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M. V. Menshykova · O. V. Menshykov · I. A. Guz ·
M. Wuensche · Ch. Zhang

A boundary integral equation method in the frequency domain for cracks under transient loading

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Abstract This paper concerns the fracture mechanics problem for elastic cracked materials under transient dynamic loading. The problem is solved by use of the boundary integral equations in the frequency domain, and the components of the solution are presented by the Fourier exponential series. The dynamic stress intensity factors are computed for different stress pulses and compared with those obtained for the case of the Heaviside loading.

1 Introduction

Nowadays, much attention is paid to the responses of solid materials with crack-like defects to impact loading. This type of loading can lead to stresses and displacements that are larger than the ones caused by static loading. Under the impact load the elastic waves are generated throughout the structure and refracted and reflected causing the local stresses to increase beyond their corresponding values under static loads of the same magnitude [1]. The unstable motion of the crack and eventually the fracture of the structure could be the result of such impact loading.

Works devoted to impact loading in the homogeneous elastic materials with cracks were published by a number of authors.

An elastic solid containing a half-plane crack subjected to impact loading was considered by Freund [2]. Solving the problem of the plane strain response to impact loading on the crack faces, particular attention was given to the elastic field near the crack tip. The problem was solved with the assumption that the crack was initially open and that the crack faces remained traction free.

In works [3,4] a centre crack in a finite elastic solid under suddenly applied crack face pressure was considered. In the study, the direct three-dimensional (3D) time domain formulation of the boundary element method (BEM) was implemented to investigate the effect of a free surface on the stress intensity factor distribution along the crack front. In the works, the straight as well as curved crack profiles were studied and the critical intersection angle of the crack front with a free surface was computed. Agrawal and Kishore [4] presented a direct time domain approach where the free surface effect for a stationary crack under impact

M. V. Menshykova (✉) · O. V. Menshykov · I. A. Guz
Centre for Micro- and Nanomechanics (CEMINACS), School of Engineering, University of Aberdeen, Fraser Noble Building, King's College, Aberdeen AB24 3UE, UK
E-mail: m.menshykova@abdn.ac.uk

M. Wuensche · Ch. Zhang
Department of Civil Engineering, University of Siegen, 57068 Siegen, Germany

M. Wuensche
Faculty D (Mechanical Engineering), University of Wuppertal, 42119 Wuppertal, Germany

loading can be easily extended to study the free surface effect for a running crack in finite elastic solids (Agrawal [3]).

A cylindrical crack in an infinite elastic medium under impact loading was considered by Itou in [5]. The 3D impact response of the crack subjected to a longitudinal stress wave was investigated. A numerical solution was obtained for a circular crack of a finite radius. Using the method employed for a solution of longitudinal stress wave incidence [5], the problem can be solved for an incident shear stress wave.

Chen and Sih [1,6] investigated the transient response of a crack to impact load, solved the problem in the Laplace transform domain, and then performed inverse Laplace transform numerically to get the time history of the dynamic stress intensity factors. A similar method was used by Itou. In [7–10], Itou investigated three-dimensional rectangular crack behaviour under impact loading. Itou used Laplace and Fourier transforms to reduce the mixed boundary value problem to a set of dual integral equations. The stress intensity factors were computed in the Laplace transform domain and inverted numerically in the physical space to obtain the time histories of the elastodynamic stress intensity factors.

In [11], Zhang considered a rectangular and a penny-shaped crack in an infinite elastic solid under incident transient elastic wave loading, whose front is assumed to be planar. The time history and the spatial variation of the dynamic stress intensity factors in dependence on the type and direction of incident waves were investigated. The response of a single penny-shaped crack to transient elastic waves also was studied in [12,13].

The crack problem in an anisotropic elastic solid under impact loading was considered in [14,15]. In the paper [14], the time domain BEM for a transient elastodynamic crack analysis in homogeneous and linear elastic solids of general anisotropy was presented. In the numerical analysis, special attention was given to the transient elastodynamic stress intensity factors. In [15], the semi-infinite crack in an unbounded anisotropic elastic body was considered. The response to a concentrated impact force on one of the crack faces was studied in the work.

Piezoelectric solids with cracks under impact mechanical and electrical load were considered in [16–19]. The problem of a piezoelectrical strip containing a crack was solved in [16–18]. The transient dynamic analysis of a crack in a piezoelectric solid under coupled electromechanical impact load was presented in [19]. In the work, the extended finite element method was used for the analysis.

Cracks in magneto-electric materials under impact loading are considered in [20–24]. The dynamic analysis of two collinear cracks in a piezoelectromagnetic material subjected to magneto-electro-mechanical impacts was performed in [21]. The study [22] was devoted to the study of the transient response of a magneto-electro-elastic material with a penny-shaped crack under in-plane impacts. A hypersingular time domain BEM was used in [23] for transient dynamic crack analysis in magneto-electro-elastic solids.

With the increasing use of composite materials in engineering applications, understanding the mechanism of dynamic fracture in different types of composites becomes more and more important.

