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Abstract

The purpose of this paper is to extend the existing literature on the relationship between international trade and climate change by introducing non-tradeable goods and examining their role in the characterization of the Pareto-efficient environmental and trade policies. It is argued that the presence of non-tradable goods does not impede the central planner from imposing the Pigovian carbon taxes.

Keywords: Environmental taxation; international trade; Pareto efficiency; Non-tradeable goods; climate change.

JEL classification: H20; F18.

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1 Introduction

Climate change, in economic terms, is a global externality: emitting countries ignore the damage they cause to others, thereby they are emitting more than it is desirable from a global perspective. But it is a particularly complex externality given the asymmetric impact of the stock of emissions on the geographical distribution and economic activity. Given the global aspect of this challenge, the literature (both theoretical and empirical) on the relationship between international trade and climate change is fairly sizeable,[1] with considerable attention being paid to the characterization of Pareto-efficient environmental and trade policies.[2] This literature, however, has neglected the role of non-tradeable goods, arguably a realistic feature of any economic environment. The importance of non-tradeable goods to the characterization of implemented policies–due to the substitution effect between tradeable and non-tradeable goods from the demand and supply side– is well known in the trade literature.[3]

The objective of this paper is to extend the existing literature by examining the role of non-tradeable goods. In particular, we ask whether the existence of non-tradeable goods alters the characterization of Pareto-efficient environmental and trade policies that are derived from a model with tradeable goods only. We find that that the answer to this question is: "*it does not*". Specifically, we find that, even with non-tradeable goods present, a Pareto-efficient environmental policy dictates that, in the presence of lump sum taxes, carbon taxes have a Pigouvian form and are uniform across all (tradeable and non-tradeable) sectors and countries while, in the absence of lump sum taxes, they are uniform across all sectors in a given country but not across countries which is consistent with the results of Keen and Kotsogiannis (2014).

2 Description of the model

The model is essentially that of Keen and Kotsogiannis (2014) appropriately modified to include non-tradeable goods. There are J countries, indexed by the superscript j, each of which produces $M = T + N$ goods: T of these goods are tradeable and N are non-tradeable. The T tradeable goods are traded at a T-vector of world prices given by $p_T \gg 0$.[4] The price vector of non-tradeable goods in country j is denoted by $p_N^j \gg 0$.

International trade is subject to trade taxes (or subsidies), the vector of which in country j is denoted by τ^j. The commodity price vector of the tradeable

[1] Recent insightful surveys are by Copeland and Taylor (2004), Chen and Woodland (2013), Jones et al. (2013).

[2] For contributions see, among others, Markusen (1975), Baumol and Oates (1988), Krutilla (1991) Hoel (1996),Copeland (1994), Lubema and Wooton (1994), Neary (2006), Hatzipanayotou et al. (2008), Keen and Kotsogianis (2014) and Tsakiris et al (2014)

[3] See among others Dornbush (1974), Fukushima (1979), Clements (1982), Rivera-Batiz and Almansi (1983).

[4] The following convention is used: if $x = (x_1, ..., x_N)$, then $x \gg 0$ means $x_n > 0$ for all $n = 1, ..., N$; $x > 0$ means $x_n \geq 0$ for all $n = 1, ..., N$ and at least one $x_n \neq 0$; and $x \geq 0$ means $x_n \geq 0$ for all $n = 1, ..., N$.

goods in country j is thus given by the T-vector $p_T^j = p_T + \tau^j$. The model is very general in allowing for all types of trade taxes and subsidies.

Within each country there is a perfectly competitive private production sector. Producers in country j use factor endowments, denoted by vector v^j, to produce the M-vector y^j of commodities. The production of each commodity generates some pollutant (such as carbon emissions), with the M-vector z^j denoting emissions produced by the M commodities in country j. Total emissions in country j are thus given by $\iota' z^j$, where ι is the M-vector of 1s and a prime indicates transposition. Each national government may impose carbon taxes on the emissions from each commodity (sector), and the M-vector of carbon taxes is given by s^j. Hence, we allow for pollution taxes to be sector-specific.

Global emissions are therefore given by

$$k = \iota' \sum_{j=1}^{J} z^j \ . \tag{1}$$

The production sector in country j is perfectly competitive and characterized by a revenue function[5]

$$r^j(p^j, s^j, v^j) = \max_{y^j,\ z^j} \{p^{j\prime} y^j - s^{j\prime} z^j : f^j(y^j, z^j, v^j) \leq 0\} \ , \tag{2}$$

where $f^j(\cdot)$ is the implicit production possibility frontier in country j. Following from (2), and as an envelope property, the net output $M-$vector, y^j, and the vector of emissions, z^j, are given by

$$r_{pM}^j(p^j, s^j, v^j) = y^j \ , \tag{3}$$

$$r_s^j(p^j, s^j, v^j) = -z^j \ . \tag{4}$$

The consumption sector in country j is characterized by the expenditure function

$$e^j(p^j, u^j, k) = \min_{x^j} \{p^{j\prime} x^j \ : \ U^j(x^j) \geq u^j\} \ , \tag{5}$$

where $U^j(x^j)$ is the utility attained by consuming vector x^j of commodities. Notice that pollution k affects utility (presumably negatively). Shephard's lemma gives the M-vector of compensated demands, $e_{pM}^j(p^j, u^j)$.

