



UNIVERSITY  
OF ABERDEEN

Business School

Department of  
Economics

# Does the Future Price Help Forecast the Spot Price?

Xin Jin

Discussion Paper in Economics No 16-8

June 2016

ISSN 0143-4543

# Does the Futures Price Help Forecast the Spot Price?

Xin Jin\*

Department of Economics, Business School, University of Aberdeen

**Abstract** This paper proposes a futures-based unobserved components model for the commodity spot price which proves to have superior forecasting ability. The commodity spot price is decomposed into long-term and short-term components, while the futures price is decomposed into expected future spot price and risk premium. Under this model, information from the whole futures curve could be utilized to improve forecasting accuracy of the spot price. Applying this model to oil market data, I find that the model forecasts outperform in multiple dimensions the benchmark of the literature optimal forecast (the random walk) as well as simple futures prices forecasts. The model forecasts overall have smaller error variation over the 20-year sample period, and are also more possible to have smaller absolute error when compared to the benchmark forecasts period by period.

**Keywords:** Forecasting; Commodities; Spot Price; Futures Price; Futures Curve; Unobserved Components Model; Stochastic Process

**JEL Classification Numbers:** G17, Q4, G13, C38.

---

\*Address: Edward Wright Building S67, Dunbar Street, Old Aberdeen AB243QY UK, e-mail: [xjin@abdn.ac.uk](mailto:xjin@abdn.ac.uk). The author is grateful to Nathan Balke, Tom Fomby, Mine Yücel, Marc Gronwald and Euan Phimister for advice and suggestions. All errors and mistakes are the author's.

# 1 Introduction

Given the high volatility of commodity prices and the importance of raw materials in production, accurate forecast of the spot price is of great interest for various purposes. Policy makers and central banks closely track the commodity prices, especially crude oil price. Price forecast is also crucial to business decisions in many industries.

One intuitive forecast of the spot price is the futures price. Efficient market hypothesis suggests the futures price as the best forecast. Studying the spot and futures prices movements is not only empirically useful, but also theoretically important, as commodity futures market plays an important role in risk transfer and price discovery. A large literature has discussed the forecasting ability of the futures prices. [French \(1986\)](#), [Fama and French \(1987\)](#), [Bowman and Husain \(2004\)](#), [Coppola \(2008\)](#), [Reichsfeld and Roache \(2009\)](#), [Reeve and Vigfusson \(2011\)](#), [Chinn and Coibion \(2013\)](#) among others find evidence confirming the forecasting ability of the futures prices for certain commodities, while equally large amount of research like [Bopp and Lady \(1991\)](#), [Moosa and Al-Loughani \(1994\)](#), [Chernenko et al. \(2004\)](#), [Alquist and Kilian \(2010\)](#), [Alquist et al. \(2013\)](#) find little evidence supporting the futures price as the best forecast. Instead, no-change forecast is suggested as a plausible measure of the expected spot price. While [Alquist et al. \(2013\)](#) provide a comprehensive overview of various forecasting models including futures-based ones, they do point out the potential of factor (unobserved components) models for forecasting.

This paper contributes to the forecasting and price dynamics literature by proposing a futures-based unobserved components forecasting model which has superior

forecasting accuracy. The improved forecasting accuracy relies on exploiting the futures curve (the term structure of the futures prices) in addition to the assumed spot price stochastic process. The shape of the futures curve is partly determined by the spot price stochastic process, if we consider the futures price as composed of the expected future spot price and risk premium as in [Pindyck \(2001\)](#). Prices quoted at the same time for immediate and future delivery of the same commodity all incorporate the same set of information under efficient market hypothesis, as argued by earlier work like [Working \(1942\)](#), [Tomex and Gray \(1970\)](#) and [Peck \(1985\)](#). However they are affected by the same set of information differently, as some information might have short-term effect while other have long-term effect. The prices with further delivery dates are much less affected by short-term effect of the information compared to the prices with nearer delivery dates, while all prices are affected by long-term effect similarly. The difference between the prices with further delivery dates (the far end of the futures curve) and nearer delivery dates (the near end of the futures curve with the nearest end being the spot price) roughly reflects the short-term effect of the information, while futures prices with further delivery dates (the far end of the futures curve) roughly reflect the long-term effect of the information. Thus futures curve, futures and spot prices together help infer the stochastic process that the spot price follows.

The model allows for flexibility to fit the rich dynamics of the spot price and the futures curves, while it is still relatively easy to estimate. Applying the model to oil market data, I show that the model forecasts outperform both the futures price and the random walk forecasts in multiple dimensions.

This paper differs from recent research that has already paid attention to the value of the futures curves in the assumption of the spot price stochastic process. [Coppola \(2008\)](#) proposes a VECM model which essentially uses futures curves for forecasting spot price, where the spot price is modeled as a random walk to which the effect of shocks will never dissipate. To the contrary, this paper argues the effect of shocks to the spot price could partly dissipate over time.

Motivated by the cost of carry model, the two-factor and three-factor models proposed in [Schwartz \(1997\)](#) and [Cortazar and Schwartz \(2003\)](#) also bear resemblance to my model as they are essentially estimated using the futures curves. However, the models intuitively imply that the spot price follows a random walk process in the discrete time. The assumption implies the effect of shocks to the spot price will not dissipate over time. This is fundamentally different from the proposed model in this paper.

This paper instead assumes the spot price dynamics to contain some temporary component. The idea that shocks to the spot price could be dissipated is not new. Both theoretical and empirical works suggest that commodity price contains both a long-term component and a short-term component (see [Fama and French \(1988\)](#), [Pindyck \(1999\)](#), [Carlson et al. \(2007\)](#) and [Stevens \(2013\)](#)). Allowing part of the shocks to dissipate, the unobserved components model proposed in this paper can then be seen as the empirical extension of the recent development in the commodity price theory by [Carlson et al. \(2007\)](#) and [Stevens \(2013\)](#). In terms of the assumed the spot price stochastic process, my model is similar to the model of price evolution in [Pindyck \(1999\)](#), but further relates the futures prices of different maturity terms

to the spot price, and thus is able to exploit the information from futures curve.

When applying the model to the crude oil market weekly data from March 1989 until November 2014, the model outperforms the literature benchmark of the random walk forecast and the futures price forecasts at most forecasting horizons from 4 to 48 weeks, as measured by  $MSE$ ,  $ME$  and  $MAE$ . The improvement at longer forecasting horizons is also statistically significant as tested by [Giacomini and White \(2006\)](#) finite-sample unconditional predictive ability test.

The plan of the paper is as follows. Section 2 develops the unobserved components model of crude oil prices and discusses the model features. Section 3 provides an overview of the data, describes the statistical tests adopted and presents the estimation results from oil market. Section 4 presents and evaluates the forecasting ability of the model compared to literature benchmark. Section 5 concludes.

## **2 Theory and Unobserved Components Model of Commodity Prices**

In this section, an unobserved components model is constructed. Unobserved components model is able to take full advantage of the information in both the spot price and the futures prices and uncover the unobserved factors affecting the prices. The model is later estimated by maximum likelihood, and the unobserved components are estimated using the Kalman filter, which also enables me to deal with the missing observations in the data.

## 2.1 The Spot and Futures Prices Stochastic Process

Shocks to commodity spot price could have effect of different time-persistence. [French \(1986\)](#) discusses the possibility that “shocks to the current price are dissipated before they can affect the expected price” and observes that in such scenario futures prices can provide “reliably better forecast”. Such observation indicates that shocks to spot price would dissipate over time, rather than as predicted from a random walk process. [Fama and French \(1988\)](#) discuss the long-term and short-term components in asset prices although the purpose there is to test market efficiency. [Pindyck \(1999\)](#) also argues that a model of price evolution should incorporate both a reversion to a long-run trend and a non-stationary stochastic long-run trend after studying both the empirical features of commodity prices and the theoretical implications of the depletable resource prices. More recently, [Carlson et al. \(2007\)](#) demonstrate that such price dynamics with both long-term and short-term components could arise from a hotelling model with production adjustment costs. The impulse response functions of the spot price to shocks in [Carlson et al. \(2007\)](#) show that part of the shocks are incorporated into price as temporary increments, rather than all shocks having long-lasting effects on the price. [Stevens \(2013\)](#) also derives similar price dynamics from a hotelling model with storage.