A number of researchers paid special attention to the crack problems in layered composites and bimaterials. Sih and Chen [25] developed a method for the solution of normal and radial impact of layered composites with embedded penny-shaped cracks. Transient response of a sub-interface crack in a bimaterial was studied in [26]. The study was focused on the dynamic interaction between the crack and the interface. To solve the problem, the combined traditional time domain displacement BEM and the non-hypersingular traction BEM were used.

Different piezoelectric bimaterial combinations containing two interfacial cracks were considered in [27]. In the paper, the dynamic behaviour of the bimaterial subjected to mechanical impact loading was investigated.

The problem of a penny-shaped interface crack was solved in [28,29]. In these papers, the torsional impact response of an interface crack in a layered composite was investigated. The difference between the two aforementioned works is that in [29] the FGM interlayer was considered. In [29], the authors focused their attention to the investigation of the FGM interlayer effect on the dynamic stress intensity factor of an interface crack in bonded materials. To solve the corresponding problem, the Laplace and Hankel integral transforms and the singular integral equation technique were used.

Elastodynamic response of an interface crack in a layered composite under anti-plane shear impact load was investigated by Li and Tai in [30]. A time domain BEM in conjunction with a multi-domain technique was developed in [31] for transient dynamic interior or interface crack analysis in 2D, layered, anisotropic elastic solids. In the paper, the effects of the crack configuration, the material anisotropy, the layer combination and the dynamic loading on the dynamic stress intensity factors and the scattered elastic wave fields were investigated.

Numerical and experimental methods were used to study the impact response of the fracture parameters of interfacial cracks in a bimaterial under impact loading [32]. Energy release rate (J integral), stress intensity factors (K), crack opening-displacements and T stress were calculated and their dependence on the time and the phase angle was investigated.

The boundary element method was used by Lei et al. [24] for transient dynamic crack analysis in magneto-electro-elastic bimaterials. In the paper, the effects of the poling directions, the volume fractions and the combined loading parameters on the dynamic field intensity factors at the interfacial crack tips were investigated.

A finite length interfacial crack between two dissimilar viscoelastic bodies was considered in [33]. In the analysed problem, an anti-plane step loading acts suddenly on the surface of the crack. To calculate the transient dynamic stress intensity factor, the integral transform method and the singular integral equation approach were used. Normal and shear impact loading of the interfacial crack in orthotropic materials was studied in [34] to calculate the stress intensity factor history around the crack tips.

In solving the crack problem, the crack face contact interaction must be taken into account. In papers [35–37], it was shown that it changes the distribution of the stresses and the displacements not only quantitatively but also qualitatively. The contact interaction between opposite crack faces has a very complex nature, and a great majority of researchers neglected the crack closure effects due to the difficulties with the problem solution. The crack closure effect for interfacial cracks subjected to dynamic loading was considered in [38–41]. To solve the problem of a crack located on the bimaterial interface under time harmonic loading, the BIEM was used. An iterative algorithm which makes it possible to take the crack closure into account was developed in [39]. The algorithm was used to solve the problem of an interface crack with allowance for crack face contact interaction under time harmonic normal tension–compression load. In [41], the distributions of the displacements and tractions at the bimaterial interface were obtained and analysed for the case of a penny-shaped crack under normal tension–compression time harmonic wave loading. The initially open interfacial crack subjected to a time-harmonic wave loading was considered in [40].

This paper presented a boundary integral equation method (BIEM) in the frequency domain for transient dynamic crack analysis in isotropic and linear elastic solids subjected to impact loading. Boundary integral equations (BIEs) are first formulated theoretically and then solved numerically for discrete frequency values in the Fourier-transformed or frequency domain by using BEM. Subsequently, time domain responses are computed by applying an inverse Fourier transform technique. To verify the developed BIEM, numerical results are presented and compared with other known reference results available in literature. It must be noted that in reality the opposite crack faces under dynamic loading would interact with each other. Under deformation of the elastic material, the initial contact area will change in time, and the shape of the contact area would be unknown beforehand and must be determined as a part of the solution. Considering this crack face interaction effect for the case of transient impact loading will be the next natural stage of the research.

2 Boundary integral equation method

Let us consider a linearly elastic material containing inter- and intra-component cracks under external loading. Inside the solid in the absence of body forces, the behaviour of the body satisfies the equations of motion

$$\partial_j \sigma_{ij}(\mathbf{x}, t) = \rho \partial_t^2 u_i(\mathbf{x}, t), \quad \mathbf{x} \in V, \quad t \in \Im = [0, T] \tag{1}$$

and generalized Hooke's law

$$\sigma_{ij}(\mathbf{x}, t) = C_{ijkl} \varepsilon_{kl}(\mathbf{x}, t), \quad \mathbf{x} \in V, \quad t \in \Im \tag{2}$$

subject to some boundary conditions. In Eqs. (1) and (2), $u_i(\mathbf{x}, t)$ and $\sigma_{ij}(\mathbf{x}, t)$ denote the displacement vector and the stress tensor, ρ is the mass density, C_{ijkl} is the elasticity tensor, and the components of the Cauchy strain tensor are given by $\varepsilon_{kl}(\mathbf{x}, t) = (\partial_k u_l(\mathbf{x}, t) + \partial_l u_k(\mathbf{x}, t))/2$. Unless otherwise stated, the conventional summation rule over repeated indices is applied.