It is convenient to define the net expenditure function in country j as[6]

$$S^j(p^j, s^j, u^j, k) \equiv e^j(p^j, u^j, k) - r^j(p^j, s^j, v^j) \ . \tag{6}$$

[5] The revenue (expenditure) function has the standard properties of homogeneity, convexity (concavity) and differentiability. For the properties of the revenue and expenditure function see Dixit and Norman (1980) and Woodland (1982).

[6] The function $S^j(\cdot)$ has the properties of the underlying expenditure and revenue functions.

The net expenditure function has the useful derivative properties that the vector of compensated import functions, denoted by $m_T^j(\cdot)$, is given by

$$m_T^j\left(p^j, s^j, v^j, u^j, k\right) \equiv S_{p_T}^j(p^j, s^j, v^j, u^j, k) = e_{p_T}^j(p^j, u^j, k) - r_{p_T}^j(p^j, s^j, v^j)\ , \tag{7}$$

where $e_{p_T}^j$ denotes the T-vector of compensated demands for the tradeable goods. Similarly, $r_{p_T}^j$ denotes the net output T-vector of the tradeable goods. The vector of emissions produced by country j is given by

$$z^j\left(p^j, s^j, v^j, k\right) \equiv S_s^j(p^j, s^j) = -r_s^j(p^j, s^j)\ . \tag{8}$$

The vector of non-tradeable goods is given by

$$m_N^j\left(p^j, s^j, v^j, u^j, k\right) \equiv S_{p_N}^j(p^j, s^j, v^j, u^j, k) = e_{p_N}^j(p^j, u^j) - r_{p_N}^j(p^j, s^j, v^j) = 0\ . \tag{9}$$

Without loss of generality, the first commodity of the tradeable goods is taken as the numeraire, with unit world price, and it is assumed to be untaxed by all countries, so $p_1^j = 1$ and $\tau_1^j = 0$ for all $j = 1, ..., J$.[7]

The equilibrium conditions for the world economy can be compactly expressed as[8]

$$p_T' S_{p_T}^j\left(p^j, s^j, u^j, k\right) + b^j = 0,\ j = 1, ..., J\ , \tag{12}$$

$$\sum_{j=1}^{J} S_q^j\left(p^j, s^j, u^j\right) = 0\ , \tag{13}$$

$$\iota' \sum_{j=1}^{J} S_s^j(p^j, s^j) = k\ , \tag{14}$$

$$\sum_{j=1}^{J} b^j = 0\ , \tag{15}$$

$$S_{p_N}^j(p^j, s^j, u^j, k) = 0\ ,\ j = 1, ..., J\ , \tag{16}$$

and S_q^j denotes the net expenditure function for the $T-1$ tradeable goods in country j. It is the presence of equation (16) that is central to the analysis here.

[7]For notational convenience, the price and trade tax vectors will be partitioned accordingly to

$$p_T' = (1, q') \quad ; \quad p_T^{j\prime} = (1, q^{j\prime}) \quad ; \quad q^{j\prime} = q' + \sigma^{j\prime} \quad ; \quad \tau^{j\prime} = (0, \sigma^{j\prime})\ , \tag{10}$$

and so domestic prices are given by

$$p_M^{j\prime} = (p_T^j, p_N^j)' = \left((1, q^{j\prime}), p_N^{j\prime}\right)\ . \tag{11}$$

[8]The vector of endowments v^j, since it is fixed, will be suppressed throughout.

The J equations in (12) are the consumers budget constraints: They simply state that the balance of trade deficit (value of net imports at world prices) plus any international transfers to (from) country j must be equal to zero. The $T-1$ equations in (13) are the world market equilibrium conditions for the non-numeraire tradeable commodities. Equation (14) specifies that global emissions are the sum of emissions produced by the J countries. Condition (15) requires that the sum of international transfers be zero. Equation (16) represents the market clearing conditions for the non-tradeable goods.

Given the tariff vectors τ^j, $j = 1, ..., J$, the carbon tax vectors s^j, $j = 1, ..., J$ and the vector $b = (b^1, ..., b^J)'$ of international transfers satisfying (15), the national budget constraints (12), the market equilibrium conditions (13), the global emissions equation (14), and the market clearing for non tradeable goods (16) may be solved for the competitive equilibrium world price vector for tradeable commodities, p_T, the equilibrium price vector in country j of the non-tradeable goods p_N^j, the level of global emissions, k, and the vector of national utility levels, $u = (u^1, ..., u^J)'$. The existence of a competitive equilibrium solution with $p_M^j \gg 0$ is assumed. The differential comparative static system is therefore[9]

