The zeta potential is a measureable electrical potential of paramount importance to understand the electrochemical properties of rocks. However, the zeta potential remains poorly understood because it takes place at the nanoscale of the electrical double layer on the mineral surface. Streaming potential measurements on quartz-rich Fontainebleau and Lochaline sandstones carried out at high salinity (above 0.1 M NaCl) yield surprisingly high zeta potential values, which cannot be correctly reproduced by a traditional surface complexation model considering that the shear plane is located at the beginning of the diffuse layer. We found that placing the shear plane, where the zeta potential is defined, slightly closer to the mineral surface than the Stern plane significantly improves the predictions of the zeta potential and surface charge density of quartz at high salinity as well as the values of the equilibrium constant describing sodium adsorption in the Stern layer and Stern layer capacitance. Therefore, they may be a non-zero flow of water and counter-ions within the Stern layer of quartz in contact with NaCl solution. Our results have strong implications for the modelling of the electrochemical properties of minerals in contact with highly saline solutions.
The zeta potential of quartz.

Surface complexation modelling to elucidate high salinity measurements

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Abstract

The zeta potential is a measureable electrical potential of paramount importance to understand the electrochemical properties of rocks. However, the zeta potential remains poorly understood because it takes place at the nanoscale of the electrical double layer on the mineral surface. Streaming potential measurements on quartz-rich Fontainebleau and Lochaline sandstones carried out at high salinity (above 0.1 M NaCl) yield surprisingly high zeta potential values, which cannot be correctly reproduced by a traditional surface complexation model considering that the shear plane is located at the beginning of the diffuse layer. We found that placing the shear plane, where the zeta potential is defined, slightly closer to the mineral surface than the Stern plane significantly improves the predictions of the zeta potential and surface charge density of quartz at high salinity as well as the values of the equilibrium constant describing sodium adsorption in the Stern layer and Stern layer capacitance. Therefore, they may be a non-zero flow of water and counter-ions within the Stern layer of quartz in contact with NaCl solution. Our results have strong implications for the modelling of the electrochemical properties of minerals in contact with highly saline solutions.

Key words: zeta potential, quartz, streaming potential, high salinity, shear plane location
1. Introduction

Quartz is a mineral that is particularly interesting to study because of its natural abundance and usefulness in the development of new technologies [1]. In contact with water, quartz develops a surface charge attracting counter-ions and repelling co-ions, thus forming the so-called electrical double layer (EDL), usually represented by a “compact” Stern layer and a diffuse layer [2, 3]. Investigating the electrochemical properties of quartz is of great interest in many applications in physics, chemistry and Earth sciences because these properties control adsorption and dissolution/precipitation reactions, and wettability on the quartz surface [4-6]. The EDL of quartz is also the source of electrokinetic and geophysical electrical (e.g., self-potential, resistivity, induced polarization) measurements that are used to map for instance geological fluid flows or biogeochemical reactions [7-12]. Studying quartz electrochemical properties, notably when quartz is in contact with highly saline brines, has a high potential in many geo-environmental and engineering applications including geo-sequestration of CO₂ in deep saline aquifers and enhanced hydrocarbon recovery [13-16].

Exploring the electrochemical properties of quartz is very challenging because of their nanoscopic nature [1, 17, 18]. Indeed, surface complexation reactions between surface sites and ions in the aqueous solution occur at the nm-scale [3, 19-21]. In addition, natural quartz has a low specific surface area (typically below 0.1 m² g⁻¹), which considerably complicates the direct experimental characterization of its EDL compared to minerals with a large specific surface area such as montmorillonite [2, 3, 22, 23]. Only few methods exist to probe the properties of the EDL on the surface of minerals in contact with brines. Among them, there is the streaming potential method, which implies application of a water pressure difference across the sample while measuring the resulting voltage, the streaming potential, due to the displaced excess counter-ions in the EDL [24-
From the measured streaming potential it is possible to obtain some relevant information on the electrochemical properties of minerals through the calculation of the electrokinetic zeta potential ($\zeta$), which is defined as the electrical potential at the shear (or slip) plane [24, 26]. The zeta potential determined experimentally can be interpreted in terms of mineral electrochemical properties by matching observed and simulated zeta potential using a relevant surface complexation model [20, 29, 30]. However, this approach relies on the assumption that the exact location of the shear plane from the mineral surface is known, which is obviously not the case because of the lack of experimental information at the molecular level [31-34]. Moreover, the zeta potential is, most of the times, the only physico-chemical quantity available to validate the predictions of electrostatic surface complexation models for low specific surface area minerals such as quartz or calcite [3, 29, 35]. In addition, the zeta potential is inferred from electro-hydrodynamic measurements while surface complexation models rely on electrostatics at thermodynamic equilibrium [24, 34, 36, 37]. Therefore, these limitations contribute to additional uncertainties when investigating mineral electrochemical properties from zeta potential measurements.

When water flow relative to the mineral surface takes place, it is widely accepted that the shear plane is located between the “stagnant” Stern layer bounded by the outer Helmholtz plane (OHP) and the diffuse layer because high water viscosity in the Stern layer prevents water flow within it [3, 24, 38]. The Stern layer of silica-based materials such as amorphous silica and quartz in contact with a NaCl solution is traditionally represented by a hydration layer followed by a layer containing hydrated sodium counter-ions [17, 20, 39]. Some molecular dynamic (MD) simulations (e.g., Zhang et al. [32]), spectroscopy measurements (e.g., Lis et al. [40]) and microfluidic studies (e.g., Saini et al. [41] and Werkhoven et al. [42]) have demonstrated that there could be a non-zero flow of water within the Stern layer of amorphous silica because some counter-ions form outer sphere...
surface complexes. This implies that there may be some water displacement within the Stern layer of amorphous silica and quartz. Therefore, the effective shear plane of these materials may be located closer to the mineral surface than the outer Helmholtz plane (Figure 1), in disagreement with the assumption accepted by most that the shear plane is located at the OHP (e.g., Sverjensky [3]) or beyond it (e.g., Alizadeh and Wang [43]).

**Figure 1.** Sketch showing water flow and ion distribution at the interface between a silica mineral and a NaCl aqueous solution (modified, from Brown et al. [39]). Circles with arrows inside represent water molecules. The shear plane is denoted by the red dashed line. Counter-ions adsorbed as outer sphere complexes form the outer Helmholtz plane.

The quartz (0001) crystal face is the most stable plane with the lowest surface energy and is often considered as a “model surface”, convenient for modelling SiO\(_2\) materials and hydrophilic surfaces in general [34]. With the improved accuracy of the streaming potential method, it is now possible to accurately measure extremely small voltages due to the displacement of the ions in the EDL of
quartz [14]. Published studies of Jaafar et al. [8], Vinogradov et al. [13], and Walker and Glover [15] observed, that at high salinities (NaCl concentrations above 0.4 M, M means mol L\(^{-1}\)), the zeta potential of sandstones appears to level off at a small constant negative value between \(-30\) and \(-10\) mV or even to increase slightly in magnitude (i.e. become more negative) with salinity. They noted that the zeta potential of sandstones stabilizes at a salinity of about 0.4 M NaCl that corresponds to a Debye length characterizing the diffuse layer thickness of approximately 0.47 nm, which is similar to the size of a hydrated sodium ion. This observation led them to suggest that the constant zeta potential of sandstones at high salinities reflected the maximum charge density in the diffuse layer which was reached when the diffuse layer thickness approached the diameter of the counterions. However, Jaafar et al. [8], Vinogradov et al. [13], and Walker and Glover [15] did not explicitly explain this behavior through a surface complexation model describing their zeta potential measurements on sandstones.

In our study, we used a surface complexation model named basic Stern model (BSM) and considered that the shear plane is at the OHP or closer to the mineral surface than the OHP to describe the zeta potential and the electrochemical properties of quartz at varying NaCl concentrations. In our model we described the effective location of the OHP and the shear plane (i.e., the average distances from topologically complex grains surfaces), hence modelling the effective zeta potential. Therefore, the developed surface complexation model accurately replicated the experimental conditions under which the streaming potential measurements on intact rock samples comprising grains of various shape and roughness were conducted. The model predictions were compared with the existing experimental zeta potential data on Fontainebleau and Lochaline rocks measured over a broad salinity range (from around \(10^{-4}\) M NaCl up to around 5.5 M NaCl). The values of the optimized model parameters were finally discussed. Our findings shed light on
the electrochemical properties of quartz and on the likelihood of non-zero water flow within the Stern layer.

2. Theoretical background

2.1. Surface complexation model for quartz

Our basic Stern model, BSM, [36, 44] describes proton (H\(^+\)) adsorption onto >SiO\(^-\) surface sites at the 0-plane (defining the mineral surface) and sodium cation (Na\(^+\)) adsorption by these surface sites at the \(\beta\)-plane (Stern plane and OHP) (Figure 2) [3, 6, 19, 20, 22]. The BSM considers that the \(\beta\)-plane coincides with the \(d\)-plane defining the start of the diffuse layer (not the shear plane). This model only needs one Stern layer capacitance as an input parameter to model the electrical potential distribution between the mineral surface and the Stern plane. Recent studies utilizing atomic force microscopy (AFM) (e.g., Siretanu et al. [45]) and X-ray photoelectron spectroscopy (XPS) (e.g., Brown et al. [39]) used BSM to model the electrochemical properties of amorphous silica in contact with a NaCl aqueous solution and demonstrated that BSM could accurately reproduce the experimental data. García et al. [6] also used BSM to match the measured electrochemical properties of quartz in contact with a NaCl aqueous solution thus confirming the validity of the approach.
Figure 2. Sketch of our basic Stern model to describe the electrochemical properties of the interface between quartz and a 1:1 electrolyte like NaCl electrolyte. The model input parameters are shown in blue and the model output parameters, including the zeta potential ($\zeta$), are shown in red.

In our BSM we used four adjustable parameters, namely the logarithms of the two adsorption equilibrium constants $K_H$ and $K_{Na}$, the Stern layer capacitance $C_1$ (F m$^{-2}$), and the distance $d$ between the shear plane (where the zeta potential is defined) and the $\beta$-plane (Figure 2). It should be noted that we considered that the doubly coordinated surface groups (>Si$_2$O$^0$) are inert [19] and that the protonated silanol sites (>SiOH$_2^+$) are not expected to form at close-to-neutral pH of the streaming potential measurements on sandstones (pH varied between 6.4 and 7.3 in Walker and Glover [15]). Therefore, these surface sites were excluded from the model. For detailed information on our BSM, the reader can refer to Appendixes A and B, and to Leroy et al. [20].
2.2. Zeta potential computation

All calculations were performed by combining the geochemical software IPhreeqc for the surface complexation modelling [46] with an in-house code implemented in Matlab for the calculation of the zeta potential and the optimization procedure [47]. The zeta potential (V) defined at the shear plane located at a distance \( d \) from the \( \beta \)-plane was determined from the computed by IPhreeqc \( \varphi_0 \) and \( \varphi_\beta \) electrical potentials by considering a linear, capacitor-like variation of the electrical potential within the Stern layer [24]

\[
\zeta = \varphi_\beta - \left( \frac{\varphi_\beta - \varphi_0}{x_\beta - x_0} \right) d,
\]

(1)

where \( x \) is the distance from the mineral surface (defined by the 0-plane, in m). Combining equation (1) with the following equation for the Stern layer capacitance [17]

\[
C_1 = \frac{\varepsilon_1}{x_\beta - x_0},
\]

(2)

where \( \varepsilon_1 \) is the water permittivity in the Stern layer (F m\(^{-1}\); we used \( \varepsilon_1 = 43\varepsilon_0 \), where \( \varepsilon_0 \) is the vacuum permittivity, in accordance with the study of Sverjensky [3]), we finally obtain an expression for the zeta potential as a function of the modelled electrochemical properties

\[
\zeta = \varphi_\beta - (\varphi_\beta - \varphi_0) \frac{C_1}{\varepsilon_1} d.
\]

(3)

We did not consider the presence of a stagnant diffuse layer (also named buffer layer), which implies that the shear plane is located further away from the mineral surface, as suggested in Alizadeh and Wang [43]. To the best of our knowledge, the stagnant diffuse layer existence has
never been directly confirmed experimentally. To the contrary, Předota et al. [34], Brkljača et al. [18], and Biriukov et al. [33] predicted no such stagnant diffuse layer from their molecular dynamic simulations of the zeta potential of the hydroxylated (110) rutile (TiO$_2$) and (0001) quartz surfaces. Furthermore, Leroy and co-workers (e.g., Leroy et al. [48], Leroy et al. [49], Leroy et al. [20], Li et al. [29]) attributed the assumption of the presence of a stagnant diffuse layer in previous studies to the misinterpretation of the zeta potentials from electrokinetic (e.g., electrophoretic mobility, streaming potential) measurements due to disregard of surface conductivity effects. Indeed, surface conductivity decreases the magnitude of the measured electrokinetic signal hence implying smaller apparent zeta potentials, the effect that can be attributed to a shift of the shear plane further away from the mineral surface when modelling the zeta potential from a surface complexation model.

The parameters of our surface complexation model (log $K_H$, log $K_{Na}$, $C_1$, $d$) were optimized by minimizing the following cost function [50]:

$$y = 1 - R^2 = \frac{\sum_{i=1}^{N} (\zeta_{mes}^i - \zeta_{mod}^i)^2}{\sum_{i=1}^{N} (\zeta_{mes}^i - \langle \zeta_{mes} \rangle)^2},$$

where $R^2$ is the coefficient of determination, $N$ is the number of zeta potential measurements, $\zeta_{mes}^i$ is the $i$-th measured zeta potential, $\langle \zeta_{mes} \rangle$ is the arithmetic mean of the measured zeta potentials, and $\zeta_{mod}^i$ is the $i$-th modelled zeta potential. The fitting procedure was realized by using the simulated annealing algorithm to find the global minimum of the cost function (equation (4)), with a refinement using the simplex method at the end of the process [47].
3. Comparison with experimental data and discussion

3.1. Considerations of impact of pore space topology and grain roughness on EDL parameters

To test our model, we used the measured zeta potentials of Fontainebleau (F2D, F3Q, F4Q) and Lochaline (L3Q, L4Q) samples in contact with NaCl solutions of increasing salinity obtained by the streaming potential method and reported in Walker and Glover [15] (temperature = 20–25°C). These two rock types were selected as they are known to consist of more than 99% quartz (by weight) [13, 51]. Unlike Fontainebleau and Lochaline samples, zeta potentials of Berea and Boise sandstones reported by Walker and Glover [15] that contained up to 6% feldspar, 2% dolomite, and 8% clays for Berea rocks [52] and up to 13% clays for Boise rocks [53], were excluded from the simulation. Despite the fact that feldspar, dolomite, and clay content in Berea and Boise samples was relatively small, clays are known to line pore walls, thus making these complex minerals a main contributor to the electrochemical processes at the mineral-water interface and causing anomalous or even positive zeta potentials [54, 55]. Therefore, the experimental zeta potential data for Boise and Berea samples were deemed unapplicable for our model that considers only surface complexation reactions on quartz surface.

All Fontainebleau and Lochaline samples exhibited a negative zeta potential with its magnitude decreasing with increasing salinity (Figures 3 and 4). The zeta potentials of Lochaline samples were found to be of a larger magnitude than those of Fontainebleau samples. Scanning electron microscopy (SEM) micrographs of the tested samples showed that Fontainebleau rock had sharper-angled grains with larger surface roughness and smaller grains than Lochaline rock (Figure 5 from Walker and Glover [15]). According to Vinogradov et al. [14], pore space topology, grain shape, surface roughness and size, all may influence streaming potential measurements. The authors
suggested that rough rocks with small grains had smaller streaming and zeta potential magnitudes than round, smooth rocks with large grains because existence of sharp grain/pore corners and larger roughness would shift the effective shear plane further away from the mineral surface (read their section 4.2). Alroudhan et al. [56] used the same argument to explain why the zeta potential of colloidal suspensions measured by the electrophoretic mobility method was larger in magnitude than the zeta potential of rocks measured by the streaming potential method (see their Figure 10 and read the related discussion in their section 5.2). Schnitzer and Ripperger [57] and Drechsler et al. [58] also showed that increasing surface roughness changes the flow velocity distribution on the solid surface shifting the shear plane further away from the solid surface and decreases the streaming and zeta potential magnitudes. According to these observations, we expected different values of the surface complexation model parameters for Fontainebleau and Lochaline samples. Specifically, for the two tested samples, we allowed different values of the Stern layer capacitance \( C_1 \) and the distance \( d \) of the shear plane from the OHP (or Stern plane), which are very sensitive to the textural properties of rocks (\( C_1 \) depends on the thickness of the Stern layer, equation (2)).
Figure 3. Zeta potentials of Fontainebleau samples as a function of NaCl concentration. Circle symbols: experimental zeta potential data with the sample name corresponding to that used by Walker and Glover [15]; curves: model predictions.
Figure 4. Zeta potentials of Lochaline samples as a function of NaCl concentration. Circle symbols: experimental zeta potential data with the sample name corresponding to that used by Walker and Glover [15]; curves: model predictions.
Figure 5. SEM micrographs of Fontainebleau (a) and Lochaline (b) rocks (modified from Walker and Glover [15]).

Figures 3 and 4 demonstrate that below the concentration thresholds of around 0.1 M NaCl (Fontainebleau samples) and 1 M NaCl (Lochaline samples) (denoted by the vertical black dotted lines), the magnitude of the negative zeta potential decreases linearly with increasing salinity. On the other hand, the rate of decrease in the zeta potential magnitude with increasing salinity became smaller above these thresholds, i.e. it became non-linear, and eventually stabilized (or even slightly increased in magnitude) at a zeta potential value of approximately -15 mV for both rock types.
Such stabilization of the zeta potential was more apparent for Fontainebleau than for Lochaline samples. These observations were also qualitatively and quantitatively consistent with the data reported by Vinogradov et al. [13], Vinogradov et al. [14] and Walker and Glover [15], who stated that at high salinities, the measured zeta potential stabilized and became equal to $-13.01 \pm 0.48$ mV for Fontainebleau samples and to $-16.81 \pm 0.68$ mV for Lochaline samples.

Considering that Fontainebleau and Lochaline sandstones did not have the same pore space topology and textural properties, we first optimized separately the parameters of the surface complexation models for these two rock types. That is, a single model was developed for F2D, F3Q, F4Q combined data (Fontainebleau rocks) and a separate model was developed for L3Q, L4Q combined data (Lochaline rocks) to match the simulated zeta potentials to the observed data. We ran the classical model (where the shear plane was assumed to coincide with OHP) denoted by CM with the parameters $\log K_H$, $\log K_{Na}$, and $C_1$, and the new model denoted by NM with the parameters $\log K_H$, $\log K_{Na}$, $C_1$, and $d$ (the distance of separation between the shear plane and OHP), to investigate the effect of the proposed inward shift of the shear plane on the simulated zeta potential while assigning measured pH values to the respective rock samples as reported by Walker and Glover [15].