We assume that the initial conditions are as follows:

$$\mathbf{u}^{(m)}(\mathbf{x}, 0) = \dot{\mathbf{u}}^{(m)}(\mathbf{x}, 0) = 0, \quad \mathbf{x} \in V. \tag{3}$$

The continuity conditions of displacements and stresses at perfectly bonding interfaces are given by:

$$\mathbf{u}^{(+)}(\mathbf{x}, t) = \mathbf{u}^{(-)}(\mathbf{x}, t), \quad \boldsymbol{\sigma}^{(+)}(\mathbf{x}, t) = \boldsymbol{\sigma}^{(-)}(\mathbf{x}, t), \quad \mathbf{x} \in \Gamma^{int}, \quad t \in \Im. \tag{4}$$

In the case of elastic solids occupying the whole space, the Sommerfeld-type radiation condition, which ensures a finite elastic energy in an infinite body, is also imposed at infinity on the displacement vector:

$$\|\mathbf{u}(\mathbf{x}, t)\| \leq c/r \tag{5}$$

where c is a constant and $r \to \infty$ is the distance from the origin of the perturbation.

Note that, for an isotropic body, Eqs. (1) and (2) lead to the following linear Lamé equations of elastodynamics for the displacement field:

$$\mu\delta_{ij}\frac{\partial}{\partial x_k}\frac{\partial}{\partial x_k}u_j(\mathbf{x},t)+(\lambda+\mu)\frac{\partial}{\partial x_i}\frac{\partial}{\partial x_j}u_j(\mathbf{x},t)=\rho\frac{\partial}{\partial t}\frac{\partial}{\partial t}u_i(\mathbf{x},t),\quad \mathbf{x}\in V,\quad t\in\Im \tag{6}$$

where δ_{ij} is the Kronecker delta, μ and λ are the Lamé's elastic constants.

It is possible to represent the components of the displacement field in terms of the boundary displacements and tractions using the dynamic Somigliana identity:

$$u_j^{(m)}(\mathbf{x},t)=\int_{\mathrm{T}}\int_{\Gamma^{(m)}}\left(t_i^{(m)}(\mathbf{y},\tau)\,U_{ij}^{(m)}(\mathbf{x},\mathbf{y},t-\tau)-u_i^{(m)}(\mathbf{y},\tau)W_{ij}^{(m)}(\mathbf{x},\mathbf{y},t-\tau)\right)\mathrm{d}\mathbf{y}\mathrm{d}\tau,$$
$$\mathbf{x}\in\Omega^{(m)},\quad t\in\Im,\quad j=\overline{1,2}. \tag{7}$$

Here, $U_{ij}^{(m)}(\mathbf{x},\mathbf{y},t-\tau)$ denotes the displacement fundamental solution:

$$U_{ij}^{(m)}(\mathbf{x},\mathbf{y},t-\tau)=\frac{1}{2\pi\mu^{(m)}}\left(\psi^{(m)}\delta_{ij}-\chi^{(m)}\frac{(y_i-x_i)}{r}\frac{(y_j-x_j)}{r}\right) \tag{8}$$

where r is the distance between the observation point and the source point, $W_{ij}^{(m)}(\mathbf{x},\mathbf{y},t-\tau)=C_{ijkl}e_j(\mathbf{y})\partial_l U_{kj}^{(m)}(\mathbf{x},\mathbf{y},t-\tau)$ is the traction fundamental solution, $\Gamma^{(m)}$ is the sub-domain boundary, which can conclude parts of the external boundary, bonding interfaces Γ^{int}, and crack faces Γ^{cr}, and the functions $\psi^{(m)}$ and $\chi^{(m)}$ for the case considered will be defined later.

Consequently, the integral kernel $W_{ij}^{(m)}(\mathbf{x},\mathbf{y},t-\tau)$ is obtained from $U_{ij}^{(m)}(\mathbf{x},\mathbf{y},t-\tau)$ by applying the differential operator

$$P_{ik}[\bullet,(\mathbf{y})]=\lambda n_i(\mathbf{y})\frac{\partial[\bullet]}{\partial y_k}+\mu\left[\delta_{ik}\frac{\partial[\bullet]}{\partial\mathbf{n}(\mathbf{y})}+n_k(\mathbf{y})\frac{\partial[\bullet]}{\partial y_i}\right] \tag{9}$$

and has the form:

$$W_{ij}^{(m)}(\mathbf{x},\mathbf{y},t-\tau)=\lambda^{(m)}n_i^{(m)}(\mathbf{y})\frac{\partial}{\partial y_k}U_{kj}^{(m)}(\mathbf{x},\mathbf{y},t-\tau)$$
$$+\mu^{(m)}n_k^{(m)}(\mathbf{y})\left[\frac{\partial}{\partial y_k}U_{ij}^{(m)}(\mathbf{x},\mathbf{y},t-\tau)+\frac{\partial}{\partial y_i}U_{kj}^{(m)}(\mathbf{x},\mathbf{y},t-\tau)\right]. \tag{10}$$