Thus this paper assumes the spot price to contain both long-term and short-term components. This is similar to [Pindyck \(1999\)](#), as both indicate dissipating shocks over time while overall the price maintains non-stationarity.

As to the the spot and futures prices interaction, there have been two views. The first is based on the cost-of-carry model which views the expected spot price as

being different from the spot price by interest rates, convenience yield and storage costs (see for example [Brennan \(1958\)](#)). This is also referred to as the theory of storage. In the empirical works built on this view, the futures price is sometimes used as the proxy of expected spot price (see for example [Fama and French \(1987\)](#)). Alternatively, the futures price is linked directly to the spot price with a slightly differently defined convenience yield (see for example [Pindyck \(2001\)](#) and [Figuerola-Ferretti and Gonzalo \(2010\)](#)). The other views the futures price as the sum of expected spot price and risk premium (see for example [Pindyck \(2001\)](#)). The two views are not exclusive of each other, also both involve the concept of the unobserved expected spot price.

Since this paper models directly the unobserved expected spot price, the futures price is modeled following the second view. The following section will discuss the modelling of the spot and futures prices based on the above assumptions.

## **2.2 An Unobserved Components Model of Spot and Futures Prices**

Like in [Pindyck \(1999\)](#), the long-term component reflects the long-run equilibrium level of price evolution, and the short-term component reflects short-term deviation from this long-run equilibrium level. Neither component is observable. The spot price generating process is summarized by the following equations:



$$p_t = \tau_t + c_t + \epsilon_t^p \quad \epsilon_t^p \sim N(0, \sigma_p^2) \quad (1)$$

$$\tau_t = \tau_{t-1} + \epsilon_t^\tau \quad \epsilon_t^\tau \sim N(0, \sigma_\tau^2) \quad (2)$$

$$c_t = \rho_1 c_{t-1} + \rho_2 c_{t-2} + \epsilon_t^c \quad \epsilon_t^c \sim N(0, \sigma_c^2) \quad (3)$$

where  $p_t$  is the spot price at time  $t$ ,  $\tau_t$  is the non-stationary long-term component,  $c_t$  is the stationary short-term component, and  $\epsilon_t^p$  is an idiosyncratic noise term in spot price.  $\epsilon_t^\tau$  and  $\epsilon_t^c$  are idiosyncratic noise terms in the long-term and short-term components respectively, and are assumed to be correlated with coefficient  $cov_{\tau,c}$ .

For the futures market, the futures prices do not contain more information about the future compared to the spot price (see for example [Working \(1942\)](#), [Tomex and Gray \(1970\)](#) and [Peck \(1985\)](#)). The information incorporated in futures prices is the same as in spot price. Assumed efficient futures market implies the following no-arbitrage condition:

$$f_t^T = E_t(p_{t+T}) + RP_t^T \quad (4)$$

where  $f_t^T$  is the price at time  $t$  for a contract maturing at  $T$ , and  $RP_t^T$  is a term incorporating all the risk at time  $t$  associated with the futures contract of maturity term  $T$ . Again, neither  $E_t(p_{t+T})$  or  $RP_t^T$  is observable. However  $E_t(p_{t+T})$  can be inferred if the spot price stochastic process is known.

I model the risk premium term  $RP_t^T$  as follows:

$$RP_t^T = \mu_{rp}^T + \beta_T rp_t + \epsilon_t^{fT} \quad \epsilon_t^{fT} \sim N(0, \sigma_{fT}^2) \quad (5)$$

$$rp_t = \rho_{rp} rp_{t-1} + \epsilon_t^{rp} \quad \epsilon_t^{rp} \sim N(0, \sigma_{rp}^2) \quad (6)$$

where  $\mu_{rp}^T$  is a constant capturing the term-dependent average risk premium level for a futures contract with maturity of  $T$ , which is time-invariant;  $rp_t$  is an AR(1) term capturing the time-varying part in the risk premium at the price-quoting time  $t$ , and is common to all futures contracts risk premiums, regardless of different maturities. However the common risk premium component  $rp_t$  is loaded into each futures price's risk premium term with term-dependent loading factor  $\beta_T$ . This allows for handling futures prices with different maturity terms, and also allows for a flexible time-varying risk premiums.  $\epsilon_t^{fT}$  is an idiosyncratic noise term in the risk premium and is specific to futures contracts with maturity term  $T$ .

An equation for the price of a futures contract with maturity term  $T$  follows through naturally:

$$f_t^T = E_t \left( p_{t+T} \right) + \mu_{rp}^T + \beta_T rp_t + \epsilon_t^{fT} \quad \epsilon_t^{fT} \sim N(0, \sigma_{fT}^2) \quad (7)$$

$$rp_t = \rho_{rp} rp_{t-1} + \epsilon_t^{rp} \quad \epsilon_t^{rp} \sim N(0, \sigma_{rp}^2) \quad (8)$$

The two sets of equations for the spot and futures prices hold simultaneously in an efficient market. Equations (1), (2), (3), (7) and (8) should be estimated as

a system with observed  $p_t$  and  $f_t^T$ . The model then can be then rewritten into a state-space form and estimated by maximum likelihood using Kalman filter. The detailed discussion is provided in Appendix A. Before moving on to estimation, I will first discuss the spot and futures price dynamics implied by this model.

## 2.3 Model Discussion

The model provides rich dynamics to fit most features of the observed price dynamics like in Figure 1 from crude oil market. One implication from the model is that it allows for lower volatility for the prices of future contracts with longer maturity lengths. Under the model, all prices contain a non-stationary long-term component as well as a stationary short-term component, but are affected by them differently. In other words, the spot and futures prices all contain the same set of information but are affected differently. The futures prices, especially those with longer terms, are less affected by the short-term shocks like temporary supply disruptions. The longer the maturity term, the more the short-term shocks would dissipate, leaving the futures prices to be subject to mostly the volatility of long-term component. This implication is consistent with observed market data.

Another implication from the model is that it allows for flexible futures curves. Futures curves simulated by the model can be upward or downward sloping, or even hump-shaped. If current spot price is lower than the long-term level, the negative short-term component's effect on future delivery would be expected to dissipate over time, resulting in higher futures price quoted today, and thus a upward-sloping futures curve. The futures curve could also be further shaped by the futures contract

risk premiums. If the risk premium for futures contracts with longer maturity lengths is more negative than the shorter ones, the different risk premiums at different maturity lengths might result in a hump-shaped futures curve.

Furthermore, the model provides an alternative interpretation of forecasting accuracy. Namely, the forecasting accuracy could be affected by changing market condition like the prevalence of short-term or long-term shocks and their sizes in the market. If the model correctly captures the price dynamics, the existence of stationary component would reduce the forecasting error variation relatively to a random walk forecast. More specifically, the reduction as measured by the relative *MSE* of the two is determined by the short-term component volatility.

## **3 Empirical Studies: the Case of Crude Oil Price**

### **3.1 Data Overview**

I use the crude oil market data for applying the model. An overview of the data is plotted in Figure 1. The main advantage of crude oil market is the availability of data. WTI spot price is available from 1986, and WTI futures contracts have been traded actively since late 1980s. Among different oil price measures, I focus on the forecasting of weekly WTI spot price. Firstly, WTI spot and futures prices have a longer history. Secondly, the concern that there is a consistent divergence between WTI and Brent prices and that WTI price is less popular as the world oil price is less relevant to this application. Using the proposed model to explore the forecasting ability of the futures prices requires only that the spot and futures prices are quoted

on the same underlying commodity, so that presumably the spot and futures prices would be affected by the same market disturbances.