$$Adu + Bdq + C_T d\widehat{q} + C_N dp_N + Dds + Edk + Fdb = 0 \ . \tag{17}$$

To characterize Pareto optimality of the initial equilibrium it proves convenient to use Tucker's Theorem of the Alternative (Mangasarian, 1969, p. 34), which states that either the system in (17) has a solution with $du > 0$ (where $du > 0$ is a semipositive vector with, that is, $du^j \geq 0$ for all $j = 1, ..., J$ and $du \neq 0$) for some perturbation $(dq, d\widehat{q}, dp_N, ds, dk, db)$—so that the initial equilibrium is Pareto *inefficient*—or there is a vector $y = (y_1, y_2, y_3, y_4, y_5) \in R^{J+(T-1)+1+T+JN}$ (where $y_1 = \left(y_1^1, ..., y_1^J\right)' \in R^J$, $y_2 = \left(y_2^2, ..., y_2^T\right)' \in R^{T-1}$, $y_4 = \left(y_4^1, ..., y_4^T\right)' \in R^T$ $y_5 = \left(y_5^1, ..., y_5^J\right)' \in R^{JN}$ with $y_5^j = \left(y_5^{j1}, ..., y_5^{jN}\right) \in R^N$) such that

$$y'[B, C_T, C_N, D, E, F] = 0, \tag{18}$$
$$y'A \ll 0, \tag{19}$$

in which case the initial equilibrium is Pareto efficient. The analysis now proceeds to derive the Pareto efficient climate and trade policies.[10]

3 Pareto-efficient climate and trade policies

This section considers the characterization of Pareto-efficient carbon tax and trade tax policies. Starting from an initial equilibrium in which the world economy is characterised by Pareto-efficient policies, no government can alter carbon

[9] The definitions of the matrixes A, B, C_T, C_N, D, E and F are given to the appendix(A).
[10] The details of the derivations are available upon request.

or trade taxes to make one country better off without making some other country worse off. In the current framework, with both tradeable and non-tradeable goods, the following results establish the two key features of *any* Pareto efficient allocation.

Proposition 1. *In the presence of lump sum transfers across countries (and with both tradeable and non-tradeable goods), Pareto efficiency requires that in every country $j = 1, ..., J$:*
(a) carbon taxes are set at $s^j = \left(\sum_{i=1}^{J} S_k^i\right)\iota$, i.e. they are uniform across production sectors within a country and also uniform across countries,
(b) trade tax vectors are equal, $\sigma^j = \sigma$, $j = 1, ..., J$, implying that domestic price vectors of the tradeable goods are all equal
(c) the prices of non-tradeable goods are country-specific and satisfy $p_N^{j\prime} = y_5^{j\prime}/y_4$.

According to Proposition 1 Pareto efficiency requires that—even in the presence of non-tradeable goods—each country sets a Pigovian carbon tax in each of the M sector to equate the marginal cost of an extra unit of carbon emissions, s^j, to the marginal global damage that the extra unit of emissions causes through climate change, $\sum_{i=1}^{J} S_k^i \iota$. The uniformity of carbon taxes within and across countries follows from the fact that carbon emissions from each sector and country contribute equally, at the margin, to the stock of carbon in the atmosphere and, hence, to climate change. Part (b) of the Proposition 1, implies equality of domestic prices (of the tradeable goods) and the collinearity of the tariff vectors across all countries. The importance of this is in emphasizing that—in the presence of lump sum transfers—production efficiency (for the tradeable goods) is part of a Pareto efficient allocation. To see this, recall that producer prices in country j are $p_T^{j\prime} = (1, q') + (0, \sigma^{j\prime})$ and so with $\sigma^j = \sigma$ for $j = 1, ..., J$, it is the case that $p_T^{j\prime} = p_T^{\prime}$ for all countries j. Parts (c) of the Proposition 1 states the Pareto efficient country-specific prices of the non-tradeable goods. Notice that, if it happens that $y_5^j = y_5$—and so the shadow value of non-tradeable goods is the same across countries[11]—then the price vectors for non-tradeable goods are collinear across countries (with a degree of collinearity $1/y_4$).

Clearly, as Proposition 1 shows, the presence of the non-tradeable goods does not change the uniformity structure of carbon taxes within and across countries nor the collinearity of the trade tax vectors. This is, perhaps, not surprising (once seen) as—with redistribution being taken care of by lump sum transfers and carbon taxes being uniform across all M sectors—non-tradeable goods have no additional role to play, at a Pareto efficient allocation, in pollution policies.

Therefore, the presence of non-tradeable goods does not change the uniformity structure of carbon taxes within and across countries nor the collinearity of the trade tax vectors. This is, perhaps, not surprising (once seen) as—with redistribution being taken care of by lump sum transfers and carbon taxes being

[11] There is no reason, of course, to suppose that this will be the case.

uniform across all M sectors—non-tradeable goods have no additional role to play, at a Pareto efficient allocation, in pollution policies.