In parallel, we also used the BSM approach, but for Fontainebleau and Lochaline samples together (all five samples, F2D, F3Q, F4Q, L3Q, L4Q) to develop a unified surface complexation model for quartz in contact with a NaCl aqueous solution, denoted by UNM for the unified new and UCM for the unified classical models.
3.2. Comparison of the computed to the observed zeta potentials and discussion

Overall, both the NM and CM reproduced well the experimental zeta potential data for the entire salinity range (Figures 3 and 4). According to the surface complexation models, the observed negative zeta potential was due to the presence of the deprotonated silanol sites >SiO\(^-\) at the 0-plane (Figure 2). The optimized values of the equilibrium constant describing protonation of >SiO\(^-\) surface sites (\(K_H\), reaction (1)) equal to \(10^{7.3}\) and \(10^{7.2}\) for Fontainebleau and Lochaline samples, respectively, were found to be close or similar to the spectroscopically determined value of \(10^{7.2\pm0.2}\) and to the theoretical value of \(10^{7.5}\) using Pauling’s definition of formal bond valence for silica [19] (Table 1). In addition, our optimized values of \(K_H\) were found to be similar to the value of \(10^{7.2}\) determined by Sverjensky [3]. He used a triple layer model (BSM with an additional \(C_2\) capacitance between the Stern plane and the external boundary of the diffuse layer) to match surface charge density measurements inferred from acid base potentiometric titration on natural quartz in contact with NaCl solutions. Our models also explained why the zeta potential magnitude of Lochaline samples was larger, for the same salinity, than the zeta potential magnitude of Fontainebleau samples. Indeed, Lochaline samples had higher pH (i.e. less protons in solution) than Fontainebleau samples (7.1 versus 6.5 in average, respectively; Walker and Glover [15]) while having essentially identical \(\log K_H\) values (Lochaline had slightly lower \(\log K_H\) value), which resulted in Lochaline samples having larger number of deprotonated >SiO\(^-\) sites per nm\(^2\) of the surface and a higher negative surface charge density \(Q_0\) (equation (A5)) (Figure 6).
Table 1. BSM parameters and estimated Stern layer thickness for Fontainebleau and Lochaline sandstones.

<table>
<thead>
<tr>
<th>Symbols</th>
<th>Range</th>
<th>Fontainebleau</th>
<th>Lochaline</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>CM</td>
<td>NM</td>
<td>CM</td>
</tr>
<tr>
<td>$\log K_H$</td>
<td>[4 10]</td>
<td>7.3</td>
<td>7.3</td>
<td>7.2</td>
</tr>
<tr>
<td>$\log K_{Na}$</td>
<td>[-15 5]</td>
<td>-16</td>
<td>0.6</td>
<td>-21</td>
</tr>
<tr>
<td>$C_1$ (F m$^{-2}$)</td>
<td>[0.5 5]</td>
<td>3.2</td>
<td>1.3</td>
<td>1.8</td>
</tr>
<tr>
<td>$d$ (Å)</td>
<td>[0 10]</td>
<td>0</td>
<td>0.5</td>
<td>0</td>
</tr>
<tr>
<td>$d_{Stern}^2$ (Å)</td>
<td></td>
<td>1.2</td>
<td>2.8</td>
<td>2.1</td>
</tr>
</tbody>
</table>

1 Hiemstra et al. [19], Kitamura et al. [22], Sonnefeld et al. [59], Sverjensky [3], García et al. [6].

2 According to Eq. (2) and fitted $C_1$ values, considering $\varepsilon_i = 43\varepsilon_0$ and $d_{Stern} = x_{j0} - x_0$. 

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1 Hiemstra et al. [19], Kitamura et al. [22], Sonnefeld et al. [59], Sverjensky [3], García et al. [6]. 

2 According to Eq. (2) and fitted $C_1$ values, considering $\varepsilon_i = 43\varepsilon_0$ and $d_{Stern} = x_{j0} - x_0$. 

---

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Figure 6. Computed surface site densities of $>\text{SiO}^-$ sites (a), $>\text{SiO}^-\text{Na}^+$ sites (b), and respective surface charge densities (c) of Fontainebleau and Lochaline samples as a function of NaCl concentration. Solid curves correspond to the calculations using the NM, dashed lines...
correspond to the calculations using the CM. The CM predicted near-zero $>\text{SiO}^- - \text{Na}^+$ surface site densities in the Stern layer (limited at $\leq 0$ sites nm$^{-2}$ in Figure 6b).

We also found that Lochaline samples had significantly lower $\log K_{\text{Na}}$ values, i.e. weaker sodium adsorption capacity, than Fontainebleau samples (-21 vs -16, respectively, for CM and 0.1 vs 0.6, respectively, for NM, Table 1). This result implies that $>\text{SiO}^- - \text{Na}^+$ surface sites of Lochaline could not counterbalance the negative surface charge density as efficiently as for Fontainebleau samples, and can also explain the larger zeta potential magnitude of Lochaline samples. Interestingly, despite Lochaline samples having lower $\log K_{\text{Na}}$ values than Fontainebleau samples, the models found that Lochaline samples, for the same salinity, had a higher surface site density of adsorbed sodium ion in the Stern layer than Fontainebleau samples due to the higher $>\text{SiO}^-$ surface site density (Figure 6b). The lower $\log K_{\text{Na}}$ values of Lochaline than Fontainebleau samples we found can be explained by Lochaline samples having smoother and larger grains and hence a smaller specific surface area available for sodium adsorption. Sverjensky [3] reported the same observation when comparing two quartz samples with different specific surface area (4.15 and 11.4 m$^2$ g$^{-1}$) in contact with NaCl solutions.

At the low salinity domain, $K_{\text{Na}}$ did not have a noticeable impact on the modelled zeta potential controlled primarily by $K_H$. With the CM, the optimization procedure required a decrease of $K_{\text{Na}}$ to extremely low value ($\log K_{\text{Na}} = -16$ or smaller) to fit the high salinity zeta potential measurements (decreasing Na$^+$ adsorption in the Stern layer results to higher modelled zeta potential magnitude). The $K_{\text{Na}}$ values inferred from our CM were extremely low and essentially
meant that there was no adsorption of Na\textsuperscript{+} at the OHP at all, and everything was controlled solely by pH. With the NM, the optimization procedure did not require to decrease $K_{Na}$ to extremely low value to fit the high salinity zeta potential measurements and it found $\log K_{Na}$ values (0.6 and 0.1 for Fontainebleau and Lochaline samples, respectively) to be of the same order of magnitude as the value reported by Sverjensky [3] for natural quartz in a contact NaCl solutions ($\log K_{Na} = 0$).

In addition, and contrary to the CM, our NM was able to reproduce most of the surface charge density measurements on Min-U-Sil 5 quartz (natural quartz with a mean grain diameter of 5 µm) at different pH and NaCl concentrations carried out by Riese [60] (Figure 7).

![Surface charge density of Min-U-Sil 5 quartz as a function of pH and NaCl concentration.](image)

**Figure 7.** Surface charge density of Min-U-Sil 5 quartz as a function of pH and NaCl concentration. Curves correspond to the predictions. Symbols correspond to the experimental surface charge density data reported by Riese [60].
Moreover, the optimized Stern layer capacitance values obtained with our NM were equal to 1.3 F m\(^{-2}\) and 2.2 F m\(^{-2}\) for Fontainebleau and Lochaline samples, respectively (Table 1), which were closer to the values of 1 F m\(^{-2}\) and 2 F m\(^{-2}\) reported by Sverjensky [3] and García et al. [6], respectively, for natural quartz in contact with NaCl solution. Conversely, the optimized Stern layer capacitance values obtained with the CM were 3.2 F m\(^{-2}\) and 1.8 F m\(^{-2}\) for Fontainebleau and Lochaline samples, respectively. Using the optimized Stern layer capacitance values from the NM, equation (2) and \(e' = 43e_0\) [3, 39], we found the Stern layer thickness to be comparable to the hydrated radius of sodium ion (\(\approx 2\) Å; Leroy et al. [61], Sverjensky [17]). Specifically, the NM predicted Fontainebleau samples to have larger Stern layer thickness (2.8 Å) than Lochaline samples (1.7 Å), the results that is explained by sharper and rougher grains of Fontainebleau samples compared to those of Lochaline [15, 62], which push further away the shear plane from the mineral surface and also explain the smaller zeta potential magnitude of Fontainebleau samples. When using the CM, the Stern layer thickness we found for Fontainebleau samples (1.2 Å) was comparable to the crystallographic radius of sodium ion (1.02 Å Sverjensky [17]). This result was not realistic for the representation of the quartz/NaCl solution interface, which, according to most studies (e.g., Brown et al. [63]), should contain mostly hydrated sodium ions in the Stern layer, corresponding to a Stern layer thickness at least equal to the hydrated radius of sodium ion. For Lochaline samples, the Stern layer thickness inferred from the CM (2.1 Å) was comparable to the hydrated radius of sodium ion.

Figures 3, 4, 7, the modelling parameters reported in Table 1 and the coefficient of determination values reported in Table 2 clearly demonstrate the importance of considering the location of the shear plane closer to the mineral surface than the OHP. Indeed, as shown in Figures 3 and 4 and reflected by the values of the coefficient of determination at high salinity \((R^2 \geq 0.5)\), the
stabilization of the zeta potential at high salinity could only be correctly predicted by the NM (red curves in Figures 3 and 4). The stabilization of the modelled zeta potential at high salinity is explained by a growing abundance of sodium ions available for adsorption in the Stern layer, and therefore the decreasing number of $\text{SiO}^-$ sites (Figures 6a and 6b), and importantly by the shear plane being located slightly closer to the mineral surface than the OHP. Moreover, the NM reproduced the surface charge density measurements on natural quartz in NaCl solutions reported in Riese [60] significantly better than the CM (Figure 7) thus independently validating our assumption on the location of the shear plane.

Table 2. Coefficient of determination values using different BSM parameter values for Fontainebleau and Lochaline sandstones.

<table>
<thead>
<tr>
<th></th>
<th>F2D</th>
<th>F3Q</th>
<th>F4Q</th>
<th>L3Q</th>
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<td>CM</td>
<td>NM</td>
<td>CM</td>
<td>NM</td>
<td>CM</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.97</td>
<td>0.98</td>
<td>0.97</td>
<td>0.99</td>
<td>0.99</td>
</tr>
<tr>
<td>$R^2_{LS}^1$</td>
<td>0.96</td>
<td>0.96</td>
<td>0.97</td>
<td>0.97</td>
<td>0.99</td>
</tr>
<tr>
<td>$R^2_{HS}^2$</td>
<td>-0.31</td>
<td>0.60</td>
<td>-0.56</td>
<td>0.60</td>
<td>0.12</td>
</tr>
</tbody>
</table>

1 Low salinity, below 0.1 M NaCl (Fontainebleau samples) and 1 M NaCl (Lochaline samples).
2 High salinity, above 0.1 M NaCl (Fontainebleau samples) and 1 M NaCl (Lochaline samples).

The measured high salinity zeta potentials were closely matched by our NM, which considers that the shear plane approaches the mineral surface with a small distance from the OHP ($d = 0.5$ Å for Fontainebleau samples and $d = 0.3$ Å for Lochaline samples; Table 1). Including such a small distance $d$ between OHP and shear plane increased the computed zeta potential magnitude compared to CM with a more pronounced impact at high salinity (Figure 8). The effective distance $d$ used in our NM was significantly smaller than the hydrated radius of Na$^+$ ($\approx 2$ Å; Leroy et al.)
[61], Sverjensky [17]), which implied that only some of Na ions were mobilized in the Stern layer, i.e., only a small portion of all ions could move inside the Stern layer. In addition, we found that $d/d_{Stern}$ (Lochalarine) = $d/d_{Stern}$ (Fontainebleau) = 0.18, which implied that regardless of rock type, 18% of the, previously considered as immobile, ions in the Stern layer will be flowing. Therefore, the thicker the Stern layer is (and we expect it to become thicker as roughness increases), the larger $d$ will become – exactly as our NM predicted.

Moreover, unlike the CM, our NM found that the shear plane of Fontainebleau samples was further away from the mineral surface than the shear plane of Lochaline samples, also explaining why the zeta potential magnitude of Fontainebleau samples was smaller than the zeta potential magnitude of Lochaline samples. Indeed, the total distance of the shear plane from the mineral surface ($d_{Stern} - d$) is larger for Fontainebleau (2.8–0.5=2.3 Å) compared with Lochaline (1.7–0.3=1.4 Å) samples. This result is consistent with our hypothesis that rougher and sharper Fontainebleau grains (Figure 5) lead to expansion of the effective EDL (averaged over large number of grains and pores) further away from the mineral surface (both, the Stern plane and the shear plane).
Figure 8. Computed zeta potential of Fontainebleau (F) and Lochaline (L) samples as a function of NaCl concentration considering or not the distance \(d\) between the OHP and the shear plane.

In the classical theory of the electrical double layer, it is assumed that only the mobile excess counter-ions in the diffuse layer contribute to the measured macroscopic streaming potential [27]. However, the diffuse layer is highly compressed at high salinity, so that there are essentially no mobile counter-ions available inside it, and such near-zero concentration of mobile counter-ions of the diffuse layer cannot explain correctly the non-zero zeta potentials in Fontainebleau and Lochaline sandstones at high salinity. Figure 9 shows the computed thicknesses of the diffuse layer and of the mobile part of the Stern layer \((d)\), as well as the \(\Gamma_{\text{Na}^+}^{d}\) surface site density in the Stern and diffuse layers, respectively. The salinity dependence of the diffuse layer thickness was evaluated by the Debye length \(\chi\):
\[
\chi = \frac{\varepsilon_w k_B T}{2e^2 1000 N_A I},
\]  

(5)

where \(\varepsilon_w\) is the water permittivity (F m\(^{-1}\); \(\varepsilon_w \approx 78.3 \varepsilon_0\) at a temperature of 25°C), \(k_B\) is the Boltzmann constant (\(\approx 1.381 \times 10^{-23} \text{ J K}^{-1}\)), \(T\) is the temperature (K), \(e\) is the electron charge (\(\approx 1.602 \times 10^{-19} \text{ C}\)), \(N_A\) is the Avogadro number (\(\approx 6.022 \times 10^{23}\) sites mol\(^{-1}\)), and \(I\) is the ionic strength (mol L\(^{-1}\)). The surface site density of adsorbed Na ion in the diffuse layer, \(\Gamma_{\text{Na}^+}^d\), was calculated using the following equations [23]:

\[
\Gamma_{\text{Na}^+}^d = 1000 N_A c_{\text{Na}^+}^\infty \int_{x=0}^{x=\chi} \left\{ \exp\left[-\frac{e \varphi_d(x)}{k_B T}\right] - 1 \right\} dx,
\]  

(6)

\[
\varphi_d(x) = \frac{4k_B T}{e} \tanh^{-1} \left[ \frac{e \varphi_d}{4k_B T} \exp\left(-x/\chi\right) \right],
\]  

(7)

where \(c_{\text{Na}^+}^\infty\) is the Na\(^+\) concentration (mol L\(^{-1}\)) in bulk water (not influenced by the charged surface), \(\varphi_d\) is the electrical potential at the start of the diffuse layer (\(\varphi_\beta = \varphi_d\)) and \(x\) is the distance from the OHP (in m).
Figure 9. Computed thickness of the diffuse layer (equal to the Debye length) and of the mobile part of the Stern layer (a); surface density of $>\text{SiO}^–\text{Na}^+$ sites in the Stern layer (at OHP) and of adsorbed Na$^+$ ions in the diffuse layer (b) as a function of NaCl concentration for Fontainebleau (F) and Lochaline (L) samples.

The computed thickness of the diffuse layer decreases significantly at high salinity to become comparable to the hydrated radius of sodium ion ($\approx$ 2 Å) but it remains considerably larger than the thickness of the mobile part of the Stern layer (0.5 Å and 0.3 Å for Fontainebleau and Lochaline samples, respectively) (Figure 9a). However, when salinity increases, the computed surface site density of adsorbed Na$^+$ ion inside the Stern layer increases considerably more than in the diffuse layer (Figure 9b), which explains the increasing contribution of the counter-ions in the mobile part of the Stern layer to the measured streaming potential.

Our new surface complexation model applied simultaneously for both Fontainebleau and Lochaline samples (all five samples together) in NaCl solution (UNM) was able to reproduce the zeta potential measurements well. Indeed, the values of the coefficient of determination were still close to 1 when calculated for the entire salinity range (Table 3). The UNM reproduced very well the low salinity
measurements, and the quality of match was similar to the results obtained using the unified classical model, UCM. Across the high salinity domain, the UNM was also found to provide a better match to the experimental data compared with the UCM (except for L4Q sample at high salinity). The values of the optimized parameters used in UNM (Table 4) agreed with the values previously reported in Table 1 for NM, and both sets were consistent with the values reported in the literature for quartz in NaCl solution. Therefore, our approach is relevant for obtaining a unified surface complexation model for quartz in a NaCl solution. However, treating different rock samples separately makes more sense because it accounts for difference in the pore space topology, so we believe that NM is better and more physically sensible than UNM.

Table 3. Coefficient of determination values using a single set of BSM parameter values for Fontainebleau and Lochaline sandstones together.

<table>
<thead>
<tr>
<th></th>
<th>F2D</th>
<th>F3Q</th>
<th>F4Q</th>
<th>L3Q</th>
<th>L4Q</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>UCM</td>
<td>UNM</td>
<td>UCM</td>
<td>UNM</td>
<td>UCM</td>
</tr>
<tr>
<td>(R^2)</td>
<td>0.98</td>
<td>0.99</td>
<td>0.97</td>
<td>0.98</td>
<td>0.99</td>
</tr>
<tr>
<td>(R^2_{LS}^1)</td>
<td>0.97</td>
<td>0.97</td>
<td>0.95</td>
<td>0.95</td>
<td>0.99</td>
</tr>
<tr>
<td>(R^2_{HS}^2)</td>
<td>-0.45</td>
<td>0.46</td>
<td>-0.60</td>
<td>0.29</td>
<td>0.05</td>
</tr>
</tbody>
</table>

\(^1\)Low salinity, below 0.1 M NaCl (Fontainebleau samples) and 1 M NaCl (Lochaline samples).

\(^2\)High salinity, above 0.1 M NaCl (Fontainebleau samples) and 1 M NaCl (Lochaline samples).
Table 4. BSM parameter values and estimated Stern layer thickness for quartz (combining Fontainebleau and Lochaline sandstones).