Weekly frequency is selected since intuitively a lot of very-short-run disturbances to the price would be averaged out at monthly frequency. Comparing the spot and futures prices at weekly frequency could help separating the short-term component in the price, which potentially could improve the forecasting accuracy.

To separate out the long-term component, I specifically select the futures contracts whose maturities are of 6 months, 12 months and 18 months to cover mid- and long-term futures prices. There are several reasons for such selection. The first is that 18-month length may be far enough to capture the long-term component as most of the short-run fluctuations in prices dissipate within this horizon ([Herce et al. \(2006\)](#)). The second consideration is the availability of futures price data as the trading of longer-term futures contracts were rare on NYMEX during 1980's.<sup>1</sup> Overall 18-month futures price would be a balance between an enough term-length for uncovering the long-term component of oil prices and a meaningful length of time series data. The third consideration is that selecting 6-month and 12-month as mid-term futures prices help alleviate the imprecision due to the expiration date of futures contracts.<sup>2</sup>

---

<sup>1</sup>For example, weekly observations of 12-month futures price started in March 1989. 18-month futures contracts first started trading in September 1989, but were not traded weekly until 1995.

<sup>2</sup>For WTI futures contracts traded on NYMEX, each contract expires on the third business day prior to the 25th calendar day of the month preceding the delivery month (If the 25th calendar day of the month is a non-business day, trading ceases on the third business day prior to the business day preceding the 25th calendar day. After a contract expires, same type of contract for the remainder of that calendar month is the second following month.). As a result, the 1-month futures prices quoted for each week of the month is not precisely associated with oil delivered 4 weeks from spot delivery. Also, since a month can be either 4 weeks or 5 weeks, the information difference between spot price and 1-month future price could reflect variable number of weeks, rather than the fixed

The empirical application of the model uses the real prices of crude oil. Although the model could work for both real and nominal prices, intuitively it seems more appropriate to measure such price in real term, as the long-term component for the equilibrium price ultimately is determined by the market fundamentals like aggregate demand and supply.

The WTI spot and futures prices are available from Energy Information Administration website and Datastream. The maturities of the futures contracts are of 6 months, 12 months and 18 months to cover mid- and long-term futures prices. The prices are the daily closing prices at the end of each week, starting from the week ending on March 31, 1989 to the week ending on November 28, 2014. US CPI data is available from U.S. Bureau of Labor Statistics at monthly frequency, which is linearly interpolated to weekly frequency. Persistence test results using ADF test are reported in Table 1. As the results show, all data series contain a unit root, confirming the view of the model that the spot and futures prices contain a long-term random-walk component.

---

4 weeks assumed by the model. Since the model assumes that the information difference between spot and futures price of a certain maturity length is fixed in order to separate the long-term and short-term components, this makes the separation less precise. However, since I use 6-month futures prices as the shortest maturity, I argue that the difference of one more week of dissipating short-term shocks on top of approximately 24 weeks is relatively small. It would be more so for 12- and 18-month futures prices.

## 3.2 Estimation Results

In this section, I estimate the model using the full sample from 1989 to 2014<sup>3</sup> with time-varying risk premiums and correlated long-term and short-term components ((9) and (12) in Appendix A). Estimates are reported in Table 2.

The point estimates of most parameters are significant at 99% significance level. Due to the small magnitude of the point estimates, I also plot the estimated unobserved components time series with 90% confidence intervals in Figure 2: long-term  $\tau_t$ , short-term  $c_t$ , and the time-varying component of risk premiums  $rp_t$ . I also plot overall risk premiums for different contracts in Figure 3. Note that the estimated risk premiums from all contracts are systematically negative throughout the sample period, and tend to be less negative post 2000s except for a short period around 2008 and towards the end of the sample period.

Keynes (1930) theory of risk premium proposes that if hedgers in the futures market are net short (i.e. producers of the physical commodity) seeking price protection, then in order to entice speculator into taking the offsetting long positions, risk premium would be negative as a reward. And the longer the maturity term, naturally the higher the risk associated with the futures contract, thus the reward for bearing the risk would tend to be larger in size. When the hedgers are net long (i.e. buyers), risk premium would be positive. It would follow from the theory that the sign and size of risk premium would logically be related to the distribution of hedgers.

---

<sup>3</sup>Although the 18-month futures price have missing observations during the earlier periods, the state space model can still be easily estimated with missing observations in data. More details are provided in Appendix A

This model enables estimating risk premiums by exploiting the spot price and futures curves, rather than approximating the unobserved risk premiums as the naive difference between the spot and futures prices. The resulting estimated risk premiums share similar pattern as in recent literature. Even without splitting the sample period into two as in [Hamilton and Wu \(2014\)](#), the estimated risk premiums plotted in [Figure 3](#) show similarly smaller on average compensation to the long position in more recent data (the risk premiums become less negative). In my results, however the very end of the sample period sees a larger compensation to the long position again.

## 4 Out-of-Sample Forecasting Performance

### 4.1 Overall Forecasting Accuracy of the Model

The out-of-sample forecasting accuracy is mainly evaluated in this paper by four measures, and the statistical significance of the improvement is also tested.  $MSE$  measures the average variation of the forecasting errors,  $ME$  measures the overall unbiasedness of the forecasts,  $MAE$  provides an overview of the average absolute error size, and the fraction of smaller absolute errors relative to alternative forecasts. Although the first three measures are widely used in the literature and provide a good overview, it could be possible that a few extremely good or bad forecasts bias the actual improvement in accuracy. Thus in addition I also provide the fourth relative measure. The benchmark alternatives are the futures prices and the random walk forecasts.



Figure ?? plots the forecasting errors at different forecasting horizons using forward recursive estimation. The first estimation uses data from March 31, 1989 to March 18, 1994. 1 to 48-week ahead forecasts are made. The estimation sample period is extended at weekly frequency until Jan 31, 2014. A total of 1038 forecasts are generated for each forecasting horizon. Figure ?? selectively plots only 4-, 12-, 24- and 48-week ahead forecasts. To check the robustness of the forecasting performance, forecasting errors from rolling-window estimation are also plotted in Figure ?. The rolling estimation uses 260 weeks (approximately 5 years) of data, first starting from March 31, 1989 to March 18, 1994. The estimation sample period is rolled forward at weekly frequency until Jan 31, 2014. Again 1038 forecasts at all forecasting horizons are generated. The rolling estimation forecasts almost look identical to the forward recursive estimation results. Judging from Figure ? and ?, the model and the two alternatives all perform similarly at all reported forecasting horizons.

Detailed summarizing statistics of the forecasting errors reported in Table 3 reveal that overall the model improves the forecasting accuracy, with slightly weaker performance at 4- to 12-week forecasting horizons by certain measures. Using the forecasting unbiasedness measure  $ME$  the model outperforms the two alternatives at all forecasting horizons. Using the average forecasting error variation and size measures  $MSE$  and  $MAE$  the model outperforms the two alternatives at all forecasting horizons longer than 12 and 8 weeks, respectively.

Among the two alternative estimation methods, forward recursive estimation is better using  $MSE$  at all but 4- and 8-week forecasting horizons, while rolling-window estimation generates better forecasts using  $ME$  and  $MAE$  at all but 4-week fore-

casting horizons.

To test statistical significance of the improvement, I carry out the [Giacomini and White \(2006\)](#) finite-sample unconditional predictive ability test constructed from the forecasts from rolling-window estimation and the two alternatives<sup>4</sup>. Column 1 in Table 3 shows that, comparing the model and the random walk forecasts by MSE, the null hypothesis of equal predictive accuracy of the two is rejected at 10% significance level at 48-week forecasting horizons, and the model outperforms with smaller MSE; by MAE the null is rejected at 10% to 1% significance levels at multiple forecasting horizons (24- to 48-week). Comparing the model and the futures price forecasts by ME, the null hypothesis of equal predictive accuracy of the two is rejected at 10% significance level at multiple forecasting horizons (28- to 48-week); by MAE the null is rejected at 10% to 5% significance level at multiple forecasting horizons (12- to 48-week). On the other hand, in no cases has evidence of statistically better forecasts by either the random walk or the futures price relative to the model been found.