It is therefore interesting to consider the possibility that international lump sum transfers are unavailable. In this case, we obtain:

Proposition 2. *In the absence of international lump sum transfers (and assuming that the substitution matrix for each j country has maximal rank), Pareto efficiency requires that in every country $j = 1, ..., J$:*
(a) carbon taxes are set such that $y_1^j s^i = \left(\sum_{i=1}^{J} y_1^i S_k^i\right) \iota$, where y_1^j is a scalar, and so carbon taxes are uniform across production sectors within a country but different across countries in the sense that for any countries j and i they satisfy $s^j = \alpha^{ij} s^i$, where $\alpha^{ij} \equiv y_1^i / y_1^j$, and
(b) trade tax vectors are collinear across countries in the sense that $\sigma^j = \alpha^{ij} \sigma^i$, $j = 1, ..., J$, implying that domestic price vectors are also collinear across countries
(c) the prices of the non-tradeable goods are country-specific and satisfy $p_N^{j\prime} = y_5^{j\prime} / y_1^j$.

Part (a) of the proposition simply states that in a Pareto efficient allocation carbon taxation reflects the global changes in utility, taking into account the cross country income implications of this (through the scalar multipliers y_1^j). Part (b) is more striking, implying that the social planner uses tariffs (and so the prices of the tradeable goods) as a redistribution device. Consistently with Keen and Kotsogiannis (2014), there is generally global production inefficiency for the tradeable goods in the allocations characterized by Proposition 2. Though increasing the net output of some good in some country without reducing the net output of any other good or increasing emissions requires that both producer prices and carbon taxes be equalized across countries, the proposition shows that Pareto efficiency allows for trade taxes, and hence domestic prices, to differ internationally. Turning now to non-tradeable goods (and their prices), one notices that these prices also reflect the redistribution motive of the central planner. Clearly, in this case (and even if the shadow prices of the non-tradeable goods where the same across countries in the sense that $y_5^j = y_5$) non-tradeable price vector will not be collinear. What Proposition 2 implies is intuitive: The central planner implicitly chooses goods prices either directly, in the case of non-tradable goods, or indirectly, in the case of tradable goods, through tariffs to correct any distributional misallocations. The presence of non-tradable goods does not impede the central planner of imposing the Pigovian carbon taxes (of part (a) of Proposition 2).

4 Concluding remarks

This paper has extend the existing literature of international trade and climate change examining the role of non-tradeable goods in the characterization of the Pareto-efficient environmental and trade policies. It is argued that the presence of non-tradable goods does not impede the central planner of imposing the Pigovian carbon taxes with the Pareto-efficient carbon and trade taxes being consistent with the findings of Keen and Kotsogiannis (2014).

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A Appendix

The matrices A, B, C_T, C_N, D, E and F are defined by

$$
Adu \equiv \begin{bmatrix}
p_T' S^1_{p_T u} & 0 & \cdots & 0 \\
0 & p_T' S^2_{p_T u} & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
0 & \cdots & \cdots & p_T' S^J_{p_T u} \\
S^1_{qu} & S^2_{qu} & \cdots & S^J_{qu} \\
0 & 0 & \cdots & 0 \\
0 & 0 & \cdots & 0 \\
S^1_{p_N u} & 0 & \cdots & 0 \\
0 & S^2_{p_N u} & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
0 & \cdots & \cdots & S^J_{p_N u}
\end{bmatrix}
\begin{bmatrix}
du^1 \\ du^2 \\ \vdots \\ du^J
\end{bmatrix},
$$

$$
Bdq \equiv \begin{bmatrix}
S_q^{1\prime} \\ S_q^{2\prime} \\ \vdots \\ S_q^{J\prime} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \vdots \\ 0
\end{bmatrix} dq \ ,
$$

[1]

$$
C_T d\widehat{q} \equiv \begin{bmatrix}
p_T' S^1_{p_T q} & 0 & \cdots & 0 \\
0 & p_T' S^2_{p_T q} & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
0 & 0 & \cdots & p_T' S^J_{p_T q} \\
S^1_{qq} & S^2_{qq} & \cdots & S^J_{qq} \\
\iota' S^1_{sq} & \iota' S^2_{sq} & \cdots & \iota' S^J_{sq} \\
0 & 0 & \cdots & 0 \\
S^1_{p_N q} & 0 & \cdots & 0 \\
0 & S^2_{p_N q} & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
0 & 0 & \cdots & S^J_{p_N q}
\end{bmatrix}
\begin{bmatrix}
dq^1 \\ dq^2 \\ \vdots \\ dq^J
\end{bmatrix},
$$

[1] Notice that the vector q is a—following the normalization of prices—$T-1$-vector.