<table>
<thead>
<tr>
<th>Symbols</th>
<th>Range$^1$</th>
<th>UCM</th>
<th>UNM</th>
</tr>
</thead>
<tbody>
<tr>
<td>log $K_H$</td>
<td>[4 10]</td>
<td>7.2</td>
<td>7.2</td>
</tr>
<tr>
<td>log $K_{Na}$</td>
<td>[-15 -5]</td>
<td>-21</td>
<td>0.1</td>
</tr>
<tr>
<td>$C_1$ (F m$^2$)</td>
<td>[0.5 -5]</td>
<td>1.8</td>
<td>2.2</td>
</tr>
<tr>
<td>d (Å)</td>
<td>[0 10]</td>
<td>0</td>
<td>0.3</td>
</tr>
<tr>
<td>$d_{Stern}$ (Å)</td>
<td></td>
<td>2.1</td>
<td>1.7</td>
</tr>
</tbody>
</table>

$^1$ Hiemstra et al. [19], Kitamura et al. [22], Sonnefeld et al. [59], Sverjensky [3], García et al. [6].

$^2$ According to Eq. (2) and fitted $C_1$ values, considering $\varepsilon_i = 43\varepsilon_0$ and $d_{Stern} = x_\beta - x_0$.

4. Conclusions

We developed a basic Stern surface complexation model to explain the zeta potential measurements on quartz in contact with NaCl solution and describe the concentration dependence of the electrochemical properties of quartz over a broad salinity range, from around $10^{-4}$ M NaCl up to around 5.5 M NaCl. These measurements were obtained using the streaming potential method. Previous surface complexation models considered that the shear plane of quartz in contact with NaCl solution was located at the Stern plane where sodium counter-ions were preferentially adsorbed or even further away from the mineral surface. In contrast to previous models, our new model considered that there could be a non-zero water flow transporting counter-ions within the Stern layer, i.e. that the shear plane where the zeta potential is defined was located closer to the mineral surface than the Stern plane.

Compared to the classical basic Stern model (BSM considering the zeta potential at the Stern plane), our new model better reproduced the zeta potential measurements on Fontainebleau and Lochaline sandstones, especially at high salinity conditions (above 0.1 M NaCl for Fontainebleau samples and 1 M NaCl for Lochaline samples) where zeta potential appeared to level off to a constant
negative value; this was particularly true for Fontainebleau samples. We explained, based on SEM micrograph images and our new model, why Fontainebleau rocks, with sharper-angle grains and larger surface roughness, had smaller in magnitude zeta potential for the same NaCl concentration compared against Lochaline data. The predicted surface charge density of quartz from the new model was also in a better agreement with the experimental data.

We found a small distance of the shear plane offset from the Stern plane (0.5 Å for Fontainebleau samples and 0.3 Å for Lochaline samples), i.e., only a small part of the Stern layer (18% independently of Fontainebleau or Lochaline samples) was mobile, confirming that the shear plane was still at a close proximity to the Stern plane. In addition, in our new model, the optimized values of the equilibrium constant describing sodium adsorption in the Stern layer ($10^{0.6}$ for Fontainebleau samples and $10^{0.1}$ for Lochaline samples) were more realistic compared with the classical BSM, which considers essentially no sodium adsorption in the Stern layer to try to match high salinity zeta potential measurements. Using our new model, we found that the Stern layer of Fontainebleau samples is thicker than the Stern layer of Lochaline samples (2.8 Å vs 1.7 Å), and that the shear plane of Fontainebleau samples is further away from the mineral surface (2.3 Å) than the shear plane of Lochaline samples (1.4 Å), in agreement with the pore space topologies given by SEM images. We also explained the increasing contribution of the counter-ions in the mobile part of the Stern layer compared to the contribution of the counter-ions in the diffuse layer to the streaming potential when salinity increases.

Our basic Stern surface complexation model with the zeta potential located closer to the mineral surface than the Stern plane can be used to interpret and predict streaming potential measurements and other types of electrokinetic measurements (e.g., electrophoretic mobility) on quartz and other minerals in contact with brines of different chemical compositions and temperatures. Therefore,
our zeta potential model, which can be confirmed by laboratory measurements looking phenomena at the microscopic scale (e.g., using microfluidics and spectroscopy methods) and atomistic simulations, may have strong implications for the modelling of the electrochemical properties of minerals in contact with highly saline brines. Our results are of crucial importance for exploring mineral-brine interactions at high salinity levels close to real subsurface conditions.

Acknowledgments

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Appendix A. Basic Stern surface complexation model

The following two surface complexation reactions were considered for the zeta potential modelling:

\[ \text{SiO}^- + \text{H}^+ \leftrightarrow \text{SiOH}, ~ K_H, \quad (A8) \]
\[ \text{SiO}^- + \text{Na}^+ \leftrightarrow \text{SiO}^- - \text{Na}^+, ~ K_{Na}, \quad (A9) \]

where \( K_H \) and \( K_{Na} \) (dimensionless) are the associated equilibrium constants, which are written as:

\[ K_H = \frac{a_{\text{SiOH}}}{\Gamma_{\text{SiOH}}} \approx \frac{\Gamma_{\text{SiOH}}}{\Gamma_{\text{SiOH}}} = \frac{\Gamma_{\text{SiOH}}}{\Gamma_{\text{SiOH}}} \exp(\frac{e\varphi_0}{k_B T}), \quad (A10) \]
\[ K_{Na} = \frac{a_{\text{SiO}^- - \text{Na}^+}}{\Gamma_{\text{SiO}^- - \text{Na}^+}} \approx \frac{\Gamma_{\text{SiO}^- - \text{Na}^+}}{\Gamma_{\text{SiO}^- - \text{Na}^+}} = \frac{\Gamma_{\text{SiO}^- - \text{Na}^+}}{\Gamma_{\text{SiO}^- - \text{Na}^+}} \exp(\frac{e\varphi_0}{k_B T}), \quad (A11) \]

where \( a_i \) is the activity (dimensionless) and \( \Gamma_i \) is the surface site density (sites m\(^{-2}\)) of species \( i \), \( e \) is the elementary charge (\( \approx 1.602 \times 10^{-19} \) C), \( \varphi \) is the electrical potential (V), \( k_B \) is the Boltzmann constant (\( \approx 1.381 \times 10^{-23} \) J K\(^{-1}\)), and \( T \) is the temperature (K). In equations (A3) and (A4), the superscript “\( \infty \)” refers to ion activities in the electroneutral free or bulk electrolyte (not influenced by the mineral surface), which were computed using Pitzer theory (Appendix B) [61].

The following determined system of equations for the surface charge density at the mineral surface, \( Q_0 \) (C m\(^{-2}\)), at the \( \beta \)-plane, \( Q_\beta \), and of the diffuse layer, \( Q_\delta \), was used to compute the electrical potential distribution at the interface between quartz and bulk NaCl solution as a function of the equilibrium constants and Stern layer capacitance [20]:
\[ Q_0 = -e(\Gamma_{\text{SiO}^-} + \Gamma_{\text{SiO}^--\text{Na}^+}) = -\frac{e\Gamma_S}{A} \left[ 1 + K_{Na} a_{Na}^{\infty} \exp\left(-\frac{e\varphi_{\beta}}{k_B T}\right) \right], \]  
(A12)

\[ Q_\beta = e\Gamma_{\text{SiO}^--\text{Na}^+} = \frac{e\Gamma_S}{A} K_{Na} a_{Na}^{\infty} \exp\left(-\frac{e\varphi_{\beta}}{k_B T}\right), \]  
(A13)

\[ A = 1 + K_{H} a_{H}^{\infty} \exp\left(-\frac{e\varphi_{0}}{k_B T}\right) + K_{Na} a_{Na}^{\infty} \exp\left(-\frac{e\varphi_{\beta}}{k_B T}\right), \]  
(A14)

\[ Q_S = \sqrt{8e_w k_B T 1000 N_A I} \sinh \left[-\left(\frac{e\varphi_{\beta}}{2k_B T}\right)\right], \]  
(A15)

\[ Q_0 + Q_\beta + Q_S = 0, \]  
(A16)

\[ \varphi_0 - \varphi_{\beta} = \frac{Q_0}{C_1}, \]  
(A17)

where \( \Gamma_S \) is the total surface site density (we took \( \Gamma_S = 4.6 \) sites nm\(^{-2}\) García et al. [6]), \( I \) is the molar ionic strength (mol L\(^{-1}\)), and \( \varphi_0 \) and \( \varphi_{\beta} \) are the electrical potentials at the 0-plane and at the \( \beta \)-plane, respectively (considering \( \varphi_{\beta} = \varphi_d \) for the BSM, where \( \varphi_d \) is the electrical potential at the start of the diffuse layer).
Appendix B. Pitzer model for ion activity coefficients in bulk electrolyte

The following equations were used to compute ion activity coefficients in bulk electrolyte [61]:

\[ a_i^\infty = \gamma_i^\infty \frac{m_i^w}{m_0}, \]  
\[ m_i^\infty = \frac{1000c_i^w}{M_wc_i^\infty}, \]  
\[ c_w^i = \frac{10^3 - \sum c_i^wV_i}{V_w}, \]

where \( \gamma_i^\infty \) is the activity coefficient (dimensionless), \( m_i^\infty \) is the molality (mol per kilogram of water, mol kg\(_w\)^{-1}, \( m_0 \) being the unit molality equal to 1 mol kg\(_w\)^{-1}), \( c_i^w \) is the molar concentration (M), and \( V_i \) is the standard partial molal volume (cm\(^3\) mol\(^{-1}\)) of ion \( i \) in bulk electrolyte. The quantity \( V_i \approx 18.07, 0, -1.13, 17.68 \text{ cm}^3 \text{ mol}^{-1} \) for H\(_2\)O, H\(^+\), Na\(^+\) (due to electrostriction) and Cl\(^-\), respectively, at a temperature of 25\(^\circ\)C. The subscript “w” in equations (B2) and (B3) refers to water molecules, and \( M_w \) refers to the molar mass of water (\( \approx 18 \text{ g mol}^{-1} \)).

Na\(^+\) activity coefficient in bulk electrolyte influences modelled Na\(^+\) adsorption in the Stern plane (\( \Gamma_{\text{SiO}^2-\text{Na}^+} = K_{\text{Na}} \Gamma_{\text{SiO}^2-\text{Na}^+} \gamma_{\text{Na}}^\infty m_{\text{Na}}^w / m_0 \exp(-e\varphi_\beta / k_BT) \) from equations (A3) and (A4)). According to Pitzer theory, which is suitable for very saline aqueous solutions (ionic strengths above 0.1 M Harvie and Weare [64]), the natural logarithm of Na\(^+\) activity coefficient in NaCl electrolyte is written as:
\[
\ln \gamma_{Na}^\infty = \sum_{Na} z_{Na}^2 F + m_{Cl}^{\infty} \left[ 2B_{Na\cdotCl} + \left( m_{Na}^{\infty} + m_{Cl}^{\infty} \right) C_{Na\cdotCl} \right] + \sum_{Na} z_{Na}^2 m_{Cl}^{\infty} C_{Na\cdotCl},
\]
(B4)

\[
F = -A_\phi \left[ \frac{\sqrt{I_m}}{1 + b \sqrt{I_m}} + \frac{2}{b \ln \left( 1 + b \sqrt{I_m} \right)} \right] + \sum_{Na} z_{Na}^2 m_{Cl}^{\infty} B_{Na\cdotCl},
\]
(B5)

\[
A_\phi = \frac{1}{3} \sqrt{\frac{2\pi N_A \rho_w}{1000}} \left( \frac{e^2}{4\pi \varepsilon_w k_B T} \right)^{3/2},
\]
(B6)

\[
B_{Na\cdotCl} = -\frac{2 \beta_{Na\cdotCl}^l}{I_m x_i^2} \left[ 1 - \left( 1 + x_i + 0.5x_i^2 \right) \exp(-x_i) \right],
\]
(B7)

\[
x_i = \alpha_i \sqrt{I_m},
\]
(B8)

\[
B_{Na\cdotCl} = \beta_{Na\cdotCl}^l + \frac{2 \beta_{Na\cdotCl}^l}{x_i^2} \left[ 1 - (1 + x_i) \exp(-x_i) \right],
\]
(B9)

\[
C_{Na\cdotCl} = \frac{C_{\phi Na\cdotCl}}{2 \sqrt{z_{Na} \cdot z_{Cl}}},
\]
(B10)

where \( z_i \) is the charge number of ion \( i \), \( b \) and \( \alpha_i \) are empirical parameters (\( b = 1.2, \alpha_i = 2 \) for 1:1 and 1:2 electrolytes), \( I_m \) is the molal ionic strength (in mol kg\(^{-1}\), \( I_m = m_{Na}^w \) here), and \( A_\phi \) is the Debye-Hückel coefficient describing long-range electrostatic interaction forces between ions (\( \approx 0.392 \) at a temperature \( T = 298 \) K). The Debye-Hückel coefficient was computed here as a function of the Avogadro number \( N_A \) (\( \approx 6.022\times10^{23} \) sites mol\(^{-1}\)), the water volumetric density \( \rho_w \) (\( \approx 997\times10^3 \) g m\(^{-3}\)), and the water permittivity \( \varepsilon_w \) (\( \approx 78.3\varepsilon_0 \) where \( \varepsilon_0 \) is the vacuum permittivity with a value of \( \approx 8.854\times10^{-12} \) F m\(^{-1}\)). The Debye-Hückel coefficient multiplied by the terms in brackets in equation (B5) is enough for computing ion activity coefficient in dilute aqueous solution (ionic
strength below 0.1 M). Pitzer and Mayorga [65] considered three additional terms (in equations (B4) and (B5)) to compute ion activity coefficients in concentrated aqueous solutions. The terms $B_{Na^+Cl^-}$ and $B_{Na^+Cl^-}$ depend on the ionic strength and describe short-range interaction forces between one cation and one anion (binary system), and the term $C_{Na^+Cl^-}$ describes short-range interaction forces between two cations and one anion, and one cation and two anions (ternary system). The Pitzer model for ion activity coefficients in 1:1 aqueous electrolyte such as NaCl depends on three parameters $\beta_{Na^+Cl^-}^0$, $\beta_{Na^+Cl^-}^1$, and $C_{\neq Na^+Cl^-}$. The Pitzer parameter values were adjusted by matching computed to measured osmotic coefficients. According to [Leroy et al. [61]] $\beta_{Na^+Cl^-}^0 = 0.0765$, $\beta_{Na^+Cl^-}^1 = 0.2664$, and $C_{\neq Na^+Cl^-} = 0.00127$. 
References


The zeta potential of quartz.

Surface complexation modelling to elucidate high salinity measurements

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Abstract

The zeta potential is a measureable electrical potential of paramount importance to understand the electrochemical properties of rocks. However, the zeta potential remains poorly understood because it takes place at the nanoscale of the electrical double layer on the mineral surface. Streaming potential measurements on quartz-rich Fontainebleau and Lochaline sandstones carried out at high salinity (above 0.1 M NaCl) yield surprisingly high zeta potential values, which cannot be correctly reproduced by a traditional surface complexation model considering that the shear plane is located at the beginning of the diffuse layer. We found that placing the shear plane, where the zeta potential is defined, slightly closer to the mineral surface than the Stern plane significantly improves the predictions of the zeta potential and surface charge density of quartz at high salinity as well as the values of the equilibrium constant describing sodium adsorption in the Stern layer. Our results have strong implications for the modelling of the electrochemical properties of minerals in contact with highly saline solutions.

Key words: zeta potential, quartz, streaming potential, high salinity, shear plane location
1. Introduction

Quartz is a mineral that is particularly interesting to study because of its natural abundance and usefulness in the development of new technologies [1]. In contact with water, quartz develops a surface charge attracting counter-ions and repelling co-ions, thus forming the so-called electrical double layer (EDL) usually represented by a “compact” Stern layer and a diffuse layer [2, 3]. Investigating the electrochemical properties of quartz is of great interest in many applications in physics, chemistry and Earth sciences because these properties control adsorption and dissolution/precipitation reactions, and wettability on the quartz surface [4-6]. The EDL of quartz is also the source of electrokinetic and geophysical electrical (e.g., self-potential, resistivity, induced polarization) measurements that are used to map for instance geological fluid flows or biogeochemical reactions [7-12]. Studying quartz electrochemical properties notably when quartz is in contact with highly saline brines has a high potential in many geo-environmental and engineering applications including geo-sequestration of CO₂ in deep saline aquifers, and oil and gas exploration and production notably enhanced hydrocarbon recovery [13-17].

Exploring the electrochemical properties of quartz is very challenging because of their nanoscopic nature [1, 18, 19]. Indeed, surface complexation reactions between surface sites and ions in the aqueous solution occur at the nm-scale [3, 20-22]. In addition, natural quartz has a low specific surface area (typically below 0.1 m² g⁻¹), which considerably complicates the experimental characterization of its EDL compared to minerals with a large specific surface area such as montmorillonite [2, 3, 23, 24]. Only few methods exist to probe the properties of the EDL on the surface of minerals in contact with brines. Among them, there is the streaming potential method, which implies application of a water pressure difference across the sample while measuring the resulting voltage, the streaming potential, due to the displaced excess counter-ions in the EDL [25-
From the measured streaming potential it is possible to obtain some relevant information on the electrochemical properties of minerals through the calculation of the electrokinetic zeta potential ($\zeta$), which is defined as the electrical potential at the shear (or slip) plane [17, 25, 27]. The zeta potential determined experimentally can be interpreted in terms of mineral electrochemical properties by matching observed and simulated zeta potential using a relevant surface complexation model [21, 30, 31]. However, this approach relies on the assumption that the exact location of the shear plane from the mineral surface is known, which is obviously not the case because of the lack of experimental information at the molecular level [32-35]. Moreover, the zeta potential is, most of the times, the only physico-chemical quantity available to validate the predictions of electrostatic surface complexation models for low specific surface area minerals such as quartz or calcite [3, 30, 36]. In addition, the zeta potential is inferred from electro-hydrodynamic measurements while surface complexation models rely on electrostatics at thermodynamic equilibrium [25, 35, 37, 38]. Therefore, these limitations contribute to additional uncertainties when investigating mineral electrochemical properties from zeta potential measurements.