In addition I also present the fraction of model improvement in Table 3d. The fraction of the model forecasts with smaller absolute errors relative to the futures prices and the random walk alternatives adds on another dimension of forecasting accuracy. Overall the model forecasts have smaller absolute errors relative to both futures prices and the random walk alternatives at longer forecasting horizons. Forward recursive estimation performs better than rolling estimation when measured relative to futures prices, while rolling estimation performs better when measured relative to the random walk.

---

<sup>4</sup>A comparison of the forecasts from forward recursive estimation and the two alternatives is not allowed using [Giacomini and White \(2006\)](#).

The forecasting evaluation in Table 3 shows strong evidence of superior performance of the model. Especially at longer (than 12-week) forecasting horizons, the model forecasts are more possible to have smaller absolute errors, on average have smaller absolute errors and variation, and are more unbiased.

## 4.2 How the Model Improves the Forecasts

The proposed unobserved components model argues that the futures curve and its changes over time provide useful information about price movements. Under the assumption of no-arbitrage, futures curves provides important information about the evolvement of the spot price in addition to the spot and futures prices individually. The addition of such information improves the forecasting accuracy relative to both the random walk and the futures prices forecasts.

Furthermore, such improvement relies on more than just a simple composite of different futures prices. For example, in Table 3, using *ME* the futures prices (9-month, corresponding to 36-week forecasting horizon) perform worse than the random walk. Nonetheless the model forecasts outperform both the random walk and futures price forecasts. Also, the improvement in forecasting accuracy is not only in terms of average absolute error size and variation, but also in terms of the chance of the forecast in each period being better than the random walk and the futures prices forecasts.

However so far the long-term and short-term view of spot price movements and the additional information from futures curves have been largely ignored in the literature of forecasting and modelling commodity price dynamics. The proposed un-

observed components model provides a way to utilize futures curves and proves to generate more accurate forecasts compared to the benchmark in the literature.

### 4.3 Time-varying Forecasting Accuracy of the Model

Although the overall forecasting accuracy improvement is convincing, one interesting question is whether the predictability has changed over time.

To have a closer look, I choose the first 1032 forecasts (starting from March 18, 1994 to December 20, 2013) and take  $MSE$  and  $ME$  every 12 weeks. The approximately 3-month average forecasting accuracy over almost 20 years is plotted out in Figure 4 and 5. In these figures, the average forecasting accuracy of the model is relatively consistent over time except for mid-2008. At longer forecasting horizon, average forecasting errors during the 2000s are slightly smaller compared to the 1990s.

However, such smaller  $MSE$  and  $ME$  do not necessarily mean that the forecasting accuracy is improved the 2000s. It could be by chance that the forecasting errors tend to be slightly smaller on average in the 2000s, regardless of what forecast is used. A comparison between the model and the random walk forecasts is also presented in Figure 6 and 7, which plot out the  $MSE$  and  $ME$  ratios of the model forecasts to the random walk benchmark. A ratio less than 1 in absolute term indicates an improvement of model relative to the random walk.

Figure 6 and 7 show that in 2000s the model forecasts more frequently have smaller  $MSE$  and  $ME$  compared to the random walk. During mid-2008 when forecasting errors are large in general regardless of what forecast is used, the model

forecasts are still comparable to the random walk counterparts.

Considering all measures for forecasting accuracy, the model forecast perform slightly better in 2000s. In general this observation is very different from [Chinn and Coibion \(2013\)](#) who document a broad decline in the predictive content of commodity futures prices since the early 2000s. More importantly, my model provides very different interpretation of the time-varying forecasting accuracy. Under my model, the reduction of forecasting error variation relative to a random walk depends on the higher short-term component volatility. The estimated short-term volatility  $\sigma_c^2$  from the 260-week (approximately 5-year) rolling estimation shows that the volatility during 2000s is indeed larger than before, however it decreases towards the end of sample period (detailed estimates over time can be provided on request). This together with the forecasting accuracy improvement relative to the random walk forecast are consistent with the model's prediction. In other words, it could be simply the changing market condition that is driving the observed pattern in forecasting accuracy. For example, long periods of strong demand with constrained supply in the 2000s might result in more temporary shortage of crude oil, and thus higher volatility of the short-term stationary component, which reduces the forecasting error variation of the model relatively to the random walk forecast.

The changing market condition has also been documented and discussed in the price discovery literature (for example [Silvério and Szklo \(2012\)](#) and [Caporale et al. \(2014\)](#)), but this model provides an alternative interpretation as the time-varying dynamics comes from the short-term component in the spot price which might be driven by market fundamentals like supply and demand, rather than from the futures

market.

## 5 Conclusion

This paper proposes a model of commodity spot and futures prices for forecasting. The improved forecasting accuracy from the model is convincing. When applying the commodity price model to 25 years of oil market data, the model forecasts outperform the benchmark of the literature optimal in multiple dimensions. The model forecasts are more unbiased on average, has smaller forecasting error size and variation on average, and are also more possible to be so individually when compared to both random walk and futures prices forecasts during the sample period.

The superior performance comes from the additional information from futures curves utilized by the unobserved components model of price dynamics and the careful approximation of expected price. The spot and futures prices quoted on the same date contains the same information, but are affected by it to different extent. Furthermore, there are information that has longer lasting effect on the prices, and also information with temporary effect. I abstract the different information as the “long-term” and “short-term” components in the spot price. This spot price dynamics model enables more careful approximation of expected future spot price, which is the model forecast, by taking into consideration of the dissipating short-term component. Thus the model forecast is able to improve upon the random walk forecast, which assumes the spot price only contains the long-term component. As the forecasting exercises show, the advantage of the model forecasts is indeed more apparent

at longer forecasting horizon. Furthermore, the expected futures spot price allows for estimating risk premiums in futures prices. Thus the model forecast is also free of the effect of risk premiums in naive futures prices forecast. As the results show, the model forecasts outperform both random walk and futures prices forecasts.

The forecasting ability of futures market has been closely related to the efficient futures market hypothesis (see for example [Malkiel \(2003\)](#) and [Chinn and Coibion \(2013\)](#)). The proposed model instead provides an alternative interpretation of the forecasting accuracy. The improved forecasting accuracy relative to the random walk guess under the efficient market hypothesis shows that the changing relative forecasting accuracy could be merely due to the changing short-term component volatility, or in other words, the changing market conditions, rather than the efficiency level of the futures market.

The model also provides an approximation of unobserved expected spot price, which is a key concept to understand the interaction between futures and spot markets. Expected spot price is also what concepts like futures risk premium and convenience yield (as defined in for example [Brennan \(1958\)](#)) hinge on. With more accurately approximated expectation using the model, the risk premiums in futures prices could also be more accurately approximated. While this paper doesn't address the fundamentals behind the changing market conditions and the micro-foundation of the structure of risk premiums directly, the resulting estimated risk premiums from the model share similarities with that of [Hamilton and Wu \(2014\)](#). Similarly this model could be used to infer the convenience yield, which is closely related to market fundamentals of storable commodities. Application of the model would prove useful

for future work in the literature on futures risk premium as well as convenience yield for storable commodity futures.

## A State-space Setting and Maximum Likelihood Estimation

This section shows that the proposed unobserved components model can be rewritten into a state-space form using the example of modeling the spot and one futures prices. It can be extended to include multiple futures prices with different maturity lengths as in the paper. It also provides more details about the maximum likelihood estimation with Kalman filter at the end.