A similar representation can be obtained for the components of the traction field by applying the differential operator (9) to the Somigliana identity (7):

$$p_j^{(m)}(\mathbf{x},t)=\int_T\int_{\Gamma^{(m)}}(p_i^{(m)}(\mathbf{y},\tau)K_{ij}^{(m)}(\mathbf{x},\mathbf{y},t-\tau)-u_i^{(m)}(\mathbf{y},\tau)F_{ij}^{(m)}(\mathbf{x},\mathbf{y},t-\tau))\mathrm{d}\mathbf{y}\mathrm{d}\tau,$$
$$\mathbf{x}\in\Omega^{(m)},\quad t\in T,\quad j=1,2 \tag{11}$$

where the integral kernels $K_{ij}^{(m)}(\mathbf{x},\mathbf{y},t-\tau)$ and $F_{ij}^{(m)}(\mathbf{x},\mathbf{y},t-\tau)$ can be obtained from the kernels $U_{ij}^{(m)}(\mathbf{x},\mathbf{y},t-\tau)$ and $W_{ij}^{(m)}(\mathbf{x},\mathbf{y},t-\tau)$ using the operator (10) with respect to $\mathbf{x}$.

Then, for the limiting case $\mathbf{x}\to\Gamma^{(m)}$ it is possible, under assumption that the boundary displacements and traction vectors are smooth enough, to deduce the boundary integral equations (BIEs) connecting the tractions and the displacements on the boundaries of all sub-domains. The resulting system of BIEs for the whole body can be obtained using the continuity conditions of the displacements and stresses at the perfectly bonding interfaces:

$$\frac{1}{2}u_j^{(m)}(\mathbf{x},t)=\int_T\int_{\Gamma^{(m)}}(p_i^{(m)}(\mathbf{y},\tau)U_{ij}^{(m)}(\mathbf{x},\mathbf{y},t-\tau)-u_i^{(m)}(\mathbf{y},\tau)W_{ij}^{(m)}(\mathbf{x},\mathbf{y},t-\tau))\mathrm{d}\mathbf{y}\mathrm{d}\tau, \tag{12}$$

$$\frac{1}{2}p_j^{(m)}(\mathbf{x},t)=\int_T\int_{\Gamma^{(m)}}(p_i^{(m)}(\mathbf{y},\tau)K_{ij}^{(m)}(\mathbf{x},\mathbf{y},t-\tau)-u_i^{(m)}(\mathbf{y},\tau)F_{ij}^{(m)}(\mathbf{x},\mathbf{y},t-\tau))\mathrm{d}\mathbf{y}\mathrm{d}\tau \tag{13}$$

where $\mathbf{x}\in\Gamma^{(m)}$, $t\in T$.

For the case of a homogenous solid with cracks the system (12)–(13) can be further simplified. It should also be noted that the efficiency of the proposed approach depends on the availability of the corresponding fundamental solutions. For instance, for the case of anisotropic materials, fundamental solutions simply cannot be expressed in closed forms [11].

In the present study, the external transient dynamic load is approximated by the Fourier exponential series, that provided an opportunity to use the methodology and results obtained by the authors previously in the frequency domain [39,41,42]. The approximations of the triangular stress pulse

$$\sigma(t)=\sigma^*\left\{\frac{t}{t^*}\left(H(t)-H\left(t-t^*\right)\right)+\left(2-\frac{t}{t^*}\right)\left(H(t)-H\left(t-t^*\right)\right)\right\}$$

and trapezoidal stress pulse

$$\sigma(t)=\sigma^*\left\{\begin{array}{l}\frac{t}{t^*}\left(H(t)-H\left(t-t^*\right)\right)+\left(H\left(t-t^*\right)-H\left(t-t^*-t_d\right)\right)\\+\left(2-\frac{t-t_d}{t^*}\right)\left(H\left(t-t^*-t_d\right)-H\left(t-2t^*-t_d\right)\right)\end{array}\right\}$$

are given in Figs. 1 and 2 for different numbers of the Fourier coefficients used.

