_1^{(1)}\left(\frac{\omega_k r}{c_1}\right)-\frac{6}{r^2}\left(H_2^{(1)}\left(\frac{\omega_k r}{c_2}\right)-\frac{\mu}{\lambda+2\mu}H_2^{(1)}\left(\frac{\omega_k r}{c_1}\right)\right)\end{array}\right\}, \tag{11}$$

where $r=|x_1-y_1|$ is the distance between the observation and load points. The detailed expressions for the real and imaginary parts of integral kernels are given in Menshykov et al (2008).

3. Contact interaction and iterative algorithm

Due to the crack's closure the traction vector at the crack surface is the superposition of the initial traction caused by the incident load and the contact force, $\mathbf{q}(\mathbf{x},t)$, that arises in the contact region, which is generally unknown beforehand, depends on the direction of the loading, changes in time under deformation of the material and must be determined as a part of the solution.

To include the contact interaction into account, the Signorini unilateral constraints and the Coulomb friction law must be imposed for the normal and tangential components of the contact force and the displacement discontinuity, Menshykov et al (2008), and Menshykova et al (2011):

$$[u_n(\mathbf{x},t)]\geq 0,\quad q_n(\mathbf{x},t)\geq 0,\quad [u_n(\mathbf{x},t)]q_n(\mathbf{x},t)=0, \tag{12}$$

$$|\, q_\tau(\mathbf{x},t)\,|<k_\tau q_n(\mathbf{x},t)\Rightarrow\frac{\partial[u_\tau(\mathbf{x},t)]}{\partial t}=0, \tag{13}$$

$$|\, q_\tau(\mathbf{x},t)\,|=k_\tau q_n(\mathbf{x},t)\Rightarrow\frac{\partial[u_\tau(\mathbf{x},t)]}{\partial t}=-\frac{q_\tau(\mathbf{x},t)}{|q_\tau(\mathbf{x},t)|}\left|\frac{\partial[u_\tau(\mathbf{x},t)]}{\partial t}\right|,\quad \mathbf{x}\in\Omega\,,t\in[0;T]. \tag{14}$$

The contact constraints above ensure that there is no interpenetration of the opposite crack faces; the normal component of the contact force is unilateral, and the opposite crack faces remain immovable with respect to each other in tangential direction while they are held by the friction force before the slipping occurs.

As the first step, the solution of elastodynamic problem for the cracked material neglecting the effect of the crack closure is obtained. Then the correction of the solution is performed applying the constraints (12)–(14) and the Fourier coefficients are changed until the solution satisfying the constraints is found. Details of the algorithm and the analysis of its convergence for different friction coefficients for the case of impact loading can be found in Menshykov et al (2008), Menshykova et al (2011), Menshykov et al (2020b).

4. Numerical results

For the validation of the numerical model the normal shear loading of unit amplitude was considered. The material has the properties of steel: E = 200 GPa, ν = 0.25, ρ = 7800 kg/m^3. The dynamic stress intensity factor (shear mode normalized by the static value) is presented in Fig. 2. As it was concluded in Menshykov et al (2020b) the magnitude of K_{II} depends on the friction and significantly decreases with the rise of the friction coefficient. At the same time, this problem may be considered as a rather artificial one since the assumption made for the normal component of the contact force being constant to test the iterative algorithm accounting for friction.