$$C_N dp_N \equiv \begin{bmatrix} p_T' S^1_{p_T p_N} & 0 & \cdots & 0 \\ 0 & p_T' S^2_{p_T p_N} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & p_T' S^J_{p_T p_N} \\ S^1_{q p_N} & S^2_{q p_N} & \cdots & S^J_{q p_N} \\ \iota' S^1_{s p_N} & \iota' S^2_{s p_N} & \cdots & \iota' S^J_{s p_N} \\ 0 & 0 & \cdots & 0 \\ S^1_{p_N p_N} & 0 & \cdots & 0 \\ 0 & S^2_{p_N p_N} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & S^J_{p_N p_N} \end{bmatrix} \begin{bmatrix} dp^1_N \\ dp^2_N \\ \vdots \\ dp^J_N \end{bmatrix},$$

$$Dds \equiv \begin{bmatrix} p_T' S^1_{p_T s} & 0 & \cdots & 0 \\ 0 & p_T' S^2_{p_T s} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & \cdots & \cdots & p_T' S^J_{p_T s} \\ S^1_{qs} & S^2_{qs} & \cdots & S^J_{qs} \\ \iota' S^1_{ss} & \iota' S^2_{ss} & \cdots & \iota' S^J_{ss} \\ 0 & 0 & \cdots & 0 \\ S^1_{p_N s} & 0 & \cdots & 0 \\ 0 & S^2_{p_N s} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & S^J_{p_N s} \end{bmatrix} \begin{bmatrix} ds^1 \\ ds^2 \\ \vdots \\ ds^J \end{bmatrix},$$

$$Edk \equiv \begin{bmatrix} p_T' S^1_{p_T k} \\ p_T' S^2_{p_T k} \\ \vdots \\ p_T' S^J_{p_T k} \\ \sum_{j=1}^{J} S^j_{q_T k} \\ -1 \\ 0 \\ S^1_{p_N k} \\ S^2_{p_N k} \\ \vdots \\ S^J_{p_N k} \end{bmatrix} dk, Fdb \equiv \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \\ 0 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \\ -1 & -1 & \cdots & -1 \\ 0 & 0 & \cdots & 0 \\ \vdots & 0 & \cdots & \vdots \\ \vdots & \vdots & 0 & \vdots \\ 0 & 0 & \cdots & 0 \end{bmatrix} \begin{bmatrix} db^1 \\ db^2 \\ \vdots \\ db^J \end{bmatrix}$$

B Appendix

Proof of Proposition 1:

Given the differentiability assumptions concerning the expenditure and revenue functions the system—re-written here again for convenience—(12), (13), (14), (15) and (16)

$$p_T' S_{p_T}^j \left(p^j, s^j, u^j, k\right) + b^j = 0, \ j = 1, ..., J, \tag{B.1}$$

$$\sum_{j=1}^{J} S_q^j \left(p^j, s^j, u^j\right) = 0, \tag{B.2}$$

$$\iota' \sum_{j=1}^{J} S_s^j (p^j, s^j) = k, \tag{B.3}$$

$$\sum_{j=1}^{J} b^j = 0, \tag{B.4}$$

$$S_{p_N}^j (p^j, s^j, u^j, k) = 0, \ j = 1, ..., J \tag{B.5}$$

Equations (18) and (19) are necessary and sufficient conditions for Pareto optimality in the present model.

The equations in (18) and (19) can be readily shown to be expressed as

$$y'A = \left[y_1^j p_T' S_{p_T u}^j + y_2' S_{qu}^j + y_5^{j\prime} S_{p_N u}^j, \qquad j = 1, ..., J \right] \ll 0', \tag{B.6}$$

$$y'B = \left[\sum_{j=1}^{J} y_1^j S_q^{j\prime} \right] = y_1' S_q' = 0', \tag{B.7}$$

$$y'C_T = \left[y_1^j p_T' S_{p_T q}^j + y_2' S_{qq}^j + y_3 \iota' S_{sq}^j + y_5^{j\prime} S_{p_N q}^j, \qquad j = 1, ..., J \right] = 0', \tag{B.8}$$

$$y'C_N = \left[y_1^j p_T' S_{p_T p_N}^j + y_2' S_{q p_N}^j + y_3 \iota' S_{s p_N}^j + y_5^{j\prime} S_{p_N p_N}^j, \qquad j = 1, ..., J \right] = 0', \tag{B.9}$$

$$y'D = \left[y_1^j p_T' S_{p_T s}^j + y_2' S_{qs}^j + y_3 \iota' S_{ss}^j + y_5^{j\prime} S_{p_N s}, \qquad j = 1, ..., J \right] = 0', \tag{B.10}$$

$$y'E = \sum_{j=1}^{J} y_1^j p_T' S_{p_T k}^j + \sum_{j=1}^{J} y_2' S_{qk}^j - y_3 + \sum_{j=1}^{J} y_5^{j\prime} S_{p_N k}^j = 0, \tag{B.11}$$

$$y'F = \left[y_1^j - y_4, \qquad j = 1, ..., J \right] = 0, \tag{B.12}$$

where we follow the convention to denote matrices with elements for each $j = 1, ..., J$ in the square brackets.