When water flow relative to the mineral surface takes place, it is widely accepted that the shear plane is located between the “stagnant” Stern layer bounded by the outer Helmholtz plane (OHP) and the diffuse layer because high water viscosity in the Stern layer prevents water flow within it [3, 25, 39] (Figure 1). The Stern layer of silica-based materials such as amorphous silica and quartz in contact with a NaCl solution is traditionally represented by a hydration layer followed by a layer containing hydrated sodium counter-ions [18, 21, 40]. Some molecular dynamic (MD) simulations (e.g., Zhang et al. [33]), spectroscopy measurements (e.g., Lis et al. [41]) and microfluidic studies (e.g., Saini et al. [42] and Werkhoven et al. [43]) have demonstrated that there could be a non-zero
flow of water within the Stern layer of silica notably because some counter-ions (such as Na\(^+\)) are not stucked to the mineral surface and form outer sphere surface complexes keeping their hydration shell. This implies that there may be some, even weak, water displacement within the Stern layer of silica, and hydrous oxide in general. Therefore, for quartz, the effective shear plane may be located slightly closer to the mineral surface than the outer Helmholtz plane, in agreement with the assumption accepted by most that the shear plane is located at the proximity of the OHP (e.g., Hunter [25], Sverjensky [3], García et al. [6]).

Figure 1. Sketch showing water flow and ion distribution at the interface between a silica mineral and a NaCl aqueous solution (modified, from Brown et al. [40]). Circles with arrows inside represent water molecules. The shear plane is denoted by the red dashed line. Counter-ions adsorbed as outer sphere complexes form the outer Helmholtz plane (OHP).

The quartz (0001) crystal face is the most stable plane with the lowest surface energy and is often considered as a “model surface”, convenient for modelling SiO\(_2\) materials and hydrophilic surfaces
in general [35]. With the improved accuracy of the streaming potential method, it is now possible
to accurately measure extremely small voltages due to the displacement of the ions in the EDL of
quartz [14]. Published studies of Jaafar et al. [8], Vinogradov et al. [13], Walker et al. [44], and
Walker and Glover [15] observed, that at high salinities (NaCl concentrations above 0.4 M, M
means mol L\(^{-1}\)), the zeta potential of sandstones appears to level off at a small constant negative
value between \(-30\) and \(-10\) mV or even to increase slightly in magnitude (i.e. become more
negative) with salinity. They noted that the zeta potential of sandstones stabilizes at a salinity of
about 0.4 M NaCl that corresponds to a Debye length characterizing the diffuse layer thickness of
approximately 0.47 nm, which is similar to the size of a hydrated sodium ion. This observation led
them to suggest that the constant zeta potential of sandstones at high salinities reflected the
maximum charge density in the diffuse layer which was reached when the diffuse layer thickness
approached the diameter of the counter-ions [45]. However, Jaafar et al. [8], Vinogradov et al. [13],
Walker et al. [44], Glover [45], and Walker and Glover [15] did not explicitly explain this behavior
through a basic Stern surface complexation model describing their zeta potential measurements on
sandstones.

In our study, we used a surface complexation model named basic Stern model (BSM) and
considered that the shear plane is at the OHP or closer to the mineral surface than the OHP to
describe the zeta potential and the electrochemical properties of quartz at varying NaCl
concentrations. In our model we described the effective location of the OHP and the shear plane,
hence modelling the effective zeta potential. Therefore, the developed surface complexation model
accurately replicated the experimental conditions under which the streaming potential
measurements on intact rock samples comprising grains of various shape and roughness were
conducted. The model predictions were compared to the existing experimental zeta potential data
measured over a broad salinity range (from around $10^{-4}$ M NaCl up to around 5.5 M NaCl). The values of the optimized parameters were finally discussed. Our findings shed light on the electrochemical properties of quartz and on the likelihood of non-zero water flow within the Stern layer.

2. Theoretical background

2.1. Surface complexation model for quartz

Our basic Stern model [37, 46] describes proton (H+) adsorption onto >SiO$^-$ surface sites at the 0-plane (defining the mineral surface) and sodium cation (Na+) adsorption by these surface sites at the $\beta$-plane (Stern plane and OHP) (Figure 2) [3, 6, 20, 21, 23]. The BSM considers that the $\beta$-plane coincides with the $d$-plane defining the start of the diffuse layer. This model only needs one Stern layer capacitance as an input parameter to model the electrical potential distribution between the mineral surface and the Stern plane. Recent studies utilizing atomic force microscopy (AFM) (e.g., Siretanu et al. [47]) and X-ray photoelectron spectroscopy (XPS) (e.g., Brown et al. [40]) used the BSM to model the electrochemical properties of amorphous silica in contact with a NaCl aqueous solution and demonstrated that the BSM could accurately reproduce the experimental data. García et al. [6] also used the BSM to match the measured electrochemical properties of quartz in contact with a NaCl aqueous solution thus confirming the validity of the approach.
Figure 2. Sketch of our basic Stern model to describe the electrochemical properties of the interface between quartz and a 1:1 electrolyte like NaCl electrolyte (the $\beta$-plane coincides with the $d$-plane). The model input parameters are shown in blue and the model output parameters, including the zeta potential ($\zeta$) at the shear plane, are shown in red.

In our BSM we used four adjustable parameters, namely the logarithms of the two adsorption equilibrium constants $K_H$ and $K_{Na}$, the Stern layer capacitance $C_1$ (F m$^{-2}$), and the distance $d$ between the shear plane (where the zeta potential is defined) and the $\beta$-plane (Figure 2). It should be noted that we considered that the doubly coordinated surface groups (>Si$\equiv$O$^0$) are inert [20] and that the protonated silanol sites (>SiOH$^+_2$) are not expected to form at close-to-neutral pH of the streaming potential measurements on sandstones (pH varied between 6.4 and 7.3 Walker and Glover [15]). Therefore, these surface sites were excluded from the model. In absence of additional
measurements, we also did not consider another type of silanol group, hence we made our model as simple as possible in order to decrease the number of optimized parameters. For more information related to our BSM, the reader can refer to Appendixes A and B, and to Leroy et al. [21].

2.2. Zeta potential computation

All calculations were performed by combining the geochemical software IPhreeqc for the surface complexation modelling [48] with an in-house code implemented in Matlab for the calculation of the zeta potential and the optimization procedure [49]. The zeta potential (V) defined at the shear plane located at a distance \( d \) from the \( \beta \)-plane was determined from the computed \( \varphi_0 \) and \( \varphi_\beta \) electrical potentials by considering a linear, capacitor-like variation of the electrical potential within the Stern layer [25]

\[
\zeta = \varphi_\beta - \frac{\varphi_\beta - \varphi_0}{x_\beta - x_0} d, \tag{1}
\]

where \( x \) is the distance from the mineral surface (defined by the 0-plane, in m). Combining equation (1) with the following equation for the Stern layer capacitance [18]

\[
C_1 = \frac{\varepsilon_1}{x_\beta - x_0}, \tag{2}
\]

where \( \varepsilon_1 \) is the water permittivity in the Stern layer (F m\(^{-1}\); we used \( \varepsilon_1 = 43\varepsilon_0 \), where \( \varepsilon_0 \) is the vacuum permittivity, in accordance with the study of Sverjensky [3]), we finally obtain an expression for the zeta potential as a function of the modelled electrochemical properties
\[ \zeta = \varphi_\beta - (\varphi_\beta - \varphi_0) \frac{C_1}{\varepsilon_1} d. \]  

We did not consider the presence of a stagnant diffuse layer (also named buffer layer), which implies that the shear plane is located further away from the mineral surface, as suggested in Alizadeh and Wang [50]. To the best of our knowledge, the stagnant diffuse layer existence has never been directly confirmed experimentally. To the contrary, Předota et al. [35], Brkljača et al. [19], and Biriukov et al. [34] predicted no such stagnant diffuse layer from their molecular dynamic simulations of the zeta potential of the hydroxylated (110) rutile (TiO₂) and (0001) quartz surfaces.

Furthermore, Leroy and co-workers. Furthermore, Leroy and co-workers (e.g., Leroy et al. [51], Leroy et al. [52], Leroy et al. [21], Li et al. [30]) attributed the assumption of the presence of a stagnant diffuse layer in previous studies to the misinterpretation of the zeta potentials from electrokinetic (e.g., electrophoretic mobility, streaming potential) measurements due to disregard of surface conductivity effects. Indeed, surface conductivity decreases the magnitude of the measured electrokinetic signal hence implying smaller apparent zeta potentials, which need to move away the shear plane from the mineral surface when modelling the zeta potential from a surface complexation model.

The parameters of our surface complexation model (\( \log K_H \), \( \log K_{Na} \), \( C_1 \), \( d \)) were optimized by minimizing the following cost function [53]:

\[ y = 1 - R^2 = \frac{\sum_{i=1}^{N} \left( \zeta_{mes}^i - \zeta_{mod}^i \right)^2}{\sum_{i=1}^{N} \left( \zeta_{mes}^i - \langle \zeta_{mes} \rangle \right)^2}, \]

where \( R^2 \) is the coefficient of determination, \( N \) is the number of zeta potential measurements, \( \zeta_{mes}^i \) is the \( i \)-th measured zeta potential, \( \langle \zeta_{mes} \rangle \) is the arithmetic mean of the measured zeta potentials,
and $\zeta_{mod}^i$ is the $i$-th modelled zeta potential. The fitting procedure was realized by using the simulated annealing algorithm to find the global minimum of the cost function (equation (4)), with a refinement using the simplex method at the end of the process [49].

3. Comparison with experimental data and discussion

3.1. Considerations of impact of pore space topology and grain roughness on EDL parameters

To test our model, we used the measured zeta potentials of Fontainebleau (F2D, F3Q, F4Q) and Lochaline (L3Q, L4Q) samples in contact with a NaCl aqueous solution of increasing salinity obtained by the streaming potential method and reported in Walker and Glover [15]. These two sample types were selected as they are known to consist of more than 99% quartz (by weight) [13, 54]. Unlike Fontainebleau and Lochaline samples, zeta potentials of Berea and Boise sandstones reported by Walker and Glover [15] that contained up to 6% feldspar, 2% dolomite, and 8% clays for Berea rocks [55] and up to 13% clays for Boise rocks [56], were excluded from the simulation. Despite the fact that feldspar, dolomite, and clay content in Berea and Boise samples is relatively small, clays are known to line pore walls, thus making these complex minerals a main contributor to the electrochemical processes at the mineral-water interface and causing anomalous or even positive zeta potentials [57, 58]. Therefore, the experimental zeta potential data for Boise and Berea samples were deemed unapplicable for our model that considers only surface complexation reactions on quartz surface.

All Fontainebleau and Lochaline samples exhibit a negative zeta potential with its magnitude decreasing with increasing salinity (Figures 3 and 4). The zeta potentials of Lochaline samples
were found to be of a larger magnitude than those of Fontainebleau samples. Scanning electron microscopy (SEM) micrographs of the tested samples showed that Fontainebleau rock has sharper-angled grains with larger surface roughness and smaller grains than Lochaline rock (Figure 5 from Walker and Glover [15]). According to Vinogradov et al. [14], pore space topology, grain shape, surface roughness and size influence streaming potential measurements. They considered that rough rocks with small grains have smaller streaming and zeta potential magnitudes than round, smooth rocks with large grains because rock sharp corners and grain roughness would shift the effective shear plane further away from the mineral surface (read their section 4.2). Alroudhan et al. [59] used the same assumption to explain that the zeta potential of colloidal suspensions measured by the electrophoretic mobility method is larger in magnitude than the zeta potential of rocks measured by the streaming potential method (see their Figure 10 and read the related discussion in their section 5.2). Schnitzer and Ripperger [60] and Drechsler et al. [61] showed that increasing surface roughness changes the flow velocity distribution on the solid surface shifting the shear plane further away from the solid surface and decreases the streaming and zeta potential magnitudes. According to these observations, we expected different values of the surface complexation model parameters between Fontainebleau and Lochaline samples, notably for the Stern layer capacitance $C_1$ and the distance $d$ of the shear plane from the OHP (or Stern plane), which are very sensitive to the textural properties of rocks ($C_1$ depends on the thickness of the Stern layer, equation (2)).
Figure 3. Zeta potentials of Fontainebleau samples as a function of NaCl concentration. Circle symbols: experimental zeta potential data with the sample name corresponding to that used by Walker and Glover [15]; curves: model predictions.
Figure 4. Zeta potentials of Lochaline samples as a function of NaCl concentration. Circle symbols: experimental zeta potential data with the sample name corresponding to that used by Walker and Glover [15]; curves: model predictions.
Figures 3 and 4 demonstrate that below the concentration thresholds of around 0.1 M NaCl (Fontainebleau samples) and 1 M NaCl (Lochaline samples) (denoted by the vertical black dotted lines), the magnitude of the negative zeta potential decreases linearly with increasing salinity. Interestingly, the rate of decrease in the zeta potential magnitude with increasing salinity became smaller above these thresholds, i.e. it became non-linear, and eventually stabilized (or even slightly...
increased in magnitude) at a zeta potential value of approximately -15 mV for both rock types.

Such stabilization of the zeta potential was more apparent for Fontainebleau than for Lochaline samples. These observations were consistent across the data reported by Vinogradov et al. [13], Vinogradov et al. [14] and Walker and Glover [15], who stated that at high salinities, the measured zeta potential stabilized and became equal to $-13.01 \pm 0.48$ mV for Fontainebleau samples and to $-16.81 \pm 0.68$ mV for Lochaline samples.

Considering that Fontainebleau and Lochaline sandstones did not have the same pore space topology and textural properties, we first optimized separately the parameters of the surface complexation models for these two rock types. That is, a single model was developed for F2D, F3Q, F4Q combined data (Fontainebleau rocks) and a separate model was developed for L3Q, L4Q combined data (Lochaline rocks) to match simulated to observed zeta potentials. We ran the classical model denoted CM with the parameters $\log K_H$, $\log K_{Na}$, and $C_1$, and the new model denoted NM with the parameters $\log K_H$, $\log K_{Na}$, $C_1$, and $d$ (the distance of separation between the shear plane and OHP), to investigate the effect of the proposed inward shift of the shear plane on the simulated zeta potential while assigning measured pH values to the respective rock samples as reported by Walker and Glover [15]. We then used the same BSM approach for Fontainebleau and Lochaline samples together (all five samples, F2D, F3Q, F4Q, L3Q, L4Q) to develop a unified surface complexation model for quartz in contact with a NaCl aqueous solution, denoted UNM for unified new model and UCM for unified classical model.
3.2. Comparison of the computed to the observed zeta potentials and discussion

Overall, both the NM and CM reproduced well the experimental zeta potential data for the entire salinity range (Figures 3 and 4, and Tables 1 and 2). To estimate the uncertainties, we fixed two/three of the three/four parameters at their optimal values and then we computed the cost function (i.e., $y=1-R^2$) for the remaining parameter which is allowed to vary. Afterwards, we computed the relative cost function associated to the varying parameter $(y-y_{opt})/y_{opt}$, where $y_{opt}$ is the value of the cost function when the three/four parameters are fixed at their optimal values (so the relative cost function associated to the varying parameter is equal to zero for the optimal set of parameters). Finally, we extracted the range of values of the varying parameter for which the relative cost function is less than 0.1. We performed this procedure for the three/four parameters.

According to the surface complexation models, the observed negative zeta potential was due to the presence of the deprotonated silanol sites $\text{SiO}^{-}$ at the $0$-plane (Figure 2). The optimized values of the equilibrium constant describing protonation of $\text{SiO}^{-}$ surface sites ($K_H$, reaction (1)) equal to $10^{7.3}$ and $10^{7.2}$ for Fontainebleau and Lochaline samples, respectively, were found to be close or similar to the spectroscopically determined value of $10^{7.2\pm0.2}$ and to the theoretical value of $10^{7.5}$ using Pauling’s definition of formal bond valence for silica [20] (Table 1). In addition, our $K_H$ optimized values were found to be close or similar to the value of $10^{7.2}$ determined by Sverjensky [3] using a triple layer model (BSM with an additional $C_2$ capacitance between the Stern plane and the start of the diffuse layer) matching surface charge density measurements inferred from acid base potentiometric titration on natural quartz in contact with a NaCl solution. The models also explained why the zeta potential magnitude of Lochaline samples was larger, for the same salinity, than the zeta potential magnitude of Fontainebleau samples. Indeed, Lochaline samples have higher
pH (i.e. less protons in solution) than Fontainebleau samples (7.1 versus 6.5 in average, respectively Walker and Glover [15]) while having essentially identical log $K_H$ values, which resulted in Lochaline samples having larger number of deprotonated $\text{SiO}^-$ sites per nm$^2$ of surface and a higher negative surface charge density $Q_0$ (equation (A5)) than Fontainebleau samples (Figure 6).

| Table 1. BSM parameter values and estimated Stern layer thickness for Fontainebleau and Lochaline sandstones. |
|---|---|---|---|---|
| Symbols | Range$^1$ | Fontainebleau | Lochaline |
| | | CM | NM | CM | NM |
| log $K_{Na}$ | [-20 5] | -20 [ND$^3$] | 0.58 [0.25 0.83] | -20 [ND$^3$] | 0.13 [-0.1 0.32] |
| $C_1$ (F m$^{-2}$) | [0.5 5] | 3.24 [2.01 6.54] | 1.34 [1.18 1.51] | 1.84 [1.62 2.10] | 2.22 [2.01 2.47] |
| $d$ (Å) | [0 10] | 0 | 0.48 [0.42 0.54] | 0 | 0.25 [0.21 0.28] |
| $d_{Stern}^2$ (Å) | | 1.18 [0.58 1.89] | 2.85 [2.52 3.23] | 2.07 [1.81 2.35] | 1.71 [1.54 1.89] |

$^1$ Hiemstra et al. [20], Kitamura et al. [23], Sonnefeld et al. [62], Sverjensky [3], García et al. [6].

$^2$ According to Eq. (2) and fitted $C_1$ values, considering $\varepsilon_i = 43\varepsilon_0$ and $d_{Stern} = x_\beta - x_0$.

$^3$ Not determined.
Figure 6. Computed surface site densities of $\text{SiO}^-$ sites (a), $\text{SiO}^- - \text{Na}^+$ sites (b), and of surface charge densities (c) of Fontainebleau and Lochaline samples as a function of NaCl concentration. Plain line curves correspond to the calculations using the NM, dotted line curves correspond to the
calculations using the CM. The CM predicted near-zero surface site densities of adsorbed sodium ion in the Stern layer (limited at \( \equiv 0 \) sites \( \text{nm}^{-2} \) in Figure 6b).