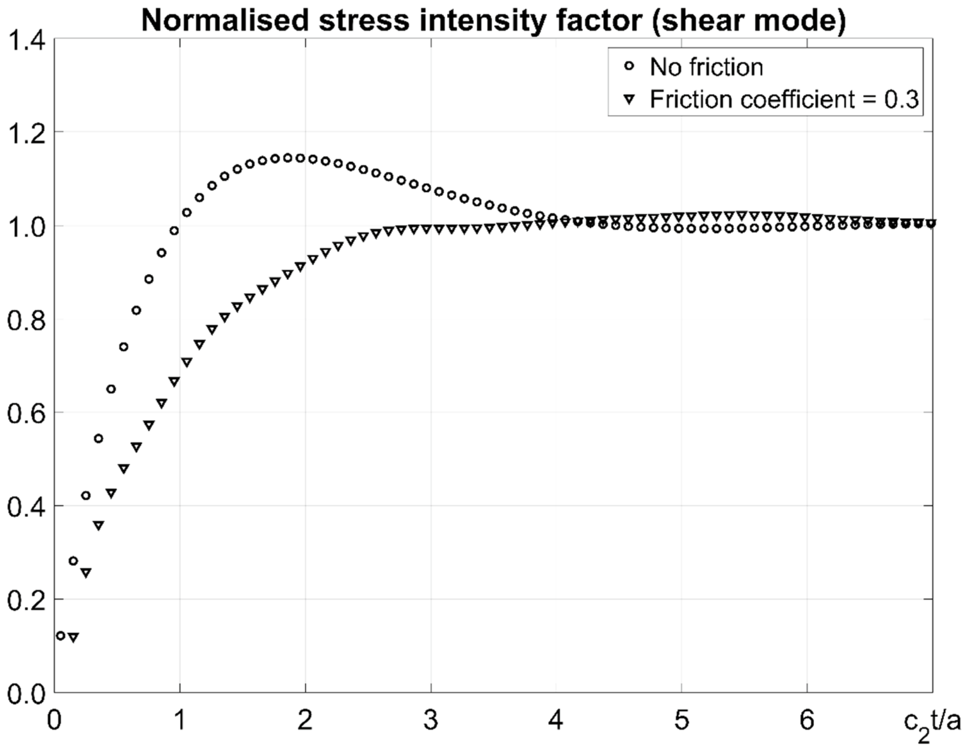


Fig. 2. Stress intensity factor (shear mode) plotted against the normalized time, normal loading.

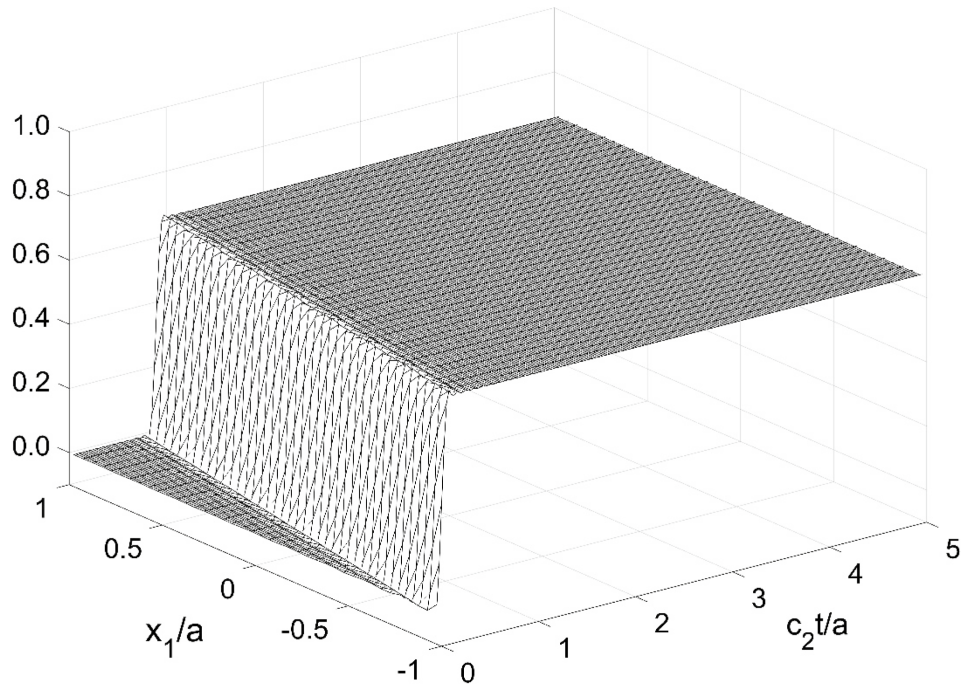
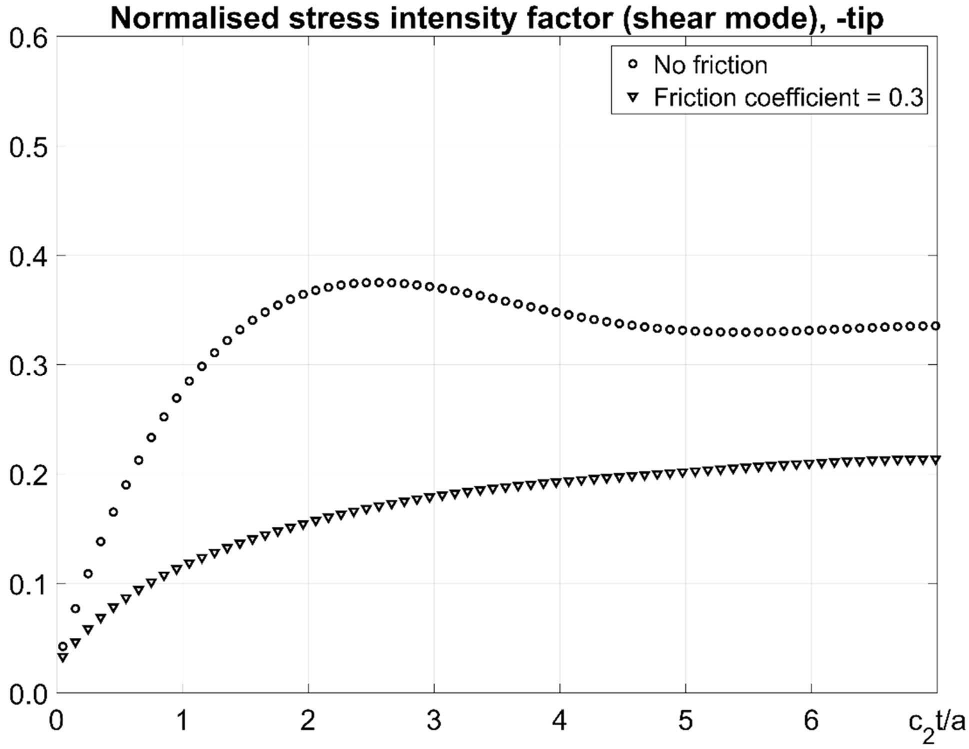