Notice that combining (B.8) and (B.9) we have that

$$y_1^j p_T' \left[S_{p_T q}^j | S_{p_T p_N}^j\right] + y_2' \left[S_{qq}^j | S_{qp_N}^j\right] + y_3 \iota' \left[S_{sq}^j | S_{sp_N}^j\right] + y_5^{j\prime} \left[S_{p_N q}^j | S_{p_N p_N}^j\right] = 0', \tag{B.13}$$

where

$$\left[S_{p_T q}^j | S_{p_T p_N}^j\right]_{T\times(M-1)}, \tag{B.14}$$

$$\left[S_{qq}^j | S_{qp_N}^j\right]_{(T-1)\times(M-1)}, \tag{B.15}$$

$$\left[S_{sq}^j | S_{sp_N}^j\right]_{M\times(M-1)}, \tag{B.16}$$

$$\left[S_{p_N q}^j | S_{p_N p_N}^j\right]_{N\times(M-1)}. \tag{B.17}$$

(B.13) can be re-written as

$$y_1^j p_T' \left[S_{p_T q}^j | S_{p_T p_N}^j\right] + (0, y_2') \left[S_{p_T q}^j | S_{p_T p_N}^j\right] + y_3 \iota' \left[S_{sq}^j | S_{sp_N}^j\right] + y_5^{j\prime} \left[S_{p_N q}^j | S_{p_N p_N}^j\right] = 0',$$

and so, after defining

$$\rho' \equiv y_4 p_T' + (0, y_2'), \tag{B.18}$$

a $1 \times T$ vector, as

$$\left(\rho', y_5^{j\prime}\right) \left[\begin{array}{c} S_{p_T q}^j | S_{p_T p_N}^j \\ S_{p_N q}^j | S_{p_N p_N}^j \end{array}\right] + y_3 \iota' \left[S_{sq}^j | S_{sp_N}^j\right] = 0'. \tag{B.19}$$

Equation (B.19) has used the fact that equation (B.12) implies that $y_1^j = y_4$ for all $j = 1, ..., J$ and so $y_1 = y_4 \iota$ (where ι is the unit vector).The implication of this is that the marginal social utilities of income—given by y_1^j for country j—are the same across all countries.

Following (B.6) we also have that

$$y' A = \left[\left(\rho, y_5^j\right)' \left(S_{p_M u}^j\right), \ j = 1, ..., J\right] \ll 0', \tag{B.20}$$

where $S_{p_M u}^j$ is an $M-1$-vector. Similarly, following equation (B.10) we have that

$$y' D = \left[y_1^j p_T' S_{p_T s}^j + y_2' S_{qs}^j + y_3 \iota' S_{ss}^j + y_5^{j\prime} S_{p_N s}, \qquad j = 1, ..., J\right] = 0', \tag{B.21}$$

$$= \left(\rho, y_5^j\right)' \left[\begin{array}{c} S_{p_T s}^j \\ S_{p_N s}^j \end{array}\right] + y_3 \iota' S_{ss}^j, \qquad j = 1, ..., J = 0'. \tag{B.22}$$

Equations (B.6)-(B.11) may therefore be re-expressed as

$$y'A = \left[\left(\rho, y_5^j \right)' S_{pu}^j, \ j = 1, ..., J \right] \ll 0', \tag{B.23}$$

$$y'B = \left[y_4 \sum_{j=1}^{J} S_q^{j\prime} \right] = 0', \tag{B.24}$$

$$y'C = \left[\left(\rho, y_5^j \right)' \left[\begin{array}{c} S_{p_T q}^j | S_{p_T p_N}^j \\ S_{p_N q}^j | S_{p_N p_N}^j \end{array} \right] + y_3 \iota' \left[S_{sq}^j | S_{s p_N}^j \right], \ j = 1, ..., J \right] = 0', \tag{B.25}$$

$$y'D = \left[\left(\rho, y_5^j \right)' \left[\begin{array}{c} S_{p_T s}^j \\ S_{p_N s}^j \end{array} \right] + y_3 \iota' S_{ss}^j, \ j = 1, ..., J \right] = 0', \tag{B.26}$$

$$y'E = \sum_{j=1}^{J} \left(\rho, y_5^j \right) \left[\begin{array}{c} S_{p_T s}^j \\ S_{p_N s}^j \end{array} \right] - y_3 = 0. \tag{B.27}$$

Following from (B.25) and (B.26), equations $y'C = 0'$ and $y'D = 0'$ may be combined together as

$$\left(\left(\rho', y_5^j \right), \quad y_3 \iota' \right) \left(\begin{array}{cc} \left[\begin{array}{c} S_{p_T q}^j | S_{p_T p_N}^j \\ S_{p_N q}^j | S_{p_N p_N}^j \end{array} \right] & \left[\begin{array}{c} S_{p_T s}^j \\ S_{p_N s}^j \end{array} \right] \\ \left[S_{sq}^j | S_{s p_N}^j \right] & \left[S_{ss}^j \right] \end{array} \right) = (0, \quad 0)'. \tag{B.28}$$