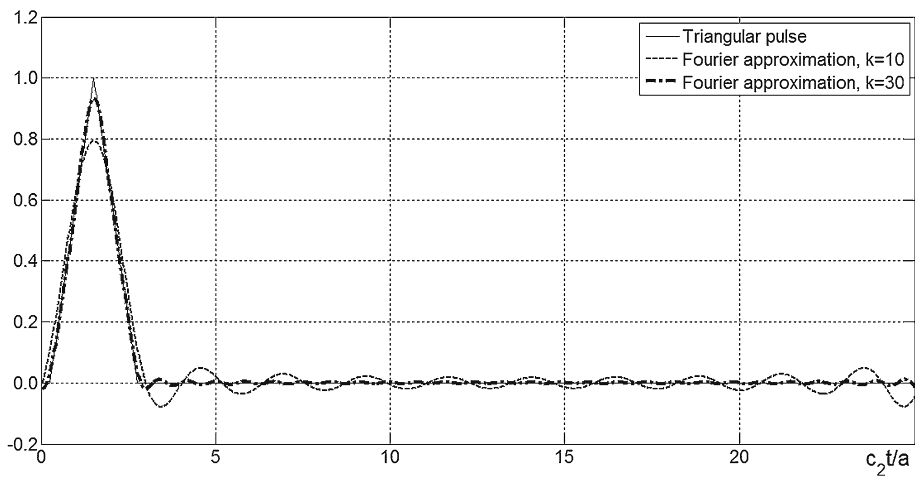

Fig. 1 Triangular pulse approximation for $c_2t^*=0.2$

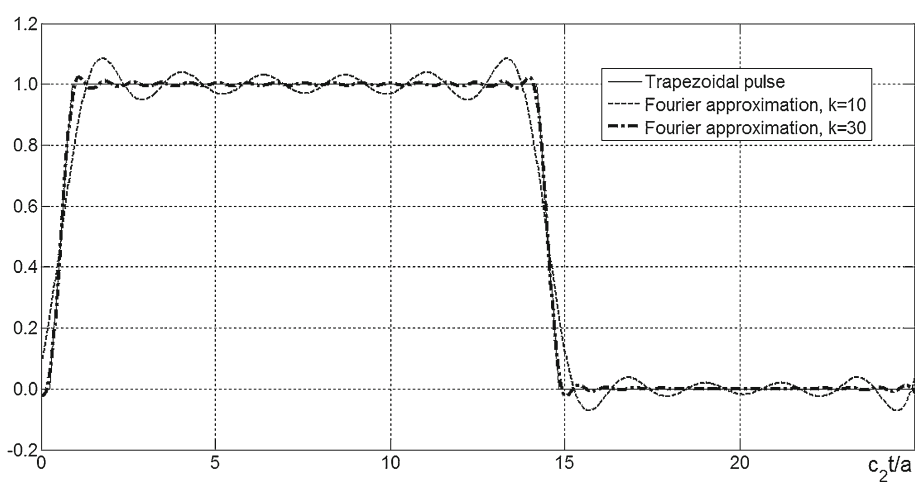

Fig. 2 Trapezoidal pulse approximation for $c_2t^*=0.1$ and $c_2t_d=12.12$

Then, following the references [39,41,42], all components of the solution should be expanded into the Fourier series:

$$f(\bullet, t) = \mathrm{Re}\left\{\sum_{k=-\infty}^{+\infty} f^k(\bullet) e^{i\omega_k t}\right\} \tag{14}$$

where $\omega_k = 2\pi k/T$, and the appropriate Fourier coefficients are given as

$$f^k(\bullet) = \frac{\omega}{2\pi}\int_0^T f(\bullet, t) e^{-i\omega_k t}\mathrm{d}t. \tag{15}$$

Hence, the system of the boundary integral equations (12)–(13) should be transformed into:

$$\frac{1}{2}u_j^{k,(m)}(\mathbf{x}) = \int_{\Gamma^{(m)}} (p_i^{k,(m)}(\mathbf{y})U_{ij}^{(m)}(\mathbf{x}, \mathbf{y}, \omega) - u_i^{k,(m)}(\mathbf{y})W_{ij}^{(m)}(\mathbf{x}, \mathbf{y}, \omega))\mathrm{d}\mathbf{y}, \tag{16}$$

$$\frac{1}{2}p_j^{k,(m)}(\mathbf{x}) = \int_{\Gamma^{(m)}} (p_i^{k,(m)}(\mathbf{y})K_{ij}^{(m)}(\mathbf{x}, \mathbf{y}, \omega) - u_i^{k,(m)}(\mathbf{y})F_{ij}^{(m)}(\mathbf{x}, \mathbf{y}, \omega))\mathrm{d}\mathbf{y}, \tag{17}$$

in the frequency domain for the given number of the Fourier coefficient k. Here, $\mathbf{x} \in \Gamma^{(m)}$, and $p_i^{k,(m)}(\mathbf{x})$ and $u_i^{k,(m)}(\mathbf{x})$ are the complex-valued Fourier coefficients of the tractions and displacements at the interface. The time-dependent values of the tractions and displacements are obtained according to Eq. (14).

The expressions for the functions $\psi^{(m)}$ and $\chi^{(m)}$ in Eq. (8) are [43]:

$$\psi^{(m)} = K_0\left(l_2^{(m)}\right) + \frac{1}{l_2^{(m)}}\left[K_1\left(l_2^{(m)}\right) - \frac{c_2^{(m)}}{c_1^{(m)}}K_1\left(l_1^{(m)}\right)\right],$$

$$\chi^{(m)} = K_2\left(l_2^{(m)}\right) - \left(\frac{c_2^{(m)}}{c_1^{(m)}}\right)^2 K_2\left(l_1^{(m)}\right)$$

where $K_n(\bullet)$ is the modified Bessel function of the second kind and order n, $l_1^{(m)} = i\omega r/c_1^{(m)}$, $l_2^{(m)} = i\omega r/c_2^{(m)}$, $c_1^{(m)} = \sqrt{(\lambda^{(m)} + 2\mu^{(m)})/\rho^{(m)}}$ and $c_2^{(m)} = \sqrt{\mu^{(m)}/\rho^{(m)}}$ are the velocities of the longitudinal and the transversal waves in the elastic material.