We also found that Lochaline samples have significantly lower log \( K_{Na} \) values, i.e. weaker sodium adsorption capacity, than Fontainebleau samples (-21 vs -16, respectively, for CM and 0.1 vs 0.6, respectively, for NM, Table 1), which could not counterbalance the negative surface charge density as efficiently as for Fontainebleau samples, and can also explain the larger zeta potential magnitude of Lochaline samples. Interestingly, despite Lochaline samples having lower log \( K_{Na} \) values than Fontainebleau samples, the models found that Lochaline samples, for the same salinity, had a higher surface site density of adsorbed sodium ion in the Stern layer than Fontainebleau samples due to the higher \( >\text{SiO}^- \) surface site density (Figure 6b). The lower log \( K_{Na} \) values of Lochaline than Fontainebleau samples we found can be explained by Lochaline samples having smoother and larger grains and hence a smaller specific surface area than Fontainebleau samples. Sverjensky [3] did the same observation when comparing two quartz with different specific surface area (4.15 and 11.4 \( \text{m}^2 \text{g}^{-1} \)) in contact with a NaCl solution. The \( K_{Na} \) values inferred from the CM are extremely low and essentially mean that there is no adsorption of \( \text{Na}^+ \) at the OHP at all and everything is controlled only by pH. With the CM, the optimization procedure decreases \( K_{Na} \) to extremely low value to fit the high salinity zeta potential measurements (decreasing \( \text{Na}^+ \) adsorption in the Stern layer results to higher zeta potential magnitude).

With the NM, the optimization procedure doesn’t need to decrease \( K_{Na} \) to extremely low value to fit the high salinity zeta potential measurements and it found log \( K_{Na} \) values (0.6 and 0.1 for Fontainebleau and Lochaline samples, respectively) within the same order of magnitude than the
value reported by Sverjensky [3] for natural quartz in a contact with a NaCl solution ($\log K_{Na} = 0$).

In addition, on the contrary to the CM, our NM was able to reproduce most of the surface charge density measurements on Min-U-Sil 5 quartz (natural quartz with a mean grain diameter of 5 µm) at different pH and NaCl concentrations carried out by Riese [63] (Figure 7).

![Figure 7. Surface charge density of Min-U-Sil 5 quartz as a function of pH and NaCl concentration. Curves correspond to the predictions. Symbols correspond to the experimental surface charge density data reported by Riese [63].](image)

With the NM, the optimized Stern layer capacitance values were equal to 1.3 F m$^{-2}$ and 2.2 F m$^{-2}$ for Fontainebleau and Lochaline samples, respectively (Table 1), which were close to the values of 1 F m$^{-2}$ and 2 F m$^{-2}$ reported by Sverjensky [3] and García et al. [6], respectively, for natural quartz in contact with a NaCl solution. With the CM, the optimized Stern layer capacitance values were equal to 3.2 F m$^{-2}$ and 1.8 F m$^{-2}$ for Fontainebleau and Lochaline samples, respectively. Using the optimized Stern layer capacitance values from the NM, equation (2) and $\varepsilon_1 = 43\varepsilon_0$ [3, 40], we
found a Stern layer thickness comparable to the hydrated radius of sodium ion ($\approx 2 \text{ Å}$ Leroy et al. [64] Sverjensky [18]), with Fontainebleau samples having larger Stern layer thickness (2.8 Å) than Lochaline samples (1.7 Å), which can be explained by Fontainebleau samples having sharper and rougher grains than Lochaline samples [15, 65]. When using the CM, the Stern layer thickness we found for Fontainebleau samples (1.2 Å) was comparable to the crystallographic radius of sodium ion (1.02 Å Sverjensky [18]). This result was not realistic regarding the representation of the quartz/NaCl solution interface containing mostly hydrated sodium ions in the Stern layer, which is accepted by most recent models (e.g., Brown et al. [66]). For Lochaline samples, the Stern layer thickness inferred from the CM was comparable to the hydrated radius of sodium ion (2.1 Å).

Figures 3, 4, 7, and the modelling results reported in Table 1 for the parameter values and in Table 2 for the coefficient of determination values clearly demonstrate the importance of considering the location of the shear plane to be closer to the mineral surface than the OHP. Indeed, as shown in Figures 3 and 4 and reflected by the values of the coefficient of determination at high salinity reported in Table 2 ($R^2 \geq 0.5$), the stabilization of the zeta potential at high salinity could only be correctly predicted by the NM (red curves in Figures 3 and 4). The stabilization of the modelled zeta potential at high salinity is explained by a growing abundance of sodium ions available for adsorption in the Stern layer, and therefore the decreasing number of $>\text{SiO}^-$ sites (Figures 6a and 6b), and importantly by the shear plane being located slightly closer to the mineral surface than the OHP. Moreover, the NM reproduced the surface charge density measurements on natural quartz in a NaCl solution reported in Riese [63] significantly better than the CM (Figure 7) thus independently validating our assumption on the location of the shear plane.
Table 2. Coefficient of determination values using different BSM parameter values for Fontainebleau and Lochaline sandstones.

<table>
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<td>NM</td>
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<td>CM</td>
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<td>0.98</td>
<td>0.97</td>
<td>0.99</td>
<td>0.99</td>
</tr>
<tr>
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<td>0.96</td>
<td>0.97</td>
<td>0.97</td>
<td>0.99</td>
</tr>
<tr>
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<td>0.60</td>
<td>-0.56</td>
<td>0.60</td>
<td>0.12</td>
</tr>
</tbody>
</table>

1 Low salinity, below 0.1 M NaCl (Fontainebleau samples) and 1 M NaCl (Lochaline samples).
2 High salinity, above 0.1 M NaCl (Fontainebleau samples) and 1 M NaCl (Lochaline samples).

The measured high salinity zeta potentials were closely matched by the BSM considering the shear plane slightly approaching the mineral surface, i.e. with a very small distance from the OHP ($d = 0.5$ Å for Fontainebleau samples and $d = 0.3$ Å for Lochaline samples; Table 1). Including such a small distance $d$ between OHP and shear plane progressively increases computed zeta potential magnitude compared to not considering it when salinity increases (Figure 8). The effective distance $d$ used in our NM was significantly smaller than the hydrated radius of Na$^+$ (≈ 2 Å Leroy et al. [64] Sverjensky [18]), which implied that only some of Na ions were mobilized in the Stern layer, i.e., only a small portion of all ions could move inside the Stern layer. In addition, $d/d_{Stern}$ (Lochaline) = $d/d_{Stern}$ (Fontainebleau) = 0.18. This means that regardless of rock type 18% of the, previously considered as immobile ions in the Stern layer will be flowing. Then, the thicker the Stern layer is (and we expect it to become thicker as roughness increases), the larger $d$ will become – exactly as NM predicts.

In addition, unlike the CM, the NM found that the shear plane of Fontainebleau samples is further away from the mineral surface than the shear plane of Lochaline samples, also explaining why the zeta potential magnitude of Fontainebleau samples is smaller than the zeta potential magnitude of Lochaline samples. Indeed, the total distance of the shear plane from the mineral surface ($d_{Stern}$ –
is larger for Fontainebleau (2.8−0.5=2.3 Å) compared with Lochaline (1.7−0.3=1.4 Å) samples, which is consistent with our hypothesis that rougher and sharper Fontainebleau grains push EDL further away from the mineral surface (both, the Stern plane and the shear plane). These findings were in agreement with the SEM micrographs showing that Fontainebleau rock has sharper-angled grains with larger surface roughness than Lochaline rock (Figure 5).

Figure 8. Computed zeta potential of Fontainebleau (F) and Lochaline (L) samples as a function of NaCl concentration considering or not the distance d between the OHP and the shear plane.

In the classical theory of the electrical double layer, it is assumed that only the mobile excess counter-ions in the diffuse layer contribute to the measured macroscopic streaming potential [28]. However, the diffuse layer is highly compressed at high salinity, so that there are essentially no mobile counter-ions available inside it, and such near-zero contribution of the diffuse layer cannot explain correctly the non-zero zeta potentials in Fontainebleau and Lochaline sandstones at high salinity. Figure 9 shows the computed thicknesses of the diffuse layer and of the mobile part of the...
Stern layer as well as the surface site density of adsorbed sodium ion in the Stern and diffuse layers, \( \Gamma_{>SiO^-Na^+} \) and \( \Gamma_{Na^+}^d \), respectively. The salinity dependence of the diffuse layer thickness was evaluated by the Debye length \( \chi \):

\[
\chi = \frac{\varepsilon_\omega k_B T}{\sqrt{2e^2 1000 N_A I}},
\]

and \( \Gamma_{Na^+}^d \) was calculated using the following equations [24]:

\[
\Gamma_{Na^+}^d = 1000 N_A c_{Na^+} \int_{x=0}^{x=\xi} \left\{ \exp\left[ -e \varphi_d (x) / k_B T \right] - 1 \right\} dx,
\]

\[
\varphi_d (x) = \frac{4k_B T}{e} \tanh^{-1} \left[ \tanh \left( \frac{e \varphi_d}{4k_B T} \right) \exp \left( -x / \chi \right) \right],
\]

where \( \varphi_d \) is the electrical potential at the start of the diffuse layer (\( \varphi_\beta = \varphi_d \)) and \( x \) is the position from the OHP (in m).
Figure 9. Computed thickness of the diffuse layer (equal to one Debye length) and of the mobile part of the Stern layer (a) and surface site density of adsorbed Na\(^+\) ion in the Stern and diffuse layers (b) as a function of NaCl concentration for Fontainebleau (F) and Lochaline (L) samples.

The computed thickness of the diffuse layer decreases significantly at high salinity to become comparable to the hydrated radius of sodium ion (\(\approx 2\ \text{Å}\)) but it remains considerably larger than the thickness of the mobile part of the Stern layer (0.5 Å and 0.3 Å for Fontainebleau and Lochaline samples, respectively) (Figure 9a). However, when salinity increases, the computed surface site density of adsorbed Na\(^+\) ion in the Stern layer increases considerably more than in the diffuse layer (Figure 9b), which explains the increasing contribution of the counter-ions in the mobile part of the Stern layer to the measured streaming potential.

Our new surface complexation model applied simultaneously for both Fontainebleau and Lochaline samples (all five samples together) in a NaCl aqueous solution (termed here the unified new model, UNM) was still able to reproduce the zeta potential measurements well. Indeed, the values of the coefficient of determination were still close to 1 when calculated for the entire salinity range (Table 3). The UNM reproduced very well the low salinity measurements, and the quality of match was
similar to the results obtained using the unified classical model, UCM. Across the high salinity domain, the UNM was also found to provide a better match to the experimental data compared with the UCM (except for L4Q sample at high salinity). The values of the optimized parameters used in UNM (Table 4) agreed with the values previously reported in Table 1, and both sets were consistent with the values reported in the literature for quartz in a NaCl aqueous solution. Therefore, our approach is relevant for obtaining a unified surface complexation model for quartz in a NaCl solution.

Table 3. Coefficient of determination values using a single set of BSM parameter values for Fontainebleau and Lochaline sandstones together.

<table>
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</table>

$^1$Low salinity, below 0.1 M NaCl (Fontainebleau samples) and 1 M NaCl (Lochaline samples).
$^2$High salinity, above 0.1 M NaCl (Fontainebleau samples) and 1 M NaCl (Lochaline samples).

Table 4. BSM parameter values and estimated Stern layer thickness for quartz (combining Fontainebleau and Lochaline sandstones).

<table>
<thead>
<tr>
<th>Symbols</th>
<th>Range$^1$</th>
<th>UCM</th>
<th>UNM</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\log K_H$</td>
<td>[4 10]</td>
<td>7.28 [7.24 7.31]</td>
<td>7.31 [7.27 7.34]</td>
</tr>
<tr>
<td>$\log K_{Na}$</td>
<td>[-20 5]</td>
<td>-20 [ND$^3$]</td>
<td>0.58 [0.27 0.83]</td>
</tr>
<tr>
<td>$C_1$ (F m$^{-2}$)</td>
<td>[0.5 5]</td>
<td>2.26 [1.78 2.96]</td>
<td>3.43 [2.92 4.02]</td>
</tr>
<tr>
<td>$d$ (Å)</td>
<td>[0 10]</td>
<td>0</td>
<td>0.20 [0.17 0.24]</td>
</tr>
<tr>
<td>$d_{Stern}^2$ (Å)</td>
<td>1.68 [1.29 2.14]</td>
<td>1.11 [0.95 1.30]</td>
<td></td>
</tr>
</tbody>
</table>

$^1$Hiemstra et al. [20], Kitamura et al. [23], Sonnefeld et al. [62], Sverjensky [3], García et al. [6].
$^2$According to Eq. (2) and fitted $C_1$ values, considering $\varepsilon_i = 43\varepsilon_0$ and $d_{Stern} = x_B - x_0$.
$^3$Not determined.
We developed a new basic Stern surface complexation model to explain the zeta potential measurements on quartz in contact with NaCl aqueous solutions and to describe the concentration dependence of the electrochemical properties of quartz over a broad salinity range (from around $10^{-4}$ M NaCl up to around 5.5 M NaCl). Previous surface complexation models considered that the shear plane of quartz in contact with a NaCl aqueous solution was located at the Stern plane where sodium counter-ions were preferentially adsorbed or even further away from the mineral surface. In contrast to previous models, our new model considered that there could be some water flow transporting counter-ions within the Stern layer, i.e. that the shear plane where the zeta potential is defined was located closer to the mineral surface than the Stern plane.

Compared to the model considering the zeta potential at the Stern plane, our new model better reproduced the zeta potential measurements on Fontainebleau and Lochaline sandstones, especially in high salinity conditions (above 0.1 M NaCl for Fontainebleau samples and 1 M NaCl for Lochaline samples) where zeta potential appeared to level off at a constant negative value. This was particularly true for Fontainebleau samples. We found a small shear plane offset distance from the Stern plane of around 0.3–0.5 Å, i.e. only a small part of the Stern layer was mobile, confirming that the shear plane was still at a close proximity to the Stern plane. In addition, the optimized value of the equilibrium constant describing sodium adsorption in the Stern layer in our new model was more realistic compared with the classical approach considering zero separation distance between the Stern and the shear planes. The predicted surface charge density of quartz of the new model was also in a better agreement with the experimental data. We also explained, based on SEM micrograph images and our new surface complexation model, why Fontainebleau rocks, with
sharper-angle grains and larger surface roughness, had smaller in magnitude zeta potential for the same NaCl concentration compared against Lochaline data.

Our approach can be used to interpret and even predict streaming potential measurements and other types of electrokinetic measurements (e.g., electrophoretic mobility) on quartz and other minerals in contact with brines of different chemical compositions and temperatures. Therefore, our results, which should be confirmed by laboratory measurements at the microscopic scale (e.g., using microfluidics and spectroscopy methods) and atomistic simulations, may have strong implications for the modelling of the electrochemical properties of minerals in contact with highly saline brines. Our results may be of crucial importance for exploring mineral-brine interactions at high salinity levels close to real subsurface conditions.

Acknowledgments

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Appendix A. Basic Stern surface complexation model

The following two surface complexation reactions were considered for the zeta potential modelling:

$$> \text{SiO}^- + \text{H}^+ \leftrightarrow \text{SiOH} \; , \; K_H ,$$  \hspace{1cm} (A8)

$$> \text{SiO}^- + \text{Na}^+ \leftrightarrow \text{SiO}^- - \text{Na}^+ \; , \; K_{Na} ,$$  \hspace{1cm} (A9)

where $K_H$ and $K_{Na}$ (dimensionless) are the associated equilibrium constants, which are written as:

$$K_H = \frac{a_{> \text{SiOH}}}{a_{> \text{SiO}^-} a_{\text{H}^+}^{-1}} \approx \frac{\Gamma_{> \text{SiOH}}}{\Gamma_{> \text{SiO}^-} a_{\text{H}^+}^{-1}} = \frac{\Gamma_{> \text{SiOH}}}{\Gamma_{> \text{SiO}^-} a_{\text{H}^+}^{-1}} \exp\left(\frac{e\varphi_0}{k_B T}\right),$$  \hspace{1cm} (A10)

$$K_{Na} = \frac{a_{> \text{SiO}^- - \text{Na}^+}}{a_{> \text{SiO}^-} a_{\text{Na}^+}^{-1} e} \approx \frac{\Gamma_{> \text{SiO}^- - \text{Na}^+}}{\Gamma_{> \text{SiO}^-} a_{\text{Na}^+}^{-1}} = \frac{\Gamma_{> \text{SiO}^- - \text{Na}^+}}{\Gamma_{> \text{SiO}^-} a_{\text{Na}^+}^{-1}} \exp\left(\frac{e\varphi_0}{k_B T}\right).$$  \hspace{1cm} (A11)

where $a_i$ is the activity (dimensionless) and $\Gamma_i$ is the surface site density (sites m$^{-2}$) of species $i$, $e$ is the elementary charge ($\approx 1.602 \times 10^{-19}$ C), $\varphi$ is the electrical potential (V), $k_B$ is the Boltzmann constant ($\approx 1.381 \times 10^{-23}$ J K$^{-1}$), and $T$ is the temperature (K). In equations (A3) and (A4), the superscript “$\infty$” refers to ion activities in the electroneutral free or bulk electrolyte (not influenced by the mineral surface), which were computed using Pitzer theory (Appendix B) [64].

The following determined system of equations for the surface charge density at the mineral surface, $Q_0$ (C m$^{-2}$), at the $\beta$-plane, $Q_\beta$, and of the diffuse layer, $Q_S$, was used to compute the electrical potential distribution at the interface between quartz and bulk NaCl solution as a function of the equilibrium constants and Stern layer capacitance [21]:
\[ Q_0 = -e \left( \Gamma_{\text{SiO}^2} + \Gamma_{\text{SiO}^-} - \Gamma_{\text{Na}^+} \right) = -\frac{e \Gamma_S}{A} \left[ 1 + K_{\text{Na}} a_{\text{Na}}^e \exp \left( -\frac{e \varphi_\beta}{k_B T} \right) \right], \]  

(A12)

\[ Q_\beta = e \Gamma_{\text{SiO}^- - \text{Na}^+} = \frac{e \Gamma_S}{A} K_{\text{Na}} a_{\text{Na}}^e \exp \left( -\frac{e \varphi_\beta}{k_B T} \right), \]  

(A13)

\[ A = 1 + K_{\text{H}} a_{\text{H}}^e \exp \left( -\frac{e \varphi_0}{k_B T} \right) + K_{\text{Na}} a_{\text{Na}}^e \exp \left( -\frac{e \varphi_\beta}{k_B T} \right), \]  

(A14)

\[ Q_S = \sqrt{8e w k_B T 1000 N_A I} \sinh \left[ -\frac{e \varphi_\beta}{2k_B T} \right], \]  

(A15)

\[ Q_0 + Q_\beta + Q_S = 0, \]  

(A16)

\[ \varphi_0 - \varphi_\beta = \frac{Q_0}{C_1}, \]  

(A17)

where \( \Gamma_S \) is the total surface site density (we took \( \Gamma_S = 4.6 \) sites nm\(^{-2}\) García et al. [6]), \( I \) is the molar ionic strength (mol L\(^{-1}\)), and \( \varphi_0 \) and \( \varphi_\beta \) are the electrical potentials at the 0-plane and at the \( \beta \)-plane, respectively (considering \( \varphi_\beta = \varphi_d \) for the BSM, where \( \varphi_d \) is the electrical potential at the start of the diffuse layer).
Appendix B. Pitzer model for ion activity coefficients in bulk electrolyte

The following equations were used to compute ion activity coefficients in bulk electrolyte [64]:

\[ a_i^\infty = \gamma_i^\infty \frac{m_i^\infty}{m_0} , \]  

(B1)

\[ m_i^\infty = \frac{1000 c_i^\infty}{M_w c_w^\infty} , \]  

(B2)

\[ c_w^\infty = \frac{10^3 - \sum_i c_i^\infty V_i}{V_w} , \]  

(B3)

where \( \gamma_i^\infty \) is the activity coefficient (dimensionless), \( m_i^\infty \) is the molality (mol per kilogram of water, mol kg\( w^{-1} \)), \( m_0 \) being the unit molality equal to 1 mol kg\( w^{-1} \), \( c_i^\infty \) is the molar concentration (M), and \( V_i \) is the standard partial molal volume (cm\(^3\) mol\(^{-1}\)) of ion \( i \) in bulk electrolyte. The quantity \( V_i \cong 18.07, 0, -1.13, 17.68 \) cm\(^3\) mol\(^{-1}\) for H\(_2\)O, H\(^+\), Na\(^+\) (due to electrostriction) and Cl\(^-\), respectively, at a temperature of 25°C. The subscript “\( w \)” in equations (B2) and (B3) refers to water molecules, and \( M_w \) refers to the molar mass of water (\( \cong 18 \) g mol\(^{-1}\)).