Denoting

$$\begin{aligned} S_{p\hat{p}}^j &\equiv \left[\begin{array}{c} S_{p_T q}^j | S_{p_T p_N}^j \\ S_{p_N q}^j | S_{p_N p_N}^j \end{array} \right], \\ S_{s\hat{p}}^j &\equiv \left[S_{sq}^j | S_{s p_N}^j \right], \\ S_{ps}^j &\equiv \left[\begin{array}{c} S_{p_T s}^j \\ S_{p_N s}^j \end{array} \right], \end{aligned}$$

then (B.28) is equal to

$$\left(\left(\rho', y_5^j \right), \quad y_3 \iota' \right) \left(\begin{array}{cc} S_{p\hat{p}}^j & S_{ps}^j \\ S_{s\hat{p}}^j & S_{ss}^j \end{array} \right) = (0, \quad 0)'. \tag{B.29}$$

Homogeneity of the net expenditure function in price vector $\left(p^{j\prime}, s^{j\prime} \right)$ implies that[2]

$$\left(p^{j\prime}, \quad s^{j\prime} \right) \left(\begin{array}{cc} S_{p\hat{p}}^j & S_{ps}^j \\ S_{s\hat{p}}^j & S_{ss}^j \end{array} \right) = (0, \quad 0)' \tag{B.30}$$

holds as an identity.

[2]This follows from the fact that $S_p^j \left(p^j, s^j \right)$ and $S_s^j \left(p^j, s^j \right)$ are homogeneous of degree zero. This implies that

$$\begin{aligned} S_{pp}^j p^{j\prime} + S_{ps}^j s^{j\prime} &= 0' \\ S_{sp}^j p^{j\prime} + S_{ss}^j s^{j\prime} &= 0'. \end{aligned}$$

Notice now that (B.29) can be written as

$$\left(\left(y_4 p_T' + (0, y_2'), y_5^j\right), \quad y_3 \iota'\right) \begin{pmatrix} S_{p\hat{p}}^j & S_{ps}^j \\ S_{s\hat{p}}^j & S_{ss}^j \end{pmatrix} = (0, \quad 0)', \tag{B.31}$$

whereas (B.30) can be written as (after multiplying by y_4)

$$\left((y_4 p_T' + y_4(0, \sigma^{j\prime}), y_4 p_N^{j\prime}), \quad y_4 s^{j\prime}\right) \begin{pmatrix} S_{p\hat{p}}^j & S_{ps}^j \\ S_{s\hat{p}}^j & S_{ss}^j \end{pmatrix} = (0, \quad 0)'. \tag{B.32}$$

Subtracting one from the other we have that

$$\left(\left(\left(y_4 \sigma^{j\prime} - y_2'\right), y_4 p_N^j - y_5^j\right), \quad \left(y_4 s^{j\prime} - y_3 \iota'\right)\right) \begin{pmatrix} S_{p\hat{p}}^j & S_{ps}^j \\ S_{s\hat{p}}^j & S_{ss}^j \end{pmatrix} = (0, \quad 0)'. \tag{B.33}$$

This implies that

$$\sigma^{j\prime} = y_2'/y_4, \tag{B.34}$$

$$s^{j\prime} = y_3 \iota'/y_4, \tag{B.35}$$

$$p_N^{j\prime} = y_5^{j\prime}/y_4. \tag{B.36}$$

Assuming that the substitution matrix for each country has maximal rank, $\left(p^{j\prime}, s^{j\prime}\right)$ is the only vector (up to a factor of proportionality) satisfying the equality in equation (B.30).

Consequently, it must be the case that

$$\left(\left(\rho', y_5^j\right), \quad y_3 \iota'\right) = \left(p^{j\prime}, s^{j\prime}\right)$$

(up to a factor of proportionality), implying that (choosing the factor of proportionality to be $y_4 \neq 0$)

$$p^{j\prime} = \left(\rho', y_5^{j\prime}\right)/y_4, \qquad j = 1, ..., J, \tag{B.37}$$

$$s^{j\prime} = (y_3/y_4)\, \iota', \qquad j = 1, ..., J. \tag{B.38}$$

Combining, we have that

$$\begin{bmatrix} S_{pp}^j & S_{ps}^j \\ S_{sp}^j & S_{ss}^j \end{bmatrix} \begin{bmatrix} p^{j\prime} \\ s^{j\prime} \end{bmatrix} = \begin{bmatrix} 0' \\ 0' \end{bmatrix},$$

and so, upon transposing,

$$\begin{bmatrix} p^{j\prime} & s^{j\prime} \end{bmatrix} \begin{bmatrix} S_{pp}^j & S_{ps}^j \\ S_{sp}^j & S_{ss}^j \end{bmatrix}' = \begin{bmatrix} 0' & 0' \end{bmatrix}.$$

This shows that all domestic prices of the international traded goods must be equal (proportional to one another)

$$p_T^j = \rho / y_4, \qquad j = 1, ..., J,$$

the prices of the non-tradeable goods

$$p_N^j = y_5^j / y_4, \qquad j = 1, ..., J,$$

and that carbon taxes are the same across countries and across sectors within each country. For domestic prices of the international traded good to be equal, the specific tariff vectors, $\sigma^{j\prime} = y_2' / y_4$, must also be equal across countries.