3 Numerical results

As a representative numerical example, we consider an incident pulse of the unit amplitude propagating in the normal direction to the interface of the straight crack with the length of $2a$. For the validation of the results, the properties of both half-planes were set identical, Young's modulus is 200 GPa, Poisson's ratio is 0.25, and the density of the material is 7800 kg/m^3.

The deformation of the cracked elastic material under the impact loading leads to displacement discontinuities of the opposite crack faces:

$$[\mathbf{u}(\mathbf{x}, t)] = \mathbf{u}^+(\mathbf{x}, t) - \mathbf{u}^-(\mathbf{x}, t), \quad \mathbf{x} \in \Gamma^{\mathrm{cr}}, \ t \in \Im \tag{18}$$

where $\mathbf{u}^+(\mathbf{x}, t)$ and $\mathbf{u}^-(\mathbf{x}, t)$ are displacements of opposite crack faces.

We expand the normal and tangential components of the displacement discontinuity vector and the traction vector into the trigonometric Fourier series with respect to time:

$$p_j(\mathbf{x}, t) = \frac{p_{j,\cos}^0(\mathbf{x})}{2} + \sum_{k=1}^{+\infty}\left(p_{j,\cos}^k(\mathbf{x})\cos(\omega_k t) + p_{j,\sin}^k(\mathbf{x})\sin(\omega_k t)\right), \tag{19}$$

$$\left[u_j(\mathbf{x}, t)\right] = \frac{\left[u_{j,\cos}^0(\mathbf{x})\right]}{2} + \sum_{k=1}^{+\infty}\left(\left[u_{j,\cos}^k(\mathbf{x})\right]\cos(\omega_k t) + \left[u_{j,\sin}^k(\mathbf{x})\right]\sin(\omega_k t)\right) \tag{20}$$

where $\mathbf{x} \in \Omega$, $\omega_k = 2\pi\, k/T$, $j = 1, 2$ and

$$p^k_{j,\cos}(\mathbf{x}) = \frac{\omega}{2\pi}\int_0^T p_j(\mathbf{x}, t)\cos(\omega_k t)\mathrm{d}t, \quad p^k_{j,\sin}(\mathbf{x}) = \frac{\omega}{2\pi}\int_0^T p_j(\mathbf{x}, t)\sin(\omega_k t)\mathrm{d}t, \tag{21}$$

$$\left[u^k_{j,\cos}(\mathbf{x})\right] = \frac{\omega}{2\pi}\int_0^T [u_j(\mathbf{x}, t)]\cos(\omega_k t)\mathrm{d}t, \quad [u^k_{j,\sin}(\mathbf{x})] = \frac{\omega}{2\pi}\int_0^T \left[u_j(\mathbf{x}, t)\right]\sin(\omega_k t)\mathrm{d}t,$$
$$k \in N_0 = 0, 1, \ldots, +\infty. \tag{22}$$

In the considered homogeneous case, the system of boundary integral equations (16)–(17) can be further reduced to:

$$\begin{aligned} &p^k_{j,\cos}(\mathbf{x}) - i p^k_{j,\sin}(\mathbf{x}) \\ &\quad = -\sum_{m=1}^{2}\int_{\Omega}(F^{\mathrm{Re}}_{mj}(\mathbf{x}, \mathbf{y}, \omega_k) + i F^{Im}_{mj}(\mathbf{x}, \mathbf{y}, \omega_k))(\Delta u^k_{m,\cos}(\mathbf{y}) - i\Delta u^k_{m,\sin}(\mathbf{y}))\,\mathrm{d}\mathbf{y}, \\ &\quad \mathbf{x} \in \Omega, \quad k \in N_0, \quad j = 1, 2 \end{aligned} \tag{23}$$

where i is the imaginary unit, $F^{\mathrm{Re}}_{mj}(\mathbf{x}, \mathbf{y}, \omega_k)$ and $F^{Im}_{mj}(\mathbf{x}, \mathbf{y}, \omega_k)$ are the real and the imaginary parts of the integral kernel $F_{mj}(\mathbf{x}, \mathbf{y}, \omega_k)$. For a straight crack, we can write them explicitly as [43]:

$$F_{12}(\mathbf{x}, \mathbf{y}, \omega_k) = F_{21}(\mathbf{x}, \mathbf{y}, \omega_k) = 0, \tag{24}$$