Na\(^+\) activity coefficient in bulk electrolyte influences modelled Na\(^+\) adsorption in the Stern plane (\( \Gamma_{\text{Na}} \Gamma_{\text{SiO}} \frac{m_{\text{Na}}^\infty}{m_0} \exp\left(-e\varphi_{\beta}/k_BT\right) \) from equations (A3) and (A4)). According to Pitzer theory, which is suitable for very saline aqueous solutions (ionic strengths above 0.1 M Harvie and Weare [67]), the natural logarithm of Na\(^+\) activity coefficient in NaCl electrolyte is written as:
\[
\ln \gamma_{Na^+} = z_{Na^+}^2 F + m_{Cl}^{\infty} \left[ 2B_{Na^+Cl^+} + (m_{Na^+}^{\infty} + m_{Cl}^{\infty}) C_{Na^+Cl^+} \right] + z_{Na^+} m_{Na^+} m_{Cl}^{\infty} C_{Na^+Cl^+}, \tag{B4}
\]

\[
F = -A_\phi \left[ \frac{\sqrt{I_m}}{1 + b \sqrt{I_m}} + \frac{2}{b} \ln \left( 1 + b \sqrt{I_m} \right) \right] + m_{Na^+} m_{Cl}^{\infty} B_{Na^+Cl^+}, \tag{B5}
\]

\[
A_\phi = \frac{1}{3} \sqrt{\frac{2\pi N_A \rho_w}{1000}} \left( \frac{e^2}{4\pi \varepsilon_w k_B T} \right)^{3/2}, \tag{B6}
\]

\[
B_{Na^+Cl^+} = -\frac{2\beta_{Na^+Cl^+}^{\infty}}{I_m x_1^2} \left[ 1 - \left( 1 + x_1 + 0.5 x_1^2 \right) \exp(-x_1) \right], \tag{B7}
\]

\[
x_1 = \alpha_1 \sqrt{I_m}, \tag{B8}
\]

\[
B_{Na^+Cl^+} = \beta_{Na^+Cl^+}^{\infty} + \frac{2\beta_{Na^+Cl^+}^{\infty}}{x_1^2} \left[ 1 - \left( 1 + x_1 \right) \exp(-x_1) \right], \tag{B9}
\]

\[
C_{Na^+Cl^+} = \frac{C_{\infty Na^+Cl^+}}{2\sqrt{|z_{Na^+} z_{Cl^+}|}}, \tag{B10}
\]

where \( z_i \) is the charge number of ion \( i \), \( b \) and \( \alpha_1 \) are empirical parameters (\( b = 1.2, \alpha_1 = 2 \) for 1:1 and 1:2 electrolytes), \( I_m \) is the molal ionic strength (in mol kg\(^{-1} \), \( I_m = m_{Na^+}^w \) here), and \( A_\phi \) is the Debye-Hückel coefficient describing long-range electrostatic interaction forces between ions (\( \equiv 0.392 \) at a temperature \( T \) of 298 K). The Debye-Hückel coefficient was computed here as a function of the Avogadro number \( N_A \) (\( \equiv 6.022 \times 10^{23} \) sites mol\(^{-1} \)), the water volumetric density \( \rho_w \) (\( \equiv 997 \times 10^3 \) g m\(^{-3} \)), and the water permittivity \( \varepsilon_w \) (\( \equiv 78.3 \varepsilon_0 \) where \( \varepsilon_0 \) is the vacuum permittivity with a value of \( \equiv 8.854 \times 10^{-12} \) F m\(^{-1} \)). The Debye-Hückel coefficient multiplied by the terms in brackets in equation (B5) is enough for computing ion activity coefficient in dilute aqueous solution (ionic
strength below 0.1 M). Pitzer and Mayorga [68] considered three additional terms (in equations (B4) and (B5)) to compute ion activity coefficients in concentrated aqueous solutions. The terms $B_{Na^+Cl^-}$ and $B'_{Na^+Cl^-}$ depend on the ionic strength and describe short-range interaction forces between one cation and one anion (binary system), and the term $C_{Na^+Cl^-}$ describes short-range interaction forces between two cations and one anion, and one cation and two anions (ternary system). The Pitzer model for ion activity coefficients in 1:1 aqueous electrolyte such as NaCl depends on three parameters $\beta_{Na^+Cl^-}^0$, $\beta_{Na^+Cl^-}^1$, and $C_{\phi Na^+Cl^-}$. The Pitzer parameter values were adjusted by matching computed to measured osmotic coefficients. According to [Leroy et al. [64]] $\beta_{Na^+Cl^-}^0 = 0.0765$, $\beta_{Na^+Cl^-}^1 = 0.2664$, and $C_{\phi Na^+Cl^-} = 0.00127$. 
References


Declaration of interests

☒ The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

☐ The authors declare the following financial interests/personal relationships which may be considered as potential competing interests:
**Philippe Leroy:** Conceptualization, Methodology, Software, Writing - Original Draft, Validation, Visualization.

**Alexis Maineult:** Software, Formal analysis.

**Shuai Li:** Investigation.

**Jan Vinogradov:** Writing- Reviewing and Editing.
The zeta potential of quartz.

Surface complexation modelling to elucidate high salinity measurements

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Abstract

The zeta potential is a measurable electrical potential of paramount importance to understand the electrochemical properties of rocks. However, the zeta potential remains poorly understood because it takes place at the nanoscale of the electrical double layer on the mineral surface.

Streaming potential measurements on quartz-rich Fontainebleau and Lochaline sandstones carried out at high salinity (above 0.1 M NaCl) yield surprisingly high zeta potential values, which cannot be correctly reproduced by a traditional surface complexation model considering that the shear plane is located at the beginning of the diffuse layer. We found that placing the shear plane, where the zeta potential is defined, slightly closer to the mineral surface than the Stern plane significantly improves the predictions of the zeta potential and surface charge density of quartz at high salinity as well as the values of the equilibrium constant describing sodium adsorption in the Stern layer.

Our results have strong implications for the modelling of the electrochemical properties of minerals in contact with highly saline solutions.

Key words: zeta potential, quartz, streaming potential, high salinity, shear plane location
1. Introduction

Quartz is a mineral that is particularly interesting to study because of its natural abundance and usefulness in the development of new technologies [1]. In contact with water, quartz develops a surface charge attracting counter-ions and repelling co-ions, thus forming the so-called electrical double layer (EDL) usually represented by a “compact” Stern layer and a diffuse layer [2, 3]. Investigating the electrochemical properties of quartz is of great interest in many applications in physics, chemistry and Earth sciences because these properties control adsorption and dissolution/precipitation reactions, and wettability on the quartz surface [4-6]. The EDL of quartz is also the source of electrokinetic and geophysical electrical (e.g., self-potential, resistivity, induced polarization) measurements that are used to map for instance geological fluid flows or biogeochemical reactions [7-12]. Studying quartz electrochemical properties notably when quartz is in contact with highly saline brines has a high potential in many geo-environmental and engineering applications including geo-sequestration of CO₂ in deep saline aquifers, and oil and gas exploration and production notably enhanced hydrocarbon recovery [13-17].

Exploring the electrochemical properties of quartz is very challenging because of their nanoscopic nature [1, 18, 19]. Indeed, surface complexation reactions between surface sites and ions in the aqueous solution occur at the nm-scale [3, 20-22]. In addition, natural quartz has a low specific surface area (typically below 0.1 m² g⁻¹), which considerably complicates the experimental characterization of its EDL compared to minerals with a large specific surface area such as montmorillonite [2, 3, 23, 24]. Only few methods exist to probe the properties of the EDL on the surface of minerals in contact with brines. Among them, there is the streaming potential method, which implies application of a water pressure difference across the sample while measuring the resulting voltage, the streaming potential, due to the displaced excess counter-ions in the EDL [25-
From the measured streaming potential it is possible to obtain some relevant information on the electrochemical properties of minerals through the calculation of the electrokinetic zeta potential ($\zeta$), which is defined as the electrical potential at the shear (or slip) plane [17, 25, 27]. The zeta potential determined experimentally can be interpreted in terms of mineral electrochemical properties by matching observed and simulated zeta potential using a relevant surface complexation model [21, 30, 31]. However, this approach relies on the assumption that the exact location of the shear plane from the mineral surface is known, which is obviously not the case because of the lack of experimental information at the molecular level [32-35]. Moreover, the zeta potential is, most of the times, the only physico-chemical quantity available to validate the predictions of electrostatic surface complexation models for low specific surface area minerals such as quartz or calcite [3, 30, 36]. In addition, the zeta potential is inferred from electro-hydrodynamic measurements while surface complexation models rely on electrostatics at thermodynamic equilibrium [25, 35, 37, 38]. Therefore, these limitations contribute to additional uncertainties when investigating mineral electrochemical properties from zeta potential measurements.

When water flow relative to the mineral surface takes place, it is widely accepted that the shear plane is located between the “stagnant” Stern layer bounded by the outer Helmholtz plane (OHP) and the diffuse layer because high water viscosity in the Stern layer prevents water flow within it [3, 25, 39] (Figure 1). The Stern layer of silica-based materials such as amorphous silica and quartz in contact with a NaCl solution is traditionally represented by a hydration layer followed by a layer containing hydrated sodium counter-ions [18, 21, 40]. Some molecular dynamic (MD) simulations (e.g., Zhang et al. [33]), spectroscopy measurements (e.g., Lis et al. [41]) and microfluidic studies (e.g., Saini et al. [42] and Werkhoven et al. [43]) have demonstrated that there could be a non-zero
flow of water within the Stern layer of silica notably because some counter-ions (such as Na\(^+\)) are not sticked to the mineral surface and form outer sphere surface complexes keeping their hydration shell. This implies that there may be some, even weak, water displacement within the Stern layer of silica, and hydrous oxide in general. Therefore, for quartz, the effective shear plane may be located slightly closer to the mineral surface than the outer Helmholtz plane, in agreement with the assumption accepted by most that the shear plane is located at the proximity of the OHP (e.g., Hunter [25], Sverjensky [3], García et al. [6]).

Figure 1. Sketch showing water flow and ion distribution at the interface between a silica mineral and a NaCl aqueous solution (modified, from Brown et al. [40]). Circles with arrows inside represent water molecules. The shear plane is denoted by the red dashed line. Counter-ions adsorbed as outer sphere complexes form the outer Helmholtz plane (OHP).

The quartz (0001) crystal face is the most stable plane with the lowest surface energy and is often considered as a “model surface”, convenient for modelling SiO\(_2\) materials and hydrophilic surfaces.
in general [35]. With the improved accuracy of the streaming potential method, it is now possible to accurately measure extremely small voltages due to the displacement of the ions in the EDL of quartz [14]. Published studies of Jaafar et al. [8], Vinogradov et al. [13], Walker et al. [44], and Walker and Glover [15] observed, that at high salinities (NaCl concentrations above 0.4 M, M means mol L\(^{-1}\)), the zeta potential of sandstones appears to level off at a small constant negative value between \(-30\) and \(-10\) mV or even to increase slightly in magnitude (i.e. become more negative) with salinity. They noted that the zeta potential of sandstones stabilizes at a salinity of about 0.4 M NaCl that corresponds to a Debye length characterizing the diffuse layer thickness of approximately 0.47 nm, which is similar to the size of a hydrated sodium ion. This observation led them to suggest that the constant zeta potential of sandstones at high salinities reflected the maximum charge density in the diffuse layer which was reached when the diffuse layer thickness approached the diameter of the counter-ions [45]. However, Jaafar et al. [8], Vinogradov et al. [13], Walker et al. [44], Glover [45], and Walker and Glover [15] did not explicitly explain this behavior through a basic Stern surface complexation model describing their zeta potential measurements on sandstones.

In our study, we used a surface complexation model named basic Stern model (BSM) and considered that the shear plane is at the OHP or closer to the mineral surface than the OHP to describe the zeta potential and the electrochemical properties of quartz at varying NaCl concentrations. In our model we described the effective location of the OHP and the shear plane, hence modelling the effective zeta potential. Therefore, the developed surface complexation model accurately replicated the experimental conditions under which the streaming potential measurements on intact rock samples comprising grains of various shape and roughness were conducted. The model predictions were compared to the existing experimental zeta potential data.
measured over a broad salinity range (from around $10^{-4}$ M NaCl up to around 5.5 M NaCl). The values of the optimized parameters were finally discussed. Our findings shed light on the electrochemical properties of quartz and on the likelihood of non-zero water flow within the Stern layer.

2. Theoretical background

2.1. Surface complexation model for quartz

Our basic Stern model [37, 46] describes proton (H+) adsorption onto $>\text{SiO}^-$ surface sites at the 0-plane (defining the mineral surface) and sodium cation (Na+) adsorption by these surface sites at the $\beta$-plane (Stern plane and OHP) (Figure 2) [3, 6, 20, 21, 23]. The BSM considers that the $\beta$-plane coincides with the $d$-plane defining the start of the diffuse layer. This model only needs one Stern layer capacitance as an input parameter to model the electrical potential distribution between the mineral surface and the Stern plane. Recent studies utilizing atomic force microscopy (AFM) (e.g., Siretanu et al. [47]) and X-ray photoelectron spectroscopy (XPS) (e.g., Brown et al. [40]) used the BSM to model the electrochemical properties of amorphous silica in contact with a NaCl aqueous solution and demonstrated that the BSM could accurately reproduce the experimental data. García et al. [6] also used the BSM to match the measured electrochemical properties of quartz in contact with a NaCl aqueous solution thus confirming the validity of the approach.
Figure 2. Sketch of our basic Stern model to describe the electrochemical properties of the interface between quartz and a 1:1 electrolyte like NaCl electrolyte (the $\beta$-plane coincides with the $d$-plane). The model input parameters are shown in blue and the model output parameters, including the zeta potential ($\zeta$) at the shear plane, are shown in red.

In our BSM we used four adjustable parameters, namely the logarithms of the two adsorption equilibrium constants $K_H$ and $K_Na$, the Stern layer capacitance $C_1$ (F m$^{-2}$), and the distance $d$ between the shear plane (where the zeta potential is defined) and the $\beta$-plane (Figure 2). It should be noted that we considered that the doubly coordinated surface groups (>Si$_2$O$^0$) are inert [20] and that the protonated silanol sites (>SiOH$_2^+$) are not expected to form at close-to-neutral pH of the streaming potential measurements on sandstones (pH varied between 6.4 and 7.3 Walker and Glover [15]). Therefore, these surface sites were excluded from the model. In absence of additional
measurements, we also did not consider another type of silanol group, hence we made our model as simple as possible in order to decrease the number of optimized parameters. For more information related to our BSM, the reader can refer to Appendixes A and B, and to Leroy et al. [21].

2.2. Zeta potential computation

All calculations were performed by combining the geochemical software IPhreeqc for the surface complexation modelling [48] with an in-house code implemented in Matlab for the calculation of the zeta potential and the optimization procedure [49]. The zeta potential (V) defined at the shear plane located at a distance $d$ from the $\beta$-plane was determined from the computed $\varphi_0$ and $\varphi_\beta$ electrical potentials by considering a linear, capacitor-like variation of the electrical potential within the Stern layer [25]

$$\zeta = \varphi_\beta - \left( \frac{\varphi_\beta - \varphi_0}{x_\beta - x_0} \right) d,$$

(1)

where $x$ is the distance from the mineral surface (defined by the 0-plane, in m). Combining equation (1) with the following equation for the Stern layer capacitance [18]

$$C_1 = \frac{\varepsilon_1}{x_\beta - x_0},$$

(2)

where $\varepsilon_1$ is the water permittivity in the Stern layer (F m$^{-1}$; we used $\varepsilon_1 = 43\varepsilon_0$, where $\varepsilon_0$ is the vacuum permittivity, in accordance with the study of Sverjensky [3]), we finally obtain an expression for the zeta potential as a function of the modelled electrochemical properties.
\[ \zeta = \varphi_\beta - (\varphi_\beta - \varphi_0) \frac{C_1}{\varepsilon_1} d. \] (3)

We did not consider the presence of a stagnant diffuse layer (also named buffer layer), which implies that the shear plane is located further away from the mineral surface, as suggested in Alizadeh and Wang [50]. To the best of our knowledge, the stagnant diffuse layer existence has never been directly confirmed experimentally. To the contrary, Předota et al. [35], Brkljača et al. [19], and Biriukov et al. [34] predicted no such stagnant diffuse layer from their molecular dynamic simulations of the zeta potential of the hydroxylated (110) rutile (TiO\(_2\)) and (0001) quartz surfaces.

Furthermore, Leroy and co-workers. Furthermore, Leroy and co-workers (e.g., Leroy et al. [51], Leroy et al. [52], Leroy et al. [21], Li et al. [30]) attributed the assumption of the presence of a stagnant diffuse layer in previous studies to the misinterpretation of the zeta potentials from electrokinetic (e.g., electrophoretic mobility, streaming potential) measurements due to disregard of surface conductivity effects. Indeed, surface conductivity decreases the magnitude of the measured electrokinetic signal hence implying smaller apparent zeta potentials, which need to move away the shear plane from the mineral surface when modelling the zeta potential from a surface complexation model.