To complete the proof, we next need to characterize carbon taxes. Following (B.11)—and upon using the fact that $\rho / y_4 = p_T^j$—we have that

$$y_3 = \sum_{j=1}^{J} \left(\rho, y_5^j \right) \begin{bmatrix} S_{p_T k}^j \\ S_{p_N k}^j \end{bmatrix} = \sum_{j=1}^{J} \left(\rho, y_5^j \right)' S_{pk}^j. \tag{B.39}$$

We now know that, following from the homogeneity property of S^j

$$p^{j\prime} S_{pk}^j \equiv S_k^j,$$

and so

$$\begin{aligned} y_3 &= \sum_{j=1}^{J} \left(\rho, y_5^j \right)' S_{pk}^j = y_4 \sum_{j=1}^{J} \left(p_T^j, p_N^j \right)' S_{pk}^j, \\ &= y_4 \sum_{j=1}^{J} p^{j\prime} S_{pk}^j = y_4 \sum_{j=1}^{J} \left(S_k^j \right). \end{aligned} \tag{B.40}$$

Substituting this expression for y_3 / y_4 into (B.38), one obtains that

$$s^j = \left(\sum_{j=1}^{J} S_k^j \right) \iota, \tag{B.41}$$

as required.

C Appendix

Proof of Proposition 2: The proof of this proposition makes use of the steps (and the equations) of the proof of Proposition 1. In the absence of international transfers, each country can only spend what it earns through production and net tax revenue and this constraint on national budgets complicates the outcomes of policy reforms. In terms of the model given by (12)-(15) and its differential system (17), the international transfers are now set to zero and left unchanged. Equation (B.12) no longer applies and so it is, therefore, no longer the case that the dual variables satisfy the condition $y_1^j = y_4,\ j = 1, ..., J$.

Accordingly, the equations $y'C = 0'$ and $y'D = 0'$ now become (since $\rho' \equiv y_1^j p_T' + (0, y_2')$),

$$\left(\left(y_1^j p_T' + (0, y_2'), y_5^j\right), \quad y_3 \iota'\right) \left(\begin{array}{cc} \left[\begin{array}{c} S_{p_T q}^j | S_{p_T p_N}^j \\ S_{p_N q}^j | S_{p_N p_N}^j \end{array}\right] & \left[\begin{array}{c} S_{p_T s}^j \\ S_{p_N s}^j \end{array}\right] \\ \left[S_{sq}^j | S_{sp_N}^j\right] & \left[S_{ss}^j\right] \end{array} \right) = (0, \quad 0)\,. \tag{C.1}$$

Equation (C.1) can be written as

$$\left(\left(y_1^j p_T' + (0, y_2'), y_5^j\right), \quad y_3 \iota'\right) \left(\begin{array}{cc} S_{p\hat{p}}^j & S_{ps}^j \\ S_{s\hat{p}}^j & S_{ss}^j \end{array} \right) = (0, \quad 0)'\,, \tag{C.2}$$

whereas (B.30) can be written as (after multiplying by y_1^j)

$$\left((y_1^j p_T' + y_1^j (0, \sigma^{j\prime}), y_1^j p_N^{j\prime}), \quad y_1^j s^{j\prime}\right) \left(\begin{array}{cc} S_{p\hat{p}}^j & S_{ps}^j \\ S_{s\hat{p}}^j & S_{ss}^j \end{array} \right) = (0, \quad 0)'\,. \tag{C.3}$$

Subtracting one from the other we have that

$$\left(\left(\left(y_1^j \sigma^{j\prime} - y_2'\right), y_1^j p_N^j - y_5^j\right), \quad \left(y_1^j s^{j\prime} - y_3 \iota'\right)\right) \left(\begin{array}{cc} S_{p\hat{p}}^j & S_{ps}^j \\ S_{s\hat{p}}^j & S_{ss}^j \end{array} \right) = (0, \quad 0)'\,. \tag{C.4}$$

This implies that

$$\sigma^{j\prime} = y_2' / y_1^j, \tag{C.5}$$

$$s^{j\prime} = y_3 \iota' / y_1^j, \tag{C.6}$$

$$p_N^{j\prime} = y_5^{j\prime} / y_1^j. \tag{C.7}$$

It then follows that

$$\begin{aligned} y_3 &= \sum_{j=1}^{J} \left(\rho, y_5^j\right)' S_{pk}^j = \sum_{j=1}^{J} y_1^j \left(p_T^j, p_N^j\right)' S_{pk}^j, \\ &= \sum_{j=1}^{J} y_1^j p^{j\prime} S_{pk}^j = \sum_{j=1}^{J} y_1^j S_k^j. \end{aligned} \tag{C.8}$$

Substituting this expression for (C.6), $s^{j\prime} = (y_3/y_1)\,\iota',\ j = 1, ..., J$ one obtains

$$s^j = \left(\sum_{i=1}^{J} y_1^i S_k^i\right) \iota / y_1^j. \tag{C.9}$$

as required.