The long-term component  $\tau_t$ , short-term component  $c_t$  described by equations (2) and (3), and the time-varying risk premium component  $rp_t$  serve as the unobserved states underlying the price dynamics. The state equation can be rewritten in matrix form as the following:

$$\begin{bmatrix} \tau_t \\ c_t \\ c_{t-1} \\ rp_t \end{bmatrix} = \mathbf{F} \begin{bmatrix} \tau_{t-1} \\ c_{t-1} \\ c_{t-2} \\ rp_{t-1} \end{bmatrix} + \mathbf{G} * 1 + \mathbf{Z}\nu_t = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \rho_1 & \rho_2 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & \rho_{rp} \end{bmatrix} \begin{bmatrix} \tau_{t-1} \\ c_{t-1} \\ c_{t-2} \\ rp_{t-1} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} \epsilon_t^\tau \\ \epsilon_t^c \\ 0 \\ \epsilon_t^{rp} \end{bmatrix} \quad (9)$$

Spot price equation can be rewritten as:



$$p_t = \tau_t + c_t + \epsilon_t^p = \begin{bmatrix} 1 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} \tau_t \\ c_t \\ c_{t-1} \\ rp_t \end{bmatrix} + \epsilon_t^p \quad (10)$$

Futures price (with maturity term  $T$ ) equation can be rewritten as:

$$\begin{aligned} f_t^T &= E_t(p_{t+T}) + \mu_{rp}^T + \beta_T rp_t + \epsilon_t^{fT} \\ &= \begin{bmatrix} 1 & 1 & 0 & 0 \end{bmatrix} (\mathbf{F})^T \begin{bmatrix} \tau_t \\ c_t \\ c_{t-1} \\ rp_t \end{bmatrix} + \mu_{rp}^T + \beta_T rp_t + \epsilon_t^{fT} \\ &= \left( \begin{bmatrix} 1 & 1 & 0 & 0 \end{bmatrix} (\mathbf{F})^T + \begin{bmatrix} 0 & 0 & 0 & \beta_T \end{bmatrix} \right) \begin{bmatrix} \tau_t \\ c_t \\ c_{t-1} \\ rp_t \end{bmatrix} + \mu_{rp}^T + \epsilon_t^{fT} \end{aligned} \quad (11)$$

where  $\mathbf{F}$  matrix is the loading matrix governing state variables dynamics in equation (9).

Equation (10) and (11) can be then fit into the observation equation as follows:



Table 1: **Overview of Crude Oil Prices Persistence**

(a) level

Series	ADF t-statistic	# of lags	ADF t-statistic	# of lags	ADF t-statistic	# of lags
WTI spot	-0.703	1	-0.705	2	-0.799	3
6-month	-0.339	1	-0.393	2	-0.550	3
12-month	-0.120	1	-0.186	2	-0.329	3
18-month	-0.314	1	-0.362	2	-0.441	3

(b) first difference

Series	ADF t-statistic	# of lags	ADF t-statistic	# of lags	ADF t-statistic	# of lags
WTI spot	-24.20***	1	-19.07***	2	-15.34***	3
6-month	-23.26***	1	-17.83***	2	-14.99***	3
12-month	-22.98***	1	-17.80***	2	-15.14***	3
18-month	-19.28***	1	-15.26***	2	-13.02***	3

*Note:* (i) The above tests are performed using log-levels of the prices; (ii) \*, \*\* and \*\*\*denote that the null of a unit root is rejected at the 10%, 5% and 1% significance levels, respectively.

Table 2: **Estimated Unobserved Model of Spot and Futures prices**

parameters	model estimates
log likelihood	2891.87
Akaike (AIC) criterion	-4.29
Bayesian (BIC) criterion	-4.23
$\rho_1$	1.2048(0.0264)***
$\rho_2$	-0.2080(0.0269)***
$\sigma_\tau^2$	0.9597(0.3611)***
$\sigma_c^2$	0.6829(0.6000)***
$\sigma_{rp}^2$	0.0483(0.0037)***
$\sigma_p^2$	0.5520(0.096)***
$\sigma_{f6}^2$	0.0000(5.5E + 10)
$\sigma_{f12}^2$	0.0042(0.0009)***
$\sigma_{f18}^2$	-0.4359(1.4239)
$\mu_{rpf6}$	0.0003(0.5221)
$\mu_{rpf12}$	0.0064(0.0005)***
$\mu_{rpf18}$	1.3787(0.0190)***
$\beta_{12}^a$	-0.2349(1.0425)
$\beta_{18}$	14020(0.0382)***
$cov_{\tau,c}$	-0.9631(0.4818)**
$\rho_{rp}$	0.9742(0.0060)***

*Note:* (i) Standard errors are in parentheses; (ii) \*, \*\* and \*\*\*denote that the point estimate is significant at the 90%, 95% and 99% confidence levels, respectively.

<sup>a</sup>The loading factor for 6-month futures prices is normalized to be 1.

Table 3: **Out-of-sample Forecast Performance: 1994 - 2014**

(a) Out-of-Sample Forecast MSE

forecasting horizon	rolling <sup>a</sup>	forward recursive <sup>b</sup>	futures prices <sup>c</sup>	random walk
4-week	5.6754	5.7471	5.3968	5.8447
8-week	12.6354	12.6434	12.3574	13.0592
12-week	21.5991	21.5326	21.4724	22.2394
16-week	30.8979	30.6886	30.9778	31.6885
20-week	40.1604	39.7054	40.4026	41.0125
24-week	48.8360	48.0105	49.1441	49.5805
28-week	55.7641	54.4636	55.9356	56.6205
32-week	60.4069	58.7147	60.4375	61.5599
36-week	63.6467	61.4869	63.3219	65.1098
40-week	65.7654	63.1187	65.0160	68.2042
44-week	67.0145	64.1373	66.1304	70.8562
48-week	68.0901* <sup>1</sup>	65.1617	67.2564	74.2680

(b) Out-of-Sample Forecast ME

forecasting horizon	rolling <sup>a</sup>	forward recursive <sup>b</sup>	futures prices <sup>c</sup>	random walk
4-week	-0.0599	-0.0262	-0.0773	-0.1261
8-week	-0.1331	-0.1380	-0.1341	-0.2470
12-week	-0.2060	-0.2506	-0.2307	-0.3666
16-week	-0.2756	-0.3605	-0.3521	-0.4820
20-week	-0.3504	-0.4759	-0.4915	-0.6016
24-week	-0.4280	-0.5942	-0.6420	-0.7230
28-week	-0.5028* <sup>2</sup>	-0.7099	-0.7967	-0.8406
32-week	-0.5684* <sup>2</sup>	-0.8164	-0.9440	-0.9483
36-week	-0.6290* <sup>2</sup>	-0.9177	-1.0861	-1.0499
40-week	-0.6758* <sup>2</sup>	-1.0050	-1.2131	-1.1370
44-week	-0.7101* <sup>2</sup>	-1.0794	-1.3266	-1.2105
48-week	-0.7166* <sup>2</sup>	-1.1258	-1.4120	-1.2555

Continued on the next page

*Note:* (i) \*, \*\* and \*\*\* denote that the null hypothesis of equal predictive accuracy of compared models in [Giacomini and White \(2006\)](#) finite-sample unconditional test can be rejected at the 90%, 95% and 99% significance levels, respectively. 1 represents when compared to the random walk forecasts, 2 represents when compared to the futures price forecasts. HAC estimators of the variance for the test statistics are computed with Barlett kernel and a bandwidth of 100.

<sup>a</sup>Forecasts are generated from rolling estimation of the model, using 260 weeks (approximately 5 years) of data. The first estimation uses data from March 29, 1989 to March 18, 1994. The estimation sample period is rolled forward at weekly frequency until Jan 31, 2014.

<sup>b</sup>Forecasts are generated from forward recursive estimation of the model. The first estimation uses data from March 29, 1989 to March 18, 1994. The estimation sample period is extended at weekly frequency until Jan 31, 2014.

<sup>c</sup>The futures prices used for forecasting at 1- to 12-month futures prices, quoted at weekly frequency.