The parameters of our surface complexation model (\( \log K_\text{H}, \log K_\text{Na}, C_1, d \)) were optimized by minimizing the following cost function [53]:

\[ y = 1 - R^2 = \frac{\sum_{i=1}^{N} (\zeta^i_{\text{mes}} - \zeta^i_{\text{mod}})^2}{\sum_{i=1}^{N} (\zeta^i_{\text{mes}} - \langle \zeta_{\text{mes}} \rangle)^2}, \] (4)

where \( R^2 \) is the coefficient of determination, \( N \) is the number of zeta potential measurements, \( \zeta^i_{\text{mes}} \) is the \( i \)-th measured zeta potential, \( \langle \zeta_{\text{mes}} \rangle \) is the arithmetic mean of the measured zeta potentials,
and $\zeta_{\text{mod}}^i$ is the $i$-th modelled zeta potential. The fitting procedure was realized by using the simulated annealing algorithm to find the global minimum of the cost function (equation (4)), with a refinement using the simplex method at the end of the process [49].

3. Comparison with experimental data and discussion

3.1. Considerations of impact of pore space topology and grain roughness on EDL parameters

To test our model, we used the measured zeta potentials of Fontainebleau (F2D, F3Q, F4Q) and Lochaline (L3Q, L4Q) samples in contact with a NaCl aqueous solution of increasing salinity obtained by the streaming potential method and reported in Walker and Glover [15]. These two sample types were selected as they are known to consist of more than 99% quartz (by weight) [13, 54]. Unlike Fontainebleau and Lochaline samples, zeta potentials of Berea and Boise sandstones reported by Walker and Glover [15] that contained up to 6% feldspar, 2% dolomite, and 8% clays for Berea rocks [55] and up to 13% clays for Boise rocks [56], were excluded from the simulation. Despite the fact that feldspar, dolomite, and clay content in Berea and Boise samples is relatively small, clays are known to line pore walls, thus making these complex minerals a main contributor to the electrochemical processes at the mineral-water interface and causing anomalous or even positive zeta potentials [57, 58]. Therefore, the experimental zeta potential data for Boise and Berea samples were deemed unapplicable for our model that considers only surface complexation reactions on quartz surface.

All Fontainebleau and Lochaline samples exhibit a negative zeta potential with its magnitude decreasing with increasing salinity (Figures 3 and 4). The zeta potentials of Lochaline samples...
were found to be of a larger magnitude than those of Fontainebleau samples. Scanning electron microscopy (SEM) micrographs of the tested samples showed that Fontainebleau rock has sharper-angled grains with larger surface roughness and smaller grains than Lochaline rock (Figure 5 from Walker and Glover [15]). According to Vinogradov et al. [14], pore space topology, grain shape, surface roughness and size influence streaming potential measurements. They considered that rough rocks with small grains have smaller streaming and zeta potential magnitudes than round, smooth rocks with large grains because rock sharp corners and grain roughness would shift the effective shear plane further away from the mineral surface (read their section 4.2). Alroudhan et al. [59] used the same assumption to explain that the zeta potential of colloidal suspensions measured by the electrophoretic mobility method is larger in magnitude than the zeta potential of rocks measured by the streaming potential method (see their Figure 10 and read the related discussion in their section 5.2). Schnitzer and Ripperger [60] and Drechsler et al. [61] showed that increasing surface roughness changes the flow velocity distribution on the solid surface shifting the shear plane further away from the solid surface and decreases the streaming and zeta potential magnitudes. According to these observations, we expected different values of the surface complexation model parameters between Fontainebleau and Lochaline samples, notably for the Stern layer capacitance $C_1$ and the distance $d$ of the shear plane from the OHP (or Stern plane), which are very sensitive to the textural properties of rocks ($C_1$ depends on the thickness of the Stern layer, equation (2)).
Figure 3. Zeta potentials of Fontainebleau samples as a function of NaCl concentration. Circle symbols: experimental zeta potential data with the sample name corresponding to that used by Walker and Glover [15]; curves: model predictions.
Figure 4. Zeta potentials of Lochaline samples as a function of NaCl concentration. Circle symbols: experimental zeta potential data with the sample name corresponding to that used by Walker and Glover [15]; curves: model predictions.
Figure 5. SEM micrographs of Fontainebleau (a) and Lochaline (b) rocks (modified from Walker and Glover [15]).

Figures 3 and 4 demonstrate that below the concentration thresholds of around 0.1 M NaCl (Fontainebleau samples) and 1 M NaCl (Lochaline samples) (denoted by the vertical black dotted lines), the magnitude of the negative zeta potential decreases linearly with increasing salinity. Interestingly, the rate of decrease in the zeta potential magnitude with increasing salinity became smaller above these thresholds, i.e. it became non-linear, and eventually stabilized (or even slightly
increased in magnitude) at a zeta potential value of approximately -15 mV for both rock types. Such stabilization of the zeta potential was more apparent for Fontainebleau than for Lochaline samples. These observations were consistent across the data reported by Vinogradov et al. [13], Vinogradov et al. [14] and Walker and Glover [15], who stated that at high salinities, the measured zeta potential stabilized and became equal to $-13.01 \pm 0.48$ mV for Fontainebleau samples and to $-16.81 \pm 0.68$ mV for Lochaline samples.

Considering that Fontainebleau and Lochaline sandstones did not have the same pore space topology and textural properties, we first optimized separately the parameters of the surface complexation models for these two rock types. That is, a single model was developed for F2D, F3Q, F4Q combined data (Fontainebleau rocks) and a separate model was developed for L3Q, L4Q combined data (Lochaline rocks) to match simulated to observed zeta potentials. We ran the classical model denoted CM with the parameters $\log K_H$, $\log K_{Na}$, and $C_1$, and the new model denoted NM with the parameters $\log K_H$, $\log K_{Na}$, $C_1$, and $d$ (the distance of separation between the shear plane and OHP), to investigate the effect of the proposed inward shift of the shear plane on the simulated zeta potential while assigning measured pH values to the respective rock samples as reported by Walker and Glover [15]. We then used the same BSM approach for Fontainebleau and Lochaline samples together (all five samples, F2D, F3Q, F4Q, L3Q, L4Q) to develop a unified surface complexation model for quartz in contact with a NaCl aqueous solution, denoted UNM for unified new model and UCM for unified classical model.
3.2. Comparison of the computed to the observed zeta potentials and discussion

Overall, both the NM and CM reproduced well the experimental zeta potential data for the entire salinity range (Figures 3 and 4, and Tables 1 and 2). To estimate the uncertainties, we fixed two/three of the three/four parameters at their optimal values and then we computed the cost function (i.e., $y=1-R^2$) for the remaining parameter which is allowed to vary. Afterwards, we computed the relative cost function associated to the varying parameter $(y-y_{opt})/y_{opt}$, where $y_{opt}$ is the value of the cost function when the three/four parameters are fixed at their optimal values (so the relative cost function associated to the varying parameter is equal to zero for the optimal set of parameters). Finally, we extracted the range of values of the varying parameter for which the relative cost function is less than 0.1. We performed this procedure for the three/four parameters.

According to the surface complexation models, the observed negative zeta potential was due to the presence of the deprotonated silanol sites $>\text{SiO}^-\text{H}_n$ at the 0-plane (Figure 2). The optimized values of the equilibrium constant describing protonation of $>\text{SiO}^-\text{H}_n$ surface sites ($K_H$, reaction (1)) equal to $10^{7.3}$ and $10^{7.2}$ for Fontainebleau and Lochaline samples, respectively, were found to be close or similar to the spectroscopically determined value of $10^{7.2\pm0.2}$ and to the theoretical value of $10^{7.5}$ using Pauling’s definition of formal bond valence for silica [20] (Table 1). In addition, our $K_H$ optimized values were found to be close or similar to the value of $10^{7.2}$ determined by Sverjensky [3] using a triple layer model (BSM with an additional $C_2$ capacitance between the Stern plane and the start of the diffuse layer) matching surface charge density measurements inferred from acid base potentiometric titration on natural quartz in contact with a NaCl solution. The models also explained why the zeta potential magnitude of Lochaline samples was larger, for the same salinity, than the zeta potential magnitude of Fontainebleau samples. Indeed, Lochaline samples have higher
pH (i.e. less protons in solution) than Fontainebleau samples (7.1 versus 6.5 in average, respectively Walker and Glover [15]) while having essentially identical log $K_H$ values, which resulted in Lochaline samples having larger number of deprotonated $>\text{SiO}^-$ sites per nm$^2$ of surface and a higher negative surface charge density $Q_0$ (equation (A5)) than Fontainebleau samples (Figure 6).

### Table 1. BSM parameter values and estimated Stern layer thickness for Fontainebleau and Lochaline sandstones.

<table>
<thead>
<tr>
<th>Symbols</th>
<th>Range$^1$</th>
<th>Fontainebleau</th>
<th>Lochaline</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\log K_{Na}$</td>
<td>[-20 5]</td>
<td>-20 [ND$^3$]</td>
<td>0.58 [0.25 0.83]</td>
</tr>
<tr>
<td>$C_1$ (F m$^{-2}$)</td>
<td>[0.5 5]</td>
<td>3.24 [2.01 6.54]</td>
<td>1.34 [1.18 1.51]</td>
</tr>
<tr>
<td>$d$ (Å)</td>
<td>[0 10]</td>
<td>0</td>
<td>0.48 [0.42 0.54]</td>
</tr>
<tr>
<td>$d_{Stern}$ (Å)</td>
<td></td>
<td>1.18 [0.58 1.89]</td>
<td>2.85 [2.52 3.23]</td>
</tr>
</tbody>
</table>

$^1$ Hiemstra et al. [20], Kitamura et al. [23], Sonnefeld et al. [62], Sverjensky [3], García et al. [6].

$^2$ According to Eq. (2) and fitted $C_1$ values, considering $\epsilon_i = 43\epsilon_0$ and $d_{Stern} = x_\beta - x_0$.

$^3$ Not determined.
Figure 6. Computed surface site densities of $>\text{SiO}^-$ sites (a), $>\text{SiO}^-\text{Na}^+$ sites (b), and of surface charge densities (c) of Fontainebleau and Lochaline samples as a function of NaCl concentration. Plain line curves correspond to the calculations using the NM, dotted line curves correspond to the
calculations using the CM. The CM predicted near-zero surface site densities of adsorbed sodium ion in the Stern layer (limited at $\equiv 0$ sites nm$^{-2}$ in Figure 6b).

We also found that Lochaline samples have significantly lower $\log K_{Na}$ values, i.e. weaker sodium adsorption capacity, than Fontainebleau samples (-21 vs -16, respectively, for CM and 0.1 vs 0.6, respectively, for NM, Table 1), which could not counterbalance the negative surface charge density as efficiently as for Fontainebleau samples, and can also explain the larger zeta potential magnitude of Lochaline samples. Interestingly, despite Lochaline samples having lower $\log K_{Na}$ values than Fontainebleau samples, the models found that Lochaline samples, for the same salinity, had a higher surface site density of adsorbed sodium ion in the Stern layer than Fontainebleau samples due to the higher $\text{SiO}^-$ surface site density (Figure 6b). The lower $\log K_{Na}$ values of Lochaline than Fontainebleau samples we found can be explained by Lochaline samples having smoother and larger grains and hence a smaller specific surface area than Fontainebleau samples. Sverjensky [3] did the same observation when comparing two quartz with different specific surface area (4.15 and 11.4 m$^2$ g$^{-1}$) in contact with a NaCl solution. The $K_{Na}$ values inferred from the CM are extremely low and essentially mean that there is no adsorption of Na$^+$ at the OHP at all and everything is controlled only by pH. With the CM, the optimization procedure decreases $K_{Na}$ to extremely low value to fit the high salinity zeta potential measurements (decreasing Na$^+$ adsorption in the Stern layer results to higher zeta potential magnitude).

With the NM, the optimization procedure doesn’t need to decrease $K_{Na}$ to extremely low value to fit the high salinity zeta potential measurements and it found $\log K_{Na}$ values (0.6 and 0.1 for Fontainebleau and Lochaline samples, respectively) within the same order of magnitude than the
value reported by Sverjensky [3] for natural quartz in a contact with a NaCl solution (log $K_{Na} = 0$).

In addition, on the contrary to the CM, our NM was able to reproduce most of the surface charge density measurements on Min-U-Sil 5 quartz (natural quartz with a mean grain diameter of 5 µm) at different pH and NaCl concentrations carried out by Riese [63] (Figure 7).

![Figure 7](image)

**Figure 7.** Surface charge density of Min-U-Sil 5 quartz as a function of pH and NaCl concentration. Curves correspond to the predictions. Symbols correspond to the experimental surface charge density data reported by Riese [63].

With the NM, the optimized Stern layer capacitance values were equal to 1.3 F m$^{-2}$ and 2.2 F m$^{-2}$ for Fontainebleau and Lochaline samples, respectively (Table 1), which were close to the values of 1 F m$^{-2}$ and 2 F m$^{-2}$ reported by Sverjensky [3] and García et al. [6], respectively, for natural quartz in contact with a NaCl solution. With the CM, the optimized Stern layer capacitance values were equal to 3.2 F m$^{-2}$ and 1.8 F m$^{-2}$ for Fontainebleau and Lochaline samples, respectively. Using the optimized Stern layer capacitance values from the NM, equation (2) and $\varepsilon_1 = 43\varepsilon_0$ [3, 40], we
found a Stern layer thickness comparable to the hydrated radius of sodium ion (≈ 2 Å) [Leroy et al. 2012] and Sverjensky [2010], with Fontainebleau samples having larger Stern layer thickness (2.8 Å) than Lochaline samples (1.7 Å), which can be explained by Fontainebleau samples having sharper and rougher grains than Lochaline samples [15, 65]. When using the CM, the Stern layer thickness we found for Fontainebleau samples (1.2 Å) was comparable to the crystallographic radius of sodium ion (1.02 Å) [Sverjensky 2010]. This result was not realistic regarding the representation of the quartz/NaCl solution interface containing mostly hydrated sodium ions in the Stern layer, which is accepted by most recent models (e.g., Brown et al. [66]). For Lochaline samples, the Stern layer thickness inferred from the CM was comparable to the hydrated radius of sodium ion (2.1 Å).

Figures 3, 4, 7, and the modelling results reported in Table 1 for the parameter values and in Table 2 for the coefficient of determination values clearly demonstrate the importance of considering the location of the shear plane to be closer to the mineral surface than the OHP. Indeed, as shown in Figures 3 and 4 and reflected by the values of the coefficient of determination at high salinity reported in Table 2 ($R^2 \geq 0.5$), the stabilization of the zeta potential at high salinity could only be correctly predicted by the NM (red curves in Figures 3 and 4). The stabilization of the modelled zeta potential at high salinity is explained by a growing abundance of sodium ions available for adsorption in the Stern layer, and therefore the decreasing number of $>\text{SiO}^-$ sites (Figures 6a and 6b), and importantly by the shear plane being located slightly closer to the mineral surface than the OHP. Moreover, the NM reproduced the surface charge density measurements on natural quartz in a NaCl solution reported in Riese [2013] significantly better than the CM (Figure 7) thus independently validating our assumption on the location of the shear plane.
Table 2. Coefficient of determination values using different BSM parameter values for Fontainebleau and Lochaline sandstones.

<table>
<thead>
<tr>
<th></th>
<th>F2D</th>
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<th>F4Q</th>
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<th>L4Q</th>
</tr>
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<td></td>
<td>CM</td>
<td>NM</td>
<td>CM</td>
<td>NM</td>
<td>CM</td>
</tr>
<tr>
<td>(R^2)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(R^2) LS(^1)</td>
<td>0.96</td>
<td>0.96</td>
<td>0.97</td>
<td>0.97</td>
<td>0.99</td>
</tr>
<tr>
<td>(R^2) HS(^2)</td>
<td>-0.31</td>
<td>0.60</td>
<td>-0.56</td>
<td>0.60</td>
<td>0.12</td>
</tr>
</tbody>
</table>

\(^1\) Low salinity, below 0.1 M NaCl (Fontainebleau samples) and 1 M NaCl (Lochaline samples).

\(^2\) High salinity, above 0.1 M NaCl (Fontainebleau samples) and 1 M NaCl (Lochaline samples).

The measured high salinity zeta potentials were closely matched by the BSM considering the shear plane slightly approaching the mineral surface, i.e. with a very small distance from the OHP \((d = 0.5 \text{ Å for Fontainebleau samples and } d = 0.3 \text{ Å for Lochaline samples; Table 1})\). Including such a small distance \(d\) between OHP and shear plane progressively increases computed zeta potential magnitude compared to not considering it when salinity increases (Figure 8). The effective distance \(d\) used in our NM was significantly smaller than the hydrated radius of Na\(^+\) (\(\approx 2 \text{ Å}\) Leroy et al. [64] Sverjensky [18]), which implied that only some of Na ions were mobilized in the Stern layer, i.e., only a small portion of all ions could move inside the Stern layer. In addition, \(d/d_{\text{Stern}}\) (Lochaline) = \(d/d_{\text{Stern}}\) (Fontainebleau) = 0.18. This means that regardless of rock type 18\% of the, previously considered as immobile ions in the Stern layer will be flowing. Then, the thicker the Stern layer is (and we expect it to become thicker as roughness increases), the larger \(d\) will become – exactly as NM predicts.

In addition, unlike the CM, the NM found that the shear plane of Fontainebleau samples is further away from the mineral surface than the shear plane of Lochaline samples, also explaining why the zeta potential magnitude of Fontainebleau samples is smaller than the zeta potential magnitude of Lochaline samples. Indeed, the total distance of the shear plane from the mineral surface \((d_{\text{Stern}} - \ldots\)
$d$ is larger for Fontainbleau ($2.8-0.5=2.3$ Å) compared with Lochaline ($1.7-0.3=1.4$ Å) samples, which is consistent with our hypothesis that rougher and sharper Fontainbleau grains push EDL further away from the mineral surface (both, the Stern plane and the shear plane). These findings were in agreement with the SEM micrographs showing that Fontainbleau rock has sharper-angled grains with larger surface roughness than Lochaline rock (Figure 5).

Figure 8. Computed zeta potential of Fontainbleau (F) and Lochaline (L) samples as a function of NaCl concentration considering or not the distance $d$ between the OHP and the shear plane.