$$\begin{aligned} F_{11}(\mathbf{x}, \mathbf{y}, \omega_k) = \frac{i\mu}{4}\Bigg[&\left(\frac{\omega_k}{c_2}\right)^2 H^{(1)}_0\left(\frac{\omega_k r}{c_2}\right) - 4\frac{\omega_k}{c_2 r}H^{(1)}_1\left(\frac{\omega_k r}{c_2}\right) \\ &+ 4\frac{\mu}{\lambda+2\mu}\frac{\omega_k}{c_1 r}H^{(1)}_1\left(\frac{\omega_k r}{c_1}\right) + \frac{12}{r^2}\left(H^{(1)}_2\left(\frac{\omega_k r}{c_2}\right) - \frac{\mu}{\lambda+2\mu}H^{(1)}_2\left(\frac{\omega_k r}{c_1}\right)\right)\Bigg], \end{aligned} \tag{25}$$

$$\begin{aligned} F_{22}(\mathbf{x}, \mathbf{y}, \omega_k) = \frac{i\mu}{2}\Bigg[&\frac{\lambda^2}{(\lambda+2\mu)^2}\left(\frac{\omega_k}{c_2}\right)^2 H^{(1)}_0\left(\frac{\omega_k r}{c_1}\right) + 2\frac{\omega_k}{c_2 r}H^{(1)}_1\left(\frac{\omega_k r}{c_2}\right) \\ &+ 2\frac{\omega_k}{c_1 r}\frac{\lambda}{\lambda+2\mu}H^{(1)}_1\left(\frac{\omega_k r}{c_1}\right) - \frac{6}{r^2}\left(H^{(1)}_2\left(\frac{\omega_k r}{c_2}\right) - \frac{\mu}{\lambda+2\mu}H^{(1)}_2\left(\frac{\omega_k r}{c_1}\right)\right)\Bigg] \end{aligned} \tag{26}$$

where $H^{(1)}_\beta$ are the Hankel functions of first kind, $r = |x_1 - y_1|$ is the distance between the points $\mathbf{x}$ and $\mathbf{y}$, and $c_2 = \sqrt{\mu/\rho}$ is the velocity of the transverse wave in the elastic material. The detailed expressions for the real and imaginary parts of the integral kernels are given in [43].

The dynamic stress intensity factors (opening mode or mode-I) obtained for trapezoidal stress pulses of different steepness and length values, and normalized by the corresponding static values are given in Figs. 3, 4, and 5, where analytical [44] and numerical [11] solutions obtained for the Heaviside stress pulse are also presented. The normalized distribution of the normal components of the displacement jump on the crack face for one of the pulses is given in Fig. 6. Note that the results obtained for the relatively long and steep trapezoidal pulse are in a very good agreement with the model ones which clearly proves the applicability of the method proposed for the case of transient impact loading (see also [45]).

4 Conclusions

In this paper, a boundary integral equation method (BIEM) in the frequency domain is presented for transient dynamic crack analysis in elastic solids under impact loading. The initial-boundary value problem is first solved for discrete frequencies in the Fourier-transformed or frequency domain by using the BIEM. Then, time-dependent solutions are obtained by performing an inverse Fourier transform. The applicability and the accuracy of the present BIEM are validated by using available reference results from literature.

It must be noted that in reality the opposite crack faces under dynamic loading would interact with each other. Under deformation of the elastic material, the initial contact area will change in time, and the shape of the

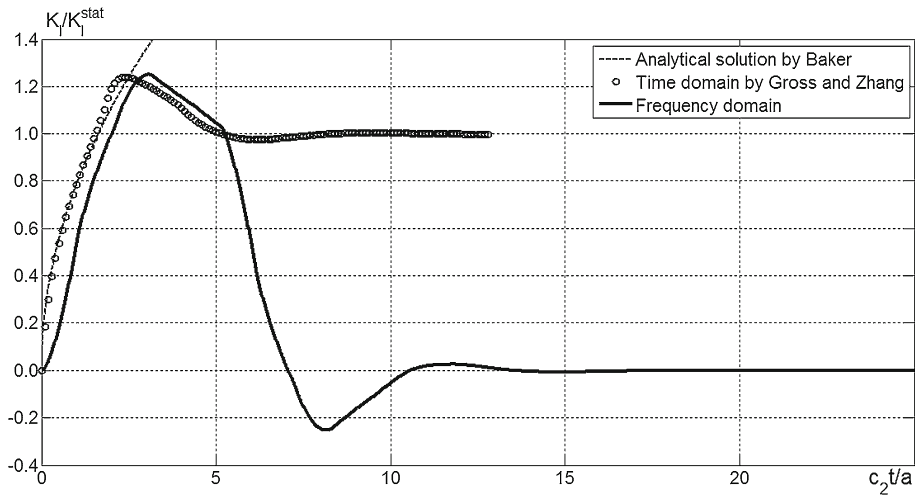

Fig. 3 Normalized stress intensity factors for trapezoidal stress pulse, $c_2t^* = 1.0$ and $c_2t_d = 4.12$