Table 3: **Out-of-sample Forecast Performance: 1994 - 2014 - Continued**

(c) Out-of-Sample Forecast MAE

forecasting horizon	rolling <sup>a</sup>	forward recursive <sup>b</sup>	futures prices <sup>c</sup>	random walk
4-week	1.6888	1.6839	1.6190	1.6847
8-week	2.3573	2.3605	2.3333	2.3849
12-week	2.9051**2	2.9244	2.9127	2.9932
16-week	3.3313**2	3.3676	3.3804	3.4448
20-week	3.6822**2	3.7544	3.7935	3.8514
24-week	4.0006*1,**2	4.1160	4.1728	4.2328
28-week	4.3275**1,**2	4.4394	4.5327	4.5949
32-week	4.5295**1,**2	4.6386	4.7561	4.8131
36-week	4.6505**1,*2	4.7754	4.9069	4.9671
40-week	4.7870***1,**2	4.9123	5.0766	5.1889
44-week	4.9441***1,**2	5.1084	5.2744	5.4196
48-week	5.1367***1,**2	5.3091	5.4649	5.7027

(d) Fraction of Smaller Absolute Forecasting Errors

forecasting horizon	benchmark: futures prices <sup>c</sup>		benchmark: random walk	
	rolling <sup>a</sup>	forward recursive <sup>b</sup>	rolling <sup>a</sup>	forward recursive <sup>b</sup>
4-week	0.4489	0.4576	0.4990	0.4817
8-week	0.4624	0.4624	0.5000	0.4884
12-week	0.4971	0.4875	0.5222	0.4798
16-week	0.5337	0.5376	0.5164	0.4692
20-week	0.5713	0.5665	0.5299	0.4846
24-week	0.5906	0.5848	0.5482	0.4817
28-week	0.5934	0.6127	0.5520	0.4942
32-week	0.5963	0.6320	0.5366	0.4913
36-week	0.6021	0.6445	0.5395	0.4855
40-week	0.6224	0.6580	0.5645	0.4904
44-week	0.6252	0.6532	0.5617	0.4904
48-week	0.6175	0.6647	0.5848	0.5010

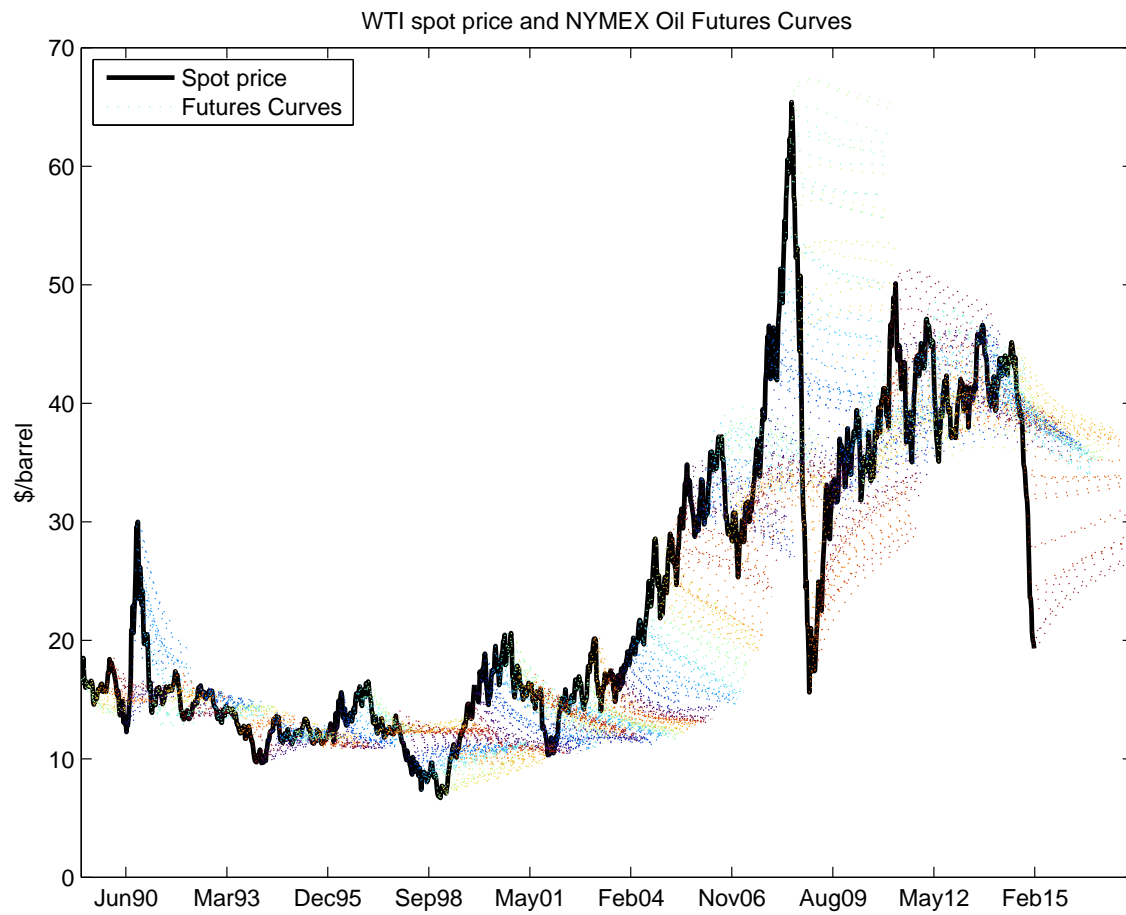


Figure 1: WTI spot price with futures curves

Source: Datastream. Weekly real prices (calculated by the author).