In the classical theory of the electrical double layer, it is assumed that only the mobile excess counter-ions in the diffuse layer contribute to the measured macroscopic streaming potential [28]. However, the diffuse layer is highly compressed at high salinity, so that there are essentially no mobile counter-ions available inside it, and such near-zero contribution of the diffuse layer cannot explain correctly the non-zero zeta potentials in Fontainbleau and Lochaline sandstones at high salinity. Figure 9 shows the computed thicknesses of the diffuse layer and of the mobile part of the
Stern layer as well as the surface site density of adsorbed sodium ion in the Stern and diffuse layers, $\Gamma_{>\text{SiO}^-\text{Na}^+}$ and $\Gamma_{\text{Na}^+}^d$, respectively. The salinity dependence of the diffuse layer thickness was evaluated by the Debye length $\chi$:

$$\chi = \sqrt{\frac{\varepsilon_0 k_B T}{2e^2 1000 N_A I}}.$$  \hspace{1cm} (5)

and $\Gamma_{\text{Na}^+}^d$ was calculated using the following equations [24]:

$$\Gamma_{\text{Na}^+}^d = 1000 N_A e N_{\text{Na}^+} \int_{x=0}^{x=x} \left\{ \exp\left[-e\varphi_d(x)/k_B T\right] - 1 \right\} dx,$$  \hspace{1cm} (6)

$$\varphi_d(x) = \frac{4k_B T}{e} \tanh^{-1} \left[ \tanh \left( \frac{e\varphi_d}{4k_B T} \right) \exp\left(-x/\chi\right) \right].$$  \hspace{1cm} (7)

where $\varphi_d$ is the electrical potential at the start of the diffuse layer ($\varphi_\beta = \varphi_d$) and $x$ is the position from the OHP (in m).
Figure 9. Computed thickness of the diffuse layer (equal to one Debye length) and of the mobile part of the Stern layer (a) and surface site density of adsorbed Na\(^+\) ion in the Stern and diffuse layers (b) as a function of NaCl concentration for Fontainebleau (F) and Lochaline (L) samples.

The computed thickness of the diffuse layer decreases significantly at high salinity to become comparable to the hydrated radius of sodium ion (\(\cong 2 \text{ Å}\)) but it remains considerably larger than the thickness of the mobile part of the Stern layer (0.5 Å and 0.3 Å for Fontainebleau and Lochaline samples, respectively) (Figure 9a). However, when salinity increases, the computed surface site density of adsorbed Na\(^+\) ion in the Stern layer increases considerably more than in the diffuse layer (Figure 9b), which explains the increasing contribution of the counter-ions in the mobile part of the Stern layer to the measured streaming potential.

Our new surface complexation model applied simultaneously for both Fontainebleau and Lochaline samples (all five samples together) in a NaCl aqueous solution (termed here the unified new model, UNM) was still able to reproduce the zeta potential measurements well. Indeed, the values of the coefficient of determination were still close to 1 when calculated for the entire salinity range (Table 3). The UNM reproduced very well the low salinity measurements, and the quality of match was
similar to the results obtained using the unified classical model, UCM. Across the high salinity
domain, the UNM was also found to provide a better match to the experimental data compared
with the UCM (except for L4Q sample at high salinity). The values of the optimized parameters
used in UNM (Table 4) agreed with the values previously reported in Table 1, and both sets were
consistent with the values reported in the literature for quartz in a NaCl aqueous solution. Therefore,
our approach is relevant for obtaining a unified surface complexation model for quartz in a NaCl
solution.

Table 3. Coefficient of determination values using a single set of BSM parameter values for
Fontainebleau and Lochaline sandstones together.

<table>
<thead>
<tr>
<th></th>
<th>F2D</th>
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<th>F4Q</th>
<th>L3Q</th>
<th>L4Q</th>
</tr>
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<tbody>
<tr>
<td></td>
<td>UCM</td>
<td>UNM</td>
<td>UCM</td>
<td>UNM</td>
<td>UCM</td>
</tr>
<tr>
<td>$R^2$</td>
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<td>0.99</td>
<td>0.97</td>
<td>0.98</td>
<td>0.99</td>
</tr>
<tr>
<td>$R^2$ LS$^1$</td>
<td>0.97</td>
<td>0.97</td>
<td>0.95</td>
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<td>0.99</td>
</tr>
<tr>
<td>$R^2$ HS$^2$</td>
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<td>0.46</td>
<td>-0.60</td>
<td>0.29</td>
<td>0.05</td>
</tr>
</tbody>
</table>

$^1$Low salinity, below 0.1 M NaCl (Fontainebleau samples) and 1 M NaCl (Lochaline samples).
$^2$High salinity, above 0.1 M NaCl (Fontainebleau samples) and 1 M NaCl (Lochaline samples).

Table 4. BSM parameter values and estimated Stern layer thickness for quartz (combining
Fontainebleau and Lochaline sandstones).

<table>
<thead>
<tr>
<th></th>
<th>Range$^1$</th>
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<th>UNM</th>
</tr>
</thead>
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<tr>
<td>$\log K_H$</td>
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<td>7.28 [7.24 7.31]</td>
<td>7.31 [7.27 7.34]</td>
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<tr>
<td>$\log K_{Na}$</td>
<td>[-20 5]</td>
<td>-20 [ND$^3$]</td>
<td>0.58 [0.27 0.83]</td>
</tr>
<tr>
<td>$C_1$ (F m$^{-2}$)</td>
<td>[0.5 5]</td>
<td>2.26 [1.78 2.96]</td>
<td>3.43 [2.92 4.02]</td>
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<tr>
<td>$d$ (Å)</td>
<td>[0 10]</td>
<td>0</td>
<td>0.20 [0.17 0.24]</td>
</tr>
<tr>
<td>$d_{Stern}$ (Å)</td>
<td>1.68 [1.29 2.14]</td>
<td>1.11 [0.95 1.30]</td>
<td></td>
</tr>
</tbody>
</table>

$^1$Hiemstra et al. [20], Kitamura et al. [23], Sonnefeld et al. [62], Sverjensky [3], García et al. [6].
$^2$According to Eq. (2) and fitted $C_1$ values, considering $\varepsilon_i = 43\varepsilon_0$ and $d_{Stern} = x_\beta - x_0$.
$^3$Not determined.
4. Conclusions

We developed a new basic Stern surface complexation model to explain the zeta potential measurements on quartz in contact with NaCl aqueous solutions and to describe the concentration dependence of the electrochemical properties of quartz over a broad salinity range (from around $10^{-4}$ M NaCl up to around 5.5 M NaCl). Previous surface complexation models considered that the shear plane of quartz in contact with a NaCl aqueous solution was located at the Stern plane where sodium counter-ions were preferentially adsorbed or even further away from the mineral surface. In contrast to previous models, our new model considered that there could be some water flow transporting counter-ions within the Stern layer, i.e. that the shear plane where the zeta potential is defined was located closer to the mineral surface than the Stern plane.

Compared to the model considering the zeta potential at the Stern plane, our new model better reproduced the zeta potential measurements on Fontainebleau and Lochaline sandstones, especially in high salinity conditions (above 0.1 M NaCl for Fontainebleau samples and 1 M NaCl for Lochaline samples) where zeta potential appeared to level off at a constant negative value. This was particularly true for Fontainebleau samples. We found a small shear plane offset distance from the Stern plane of around 0.3–0.5 Å, i.e. only a small part of the Stern layer was mobile, confirming that the shear plane was still at a close proximity to the Stern plane. In addition, the optimized value of the equilibrium constant describing sodium adsorption in the Stern layer in our new model was more realistic compared with the classical approach considering zero separation distance between the Stern and the shear planes. The predicted surface charge density of quartz of the new model was also in a better agreement with the experimental data. We also explained, based on SEM micrograph images and our new surface complexation model, why Fontainebleau rocks, with
sharper-angle grains and larger surface roughness, had smaller in magnitude zeta potential for the same NaCl concentration compared against Lochaline data.

Our approach can be used to interpret and even predict streaming potential measurements and other types of electrokinetic measurements (e.g., electrophoretic mobility) on quartz and other minerals in contact with brines of different chemical compositions and temperatures. Therefore, our results, which should be confirmed by laboratory measurements at the microscopic scale (e.g., using microfluidics and spectroscopy methods) and atomistic simulations, may have strong implications for the modelling of the electrochemical properties of minerals in contact with highly saline brines. Our results may be of crucial importance for exploring mineral-brine interactions at high salinity levels close to real subsurface conditions.

Acknowledgments

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Appendix A. Basic Stern surface complexation model

The following two surface complexation reactions were considered for the zeta potential modelling:

\[ \text{SiO}^{-} + \text{H}^{+} \leftrightarrow \text{SiOH}^{-}, \quad K_{H}, \quad (A8) \]

\[ \text{SiO}^{-} + \text{Na}^{+} \leftrightarrow \text{SiO}^{-} - \text{Na}^{+}, \quad K_{Na}, \quad (A9) \]

where \( K_{H} \) and \( K_{Na} \) (dimensionless) are the associated equilibrium constants, which are written as:

\[ K_{H} = \frac{a_{\text{SiOH}^{-}}}{a_{\text{SiO}^{-}} a_{\text{H}^{+}}} \approx \frac{\Gamma_{\text{SiOH}^{-}}}{\Gamma_{\text{SiO}^{-}} a_{\text{H}^{+}}} = \frac{\Gamma_{\text{SiOH}^{-}}}{\Gamma_{\text{SiO}^{-}} a_{\text{H}^{+}}^{\infty}} \exp \left( \frac{e\phi_{0}}{k_{B}T} \right), \quad (A10) \]

\[ K_{Na} = \frac{a_{\text{SiO}^{-} - \text{Na}^{+}}}{a_{\text{SiO}^{-}} a_{\text{Na}^{+}}} \approx \frac{\Gamma_{\text{SiO}^{-} - \text{Na}^{+}}}{\Gamma_{\text{SiO}^{-}} a_{\text{Na}^{+}}} = \frac{\Gamma_{\text{SiO}^{-} - \text{Na}^{+}}}{\Gamma_{\text{SiO}^{-}} a_{\text{Na}^{+}}^{\infty}} \exp \left( \frac{e\phi_{\beta}}{k_{B}T} \right), \quad (A11) \]

where \( a_{i} \) is the activity (dimensionless) and \( \Gamma_{i} \) is the surface site density (sites m\(^{-2}\)) of species \( i \), \( e \) is the elementary charge (\( \approx 1.602\times10^{-19} \) C), \( \phi \) is the electrical potential (V), \( k_{B} \) is the Boltzmann constant (\( \approx 1.381\times10^{-23} \) J K\(^{-1}\)), and \( T \) is the temperature (K). In equations (A3) and (A4), the superscript “\( \infty \)” refers to ion activities in the electroneutral free or bulk electrolyte (not influenced by the mineral surface), which were computed using Pitzer theory (Appendix B) [64].

The following determined system of equations for the surface charge density at the mineral surface, \( Q_{0} \) (C m\(^{-2}\)), at the \( \beta \)-plane, \( Q_{\beta} \), and of the diffuse layer, \( Q_{S} \), was used to compute the electrical potential distribution at the interface between quartz and bulk NaCl solution as a function of the equilibrium constants and Stern layer capacitance [21]:

...
\[ Q_0 = -e \left( \Gamma_{\text{SiO}^-} + \Gamma_{\text{SiO}^- - \text{Na}^+} \right) = -\frac{e\Gamma_S}{A} \left[ 1 + K_{\text{Na}} a_{\text{Na}}^{\infty} \exp \left( -\frac{e\varphi_\beta}{k_B T} \right) \right], \]  
(A12)

\[ Q_\beta = e\Gamma_{\text{SiO}^- - \text{Na}^+} = \frac{e\Gamma_S}{A} K_{\text{Na}} a_{\text{Na}}^{\infty} \exp \left( -\frac{e\varphi_\beta}{k_B T} \right), \]  
(A13)

\[ A = 1 + K_H a_H^{\infty} \exp \left( -\frac{e\varphi_0}{k_B T} \right) + K_{\text{Na}} a_{\text{Na}}^{\infty} \exp \left( -\frac{e\varphi_\beta}{k_B T} \right), \]  
(A14)

\[ Q_S = \sqrt{8e k_B T 1000 N_A I} \sinh \left[ -\left( \frac{e\varphi_\beta}{2k_B T} \right) \right], \]  
(A15)

\[ Q_0 + Q_\beta + Q_S = 0, \]  
(A16)

\[ \varphi_0 - \varphi_\beta = \frac{Q_0}{C_1}, \]  
(A17)

where \( \Gamma_S \) is the total surface site density (we took \( \Gamma_S = 4.6 \) sites nm\(^{-2}\) García et al. [6]), \( I \) is the molar ionic strength (mol L\(^{-1}\)), and \( \varphi_0 \) and \( \varphi_\beta \) are the electrical potentials at the 0-plane and at the \( \beta \)-plane, respectively (considering \( \varphi_\beta = \varphi_d \) for the BSM, where \( \varphi_d \) is the electrical potential at the start of the diffuse layer).
The following equations were used to compute ion activity coefficients in bulk electrolyte [64]:

\begin{align}
    a_i^\infty &= \gamma_i^\infty \frac{m_i^\infty}{m_0}, \\
    m_i^\infty &= \frac{1000c_i^\infty}{M_w c_w^w}, \\
    c_w^w &= \frac{10^3 - \sum c_i^\infty V_i}{V_w},
\end{align}

where \( \gamma_i^\infty \) is the activity coefficient (dimensionless), \( m_i^\infty \) is the molality (mol per kilogram of water, mol kg\(^{-1}\)), \( m_0 \) being the unit molality equal to 1 mol kg\(^{-1}\)), \( c_i^\infty \) is the molar concentration (M), and \( V_i \) is the standard partial molal volume (cm\(^3\) mol\(^{-1}\)) of ion \( i \) in bulk electrolyte. The quantity \( V_i \approx 18.07, 0, -1.13, 17.68 \text{ cm}^3 \text{ mol}^{-1} \) for H\(_2\)O, H\(^+\), Na\(^+\) (due to electrostriction) and Cl\(^-\), respectively, at a temperature of 25°C. The subscript “\( w \)” in equations (B2) and (B3) refers to water molecules, and \( M_w \) refers to the molar mass of water (\( \approx 18 \text{ g mol}^{-1} \)).

Na\(^+\) activity coefficient in bulk electrolyte influences modelled Na\(^+\) adsorption in the Stern plane

\begin{equation}
    (\Gamma_{SiO^+ - Na^+}) = K_{Na} \Gamma_{SiO} \gamma_{Na^+}^\infty \frac{m_{Na^+}^\infty}{m_0} \exp(-e\varphi / k_BT) \text{ from equations (A3) and (A4)). According to Pitzer theory, which is suitable for very saline aqueous solutions (ionic strengths above 0.1 M Harvie and Weare [67]), the natural logarithm of Na\(^+\) activity coefficient in NaCl electrolyte is written as:}
\[
\ln \gamma_{\text{Na}^+} = z_{\text{Na}^+}^2 F + m_{\text{Cl}^-}^e \left[ 2 B_{\text{Na}^+\text{Cl}^-} + (m_{\text{Na}^+}^e + m_{\text{Cl}^-}^e) C_{\text{Na}^+\text{Cl}^-} \right] + z_{\text{Na}^+} m_{\text{Na}^+}^e m_{\text{Cl}^-}^e C_{\text{Na}^+\text{Cl}^-},
\]

(B4)

\[
F = -A_\phi \left[ \frac{\sqrt{I_m}}{1 + b \sqrt{I_m}} + \frac{2}{b} \ln \left(1 + b \sqrt{I_m}\right) \right] + m_{\text{Na}^+} m_{\text{Cl}^-} B_{\text{Na}^+\text{Cl}^-},
\]

(B5)

\[
A_\phi = \frac{1}{3} \sqrt{\frac{2\pi N_A \rho_w}{1000}} \left( \frac{e^2}{4\pi\varepsilon_w k_B T} \right)^{3/2},
\]

(B6)

\[
B_{\text{Na}^+\text{Cl}^-} = - \frac{2\beta_{\text{Na}^+\text{Cl}^-}}{I_m^2 I_m^2} \left[ 1 - (1 + x_i + 0.5x_i^2) \exp(-x_i) \right],
\]

(B7)

\[
x_i = \alpha_1 \sqrt{I_m},
\]

(B8)

\[
B_{\text{Na}^+\text{Cl}^-} = \beta_{\text{Na}^+\text{Cl}^-}^0 + \frac{2\beta_{\text{Na}^+\text{Cl}^-}}{x_i^2} \left[ 1 - (1 + x_i) \exp(-x_i) \right],
\]

(B9)

\[
C_{\text{Na}^+\text{Cl}^-} = \frac{C_{\phi\text{Na}^+\text{Cl}^-}}{2\sqrt{z_{\text{Na}^+} z_{\text{Cl}^-}}},
\]

(B10)

where \( z_i \) is the charge number of ion \( i \), \( b \) and \( \alpha_1 \) are empirical parameters (\( b = 1.2 \), \( \alpha_1 = 2 \) for 1:1 and 1:2 electrolytes), \( I_m \) is the molal ionic strength (in mol kg\(^{-1}\), \( I_m = m_{\text{Na}^+}^w \) here), and \( A_\phi \) is the Debye-Hückel coefficient describing long-range electrostatic interaction forces between ions (\( \approx 0.392 \) at a temperature \( T \) of 298 K). The Debye-Hückel coefficient was computed here as a function of the Avogadro number \( N_A \) (\( \approx 6.022 \times 10^{23} \) sites mol\(^{-1}\)), the water volumetric density \( \rho_w \) (\( \approx 997 \times 10^3 \) g m\(^{-3}\)), and the water permittivity \( \varepsilon_w \) (\( \approx 78.3\varepsilon_0 \) where \( \varepsilon_0 \) is the vacuum permittivity with a value of \( \approx 8.854 \times 10^{12} \) F m\(^{-1}\)). The Debye-Hückel coefficient multiplied by the terms in brackets in equation (B5) is enough for computing ion activity coefficient in dilute aqueous solution (ionic...
strength below 0.1 M). Pitzer and Mayorga [68] considered three additional terms (in equations (B4) and (B5)) to compute ion activity coefficients in concentrated aqueous solutions. The terms $B_{\text{Na}^+\text{Cl}^-}$ and $B_{\text{Na}^+\text{Cl}^-}^\prime$ depend on the ionic strength and describe short-range interaction forces between one cation and one anion (binary system), and the term $C_{\text{Na}^+\text{Cl}^-}$ describes short-range interaction forces between two cations and one anion, and one cation and two anions (ternary system). The Pitzer model for ion activity coefficients in 1:1 aqueous electrolyte such as NaCl depends on three parameters $\beta_{\text{Na}^+\text{Cl}^-}^0$, $\beta_{\text{Na}^+\text{Cl}^-}^1$, and $C_{\text{Na}^+\text{Cl}^-}$. The Pitzer parameter values were adjusted by matching computed to measured osmotic coefficients. According to [Leroy et al. [64]] $\beta_{\text{Na}^+\text{Cl}^-}^0 = 0.0765$, $\beta_{\text{Na}^+\text{Cl}^-}^1 = 0.2664$, and $C_{\text{Na}^+\text{Cl}^-} = 0.00127$. 


References