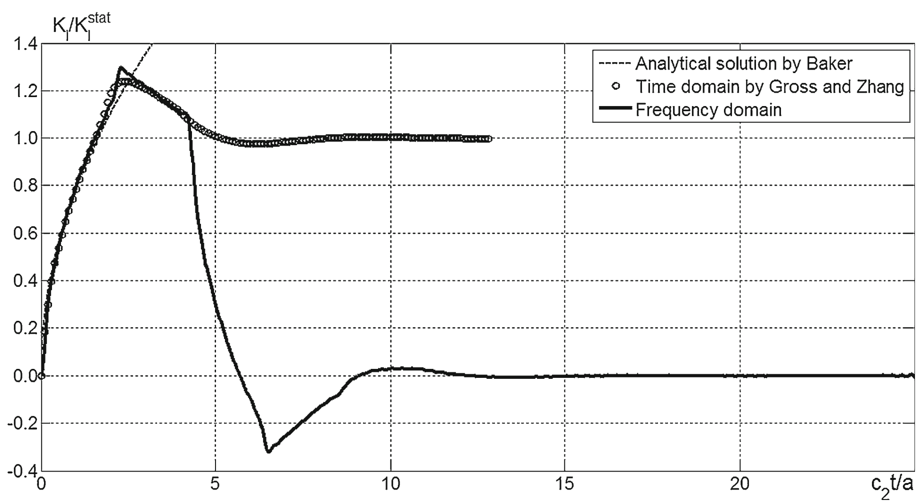

Fig. 4 Normalized stress intensity factors for trapezoidal stress pulse, $c_2t^* = 0.1$ and $c_2t_d = 4.12$

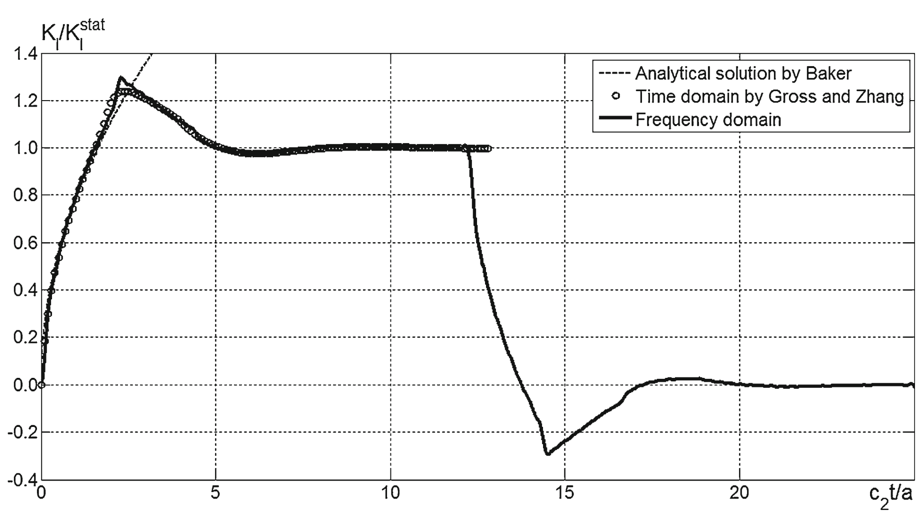

Fig. 5 Normalized stress intensity factors for trapezoidal pulse, $c_2t^* = 0.1$ and $c_2t_d = 12.12$ (approximately Heaviside pulse)

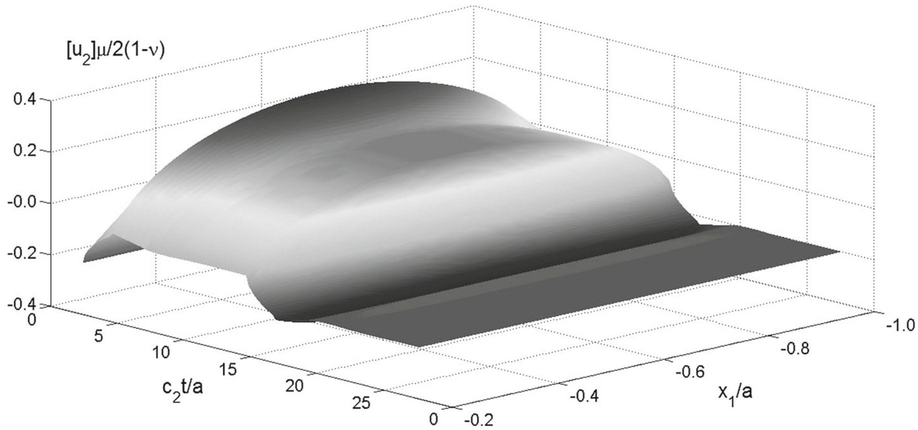

Fig. 6 Normalized displacement jump versus time for trapezoidal pulse, $c_2t^* = 0.1$ and $c_2t_d = 12.12$

contact area would be unknown beforehand and must be determined as a part of the solution, see, for example, [39,42,43], where the nonlinear effects due to the crack closure were analysed for cracks under time-harmonic loading in homogeneous and heterogeneous elastic materials. Considering this crack face interaction effect for the case of transient impact loading will be the next natural stage of the research.

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