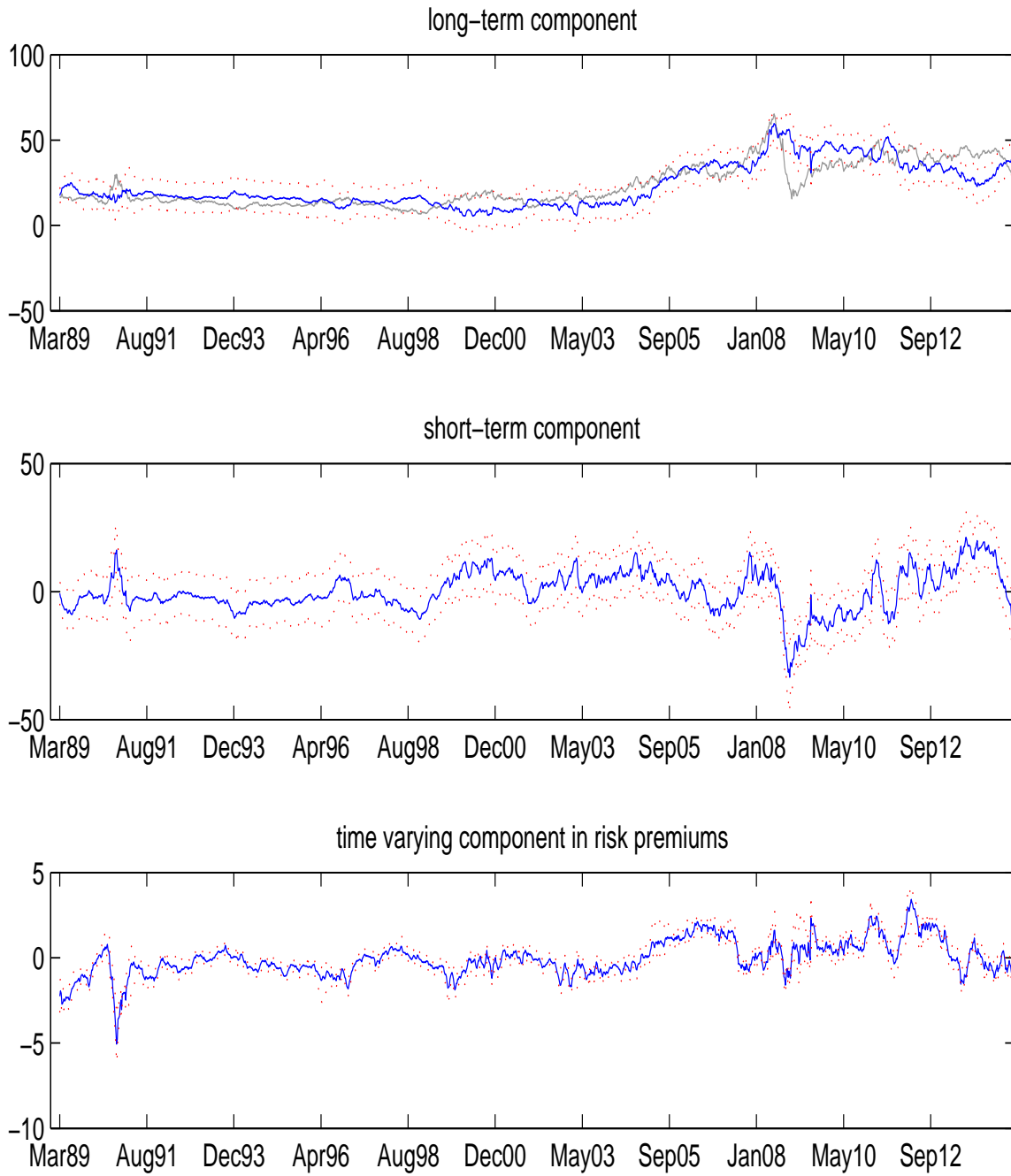


Figure 2: Estimated Unobserved Components

*Note:* The grey line plotted along with the long-term component is WTI spot price.