Fig. 3. Normal contact forces at the crack surface plotted against the normalized time.



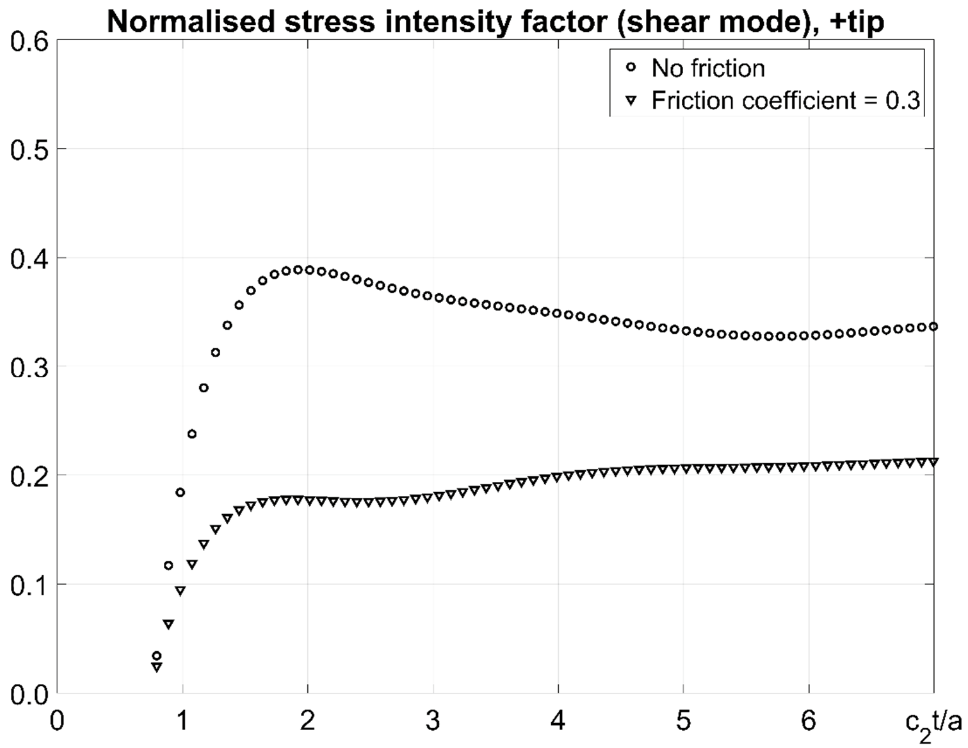


Fig. 4. Stress intensity factor (shear mode) plotted against the normalized time both crack tips.

As a numerical example, let us consider an incident pulse of unit amplitude propagating at $\alpha = 45^0$. As discussed above, in the current study the actual distribution of the contact forces was obtained, see Fig. 3, and used.

Due to the nature of the problem the crack is constantly closed, so the first mode of the stress intensity factor is absent. The normalized shear modes of dynamic stress intensity factors at both crack tips are presented in Fig. 4. The maximal values of K_{II} do not coincide and are achieved at different times, so the responses at the leading and trailing crack tips are very different due to the non-symmetry of the problem (similarly to the case of oblique harmonic loading, considered in Menshykov et al (2008). It should be also noted that the friction significantly affected the solution, especially when comparing to the case of the normal shear loading.

Considering the crack closure and friction for the case of interface cracks, where both normal opening and shear modes of the stress intensity factor are present for any type and direction of the loading, will be the next stage of this study.

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