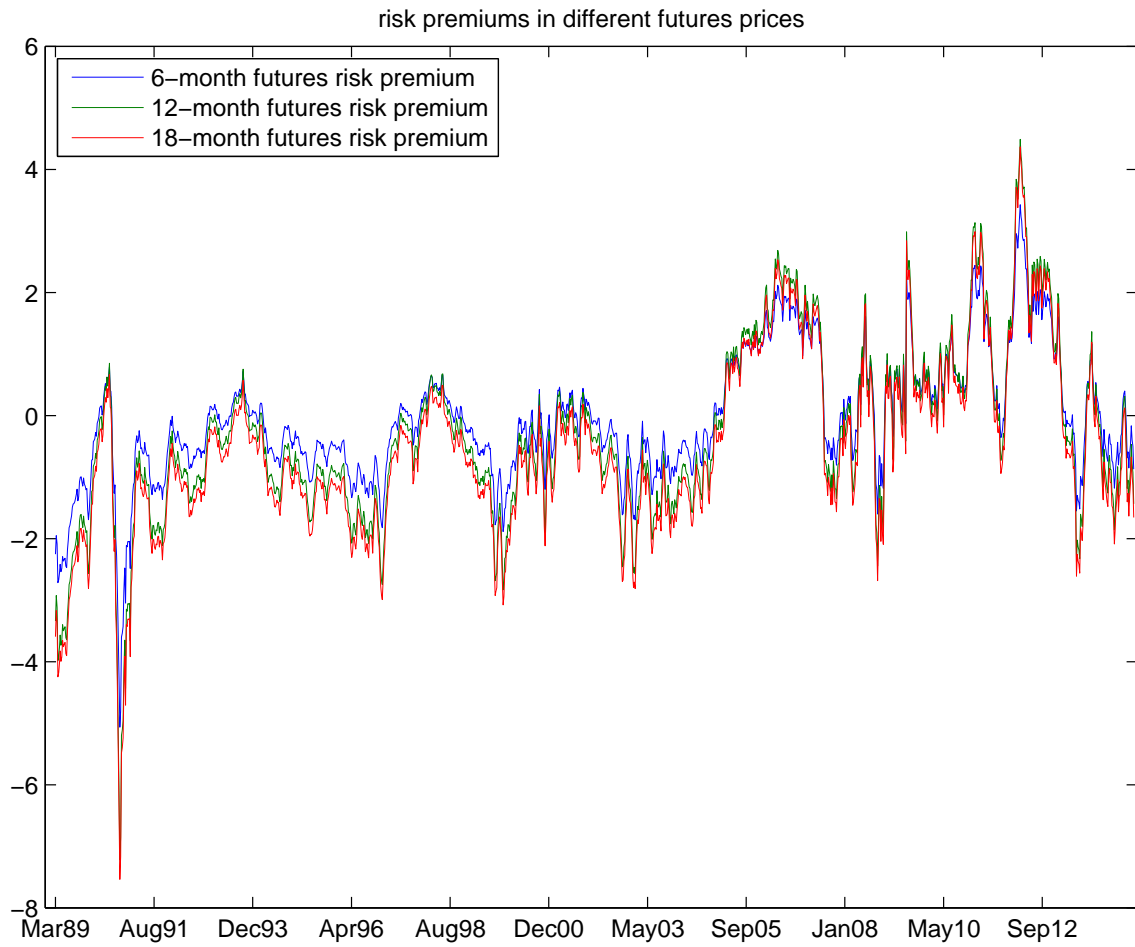


Figure 3: Estimated Risk Premiums of Different Futures Prices

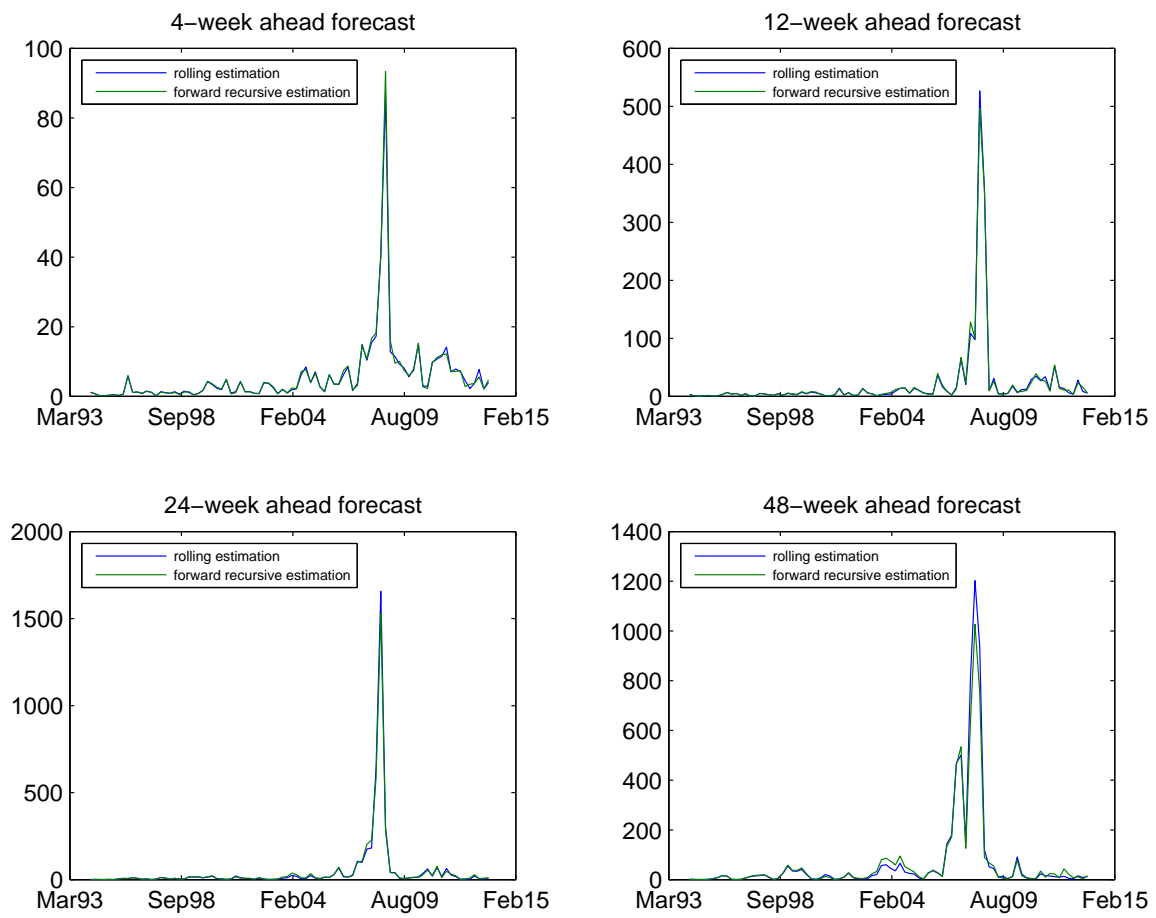


Figure 4: 12-week  $MSE$  at Different Horizons using Model Forecasts

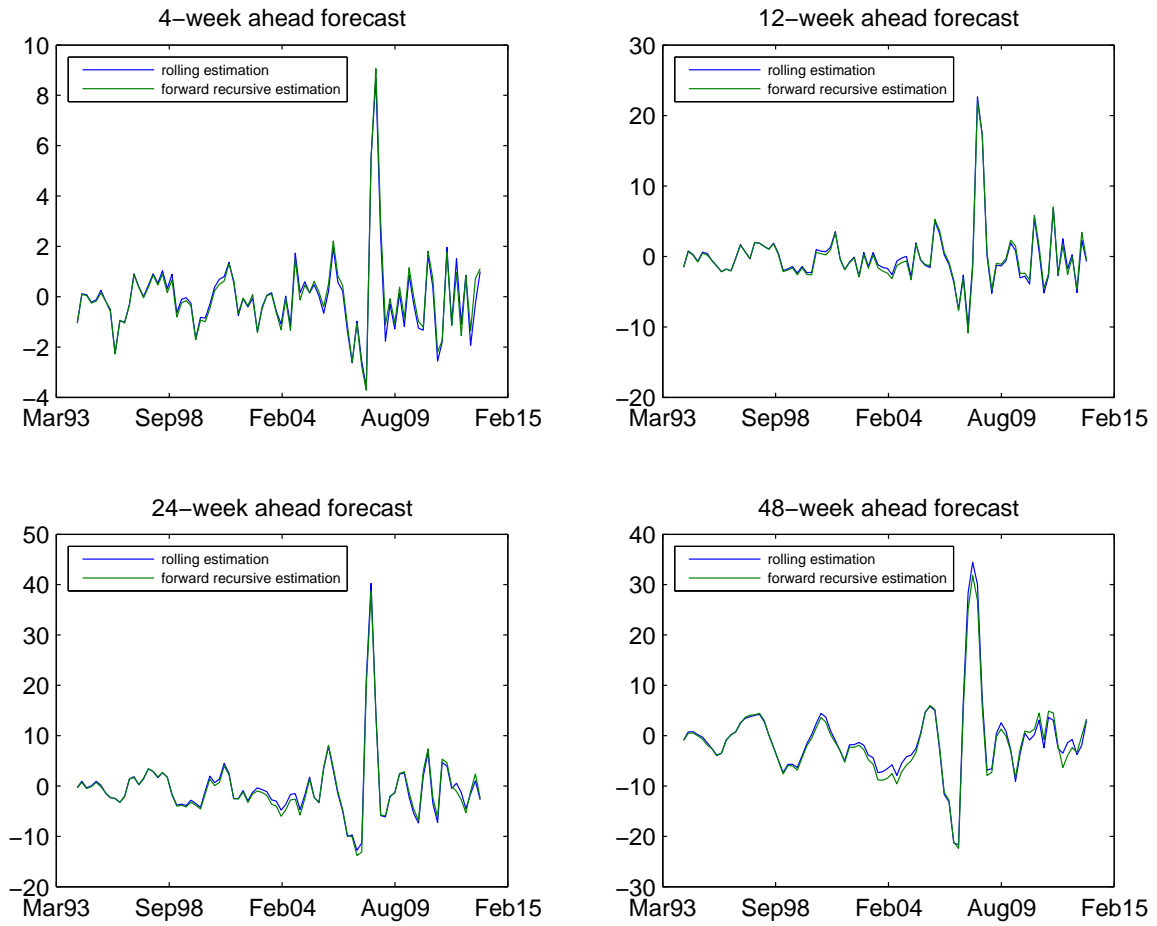


Figure 5: 12-week  $ME$  at Different Horizons using Model Forecasts

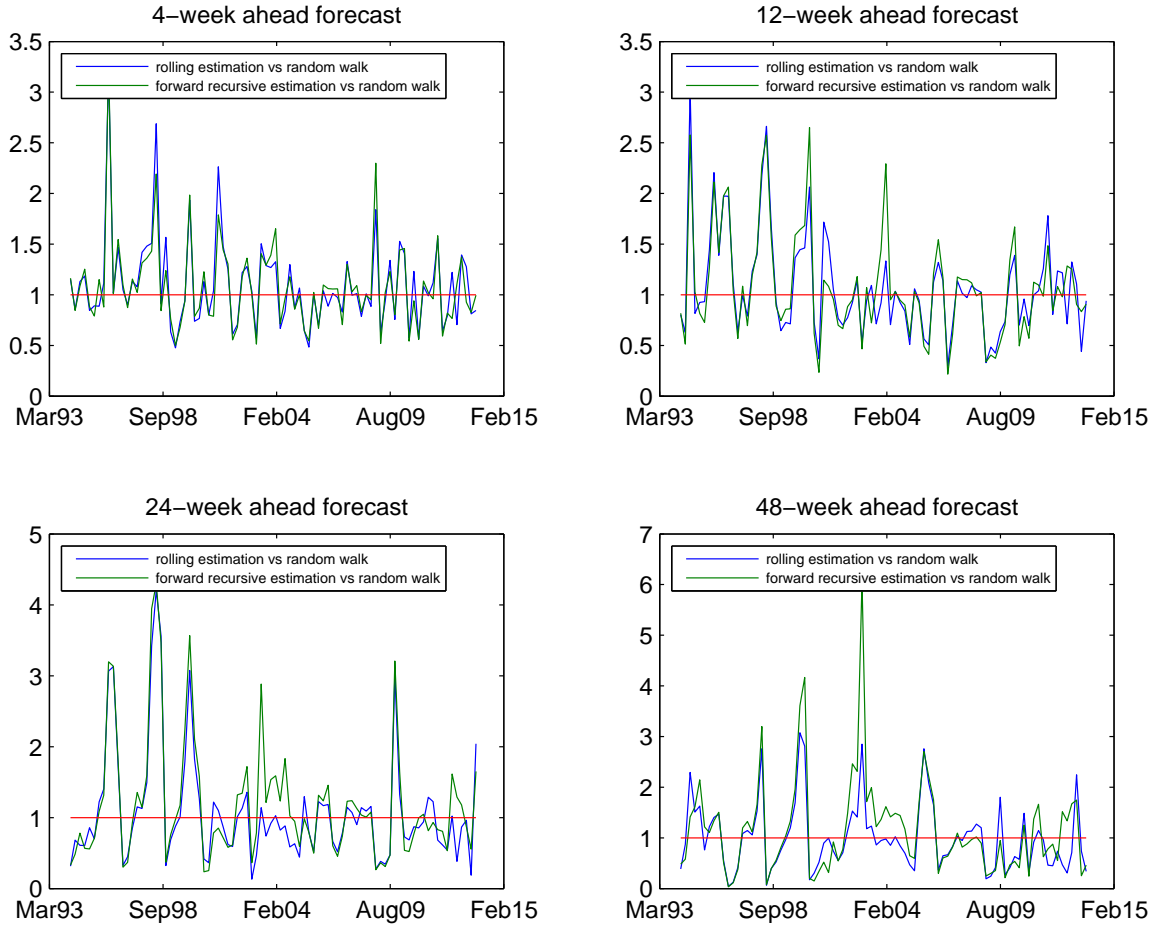


Figure 6: The model relative to the random walk: 12-week  $MSE$  ratios at Different Horizons

*Note:* Model forecasts 12-week  $MSE$  to random walk counterpart ratio is plotted with the red line =1.

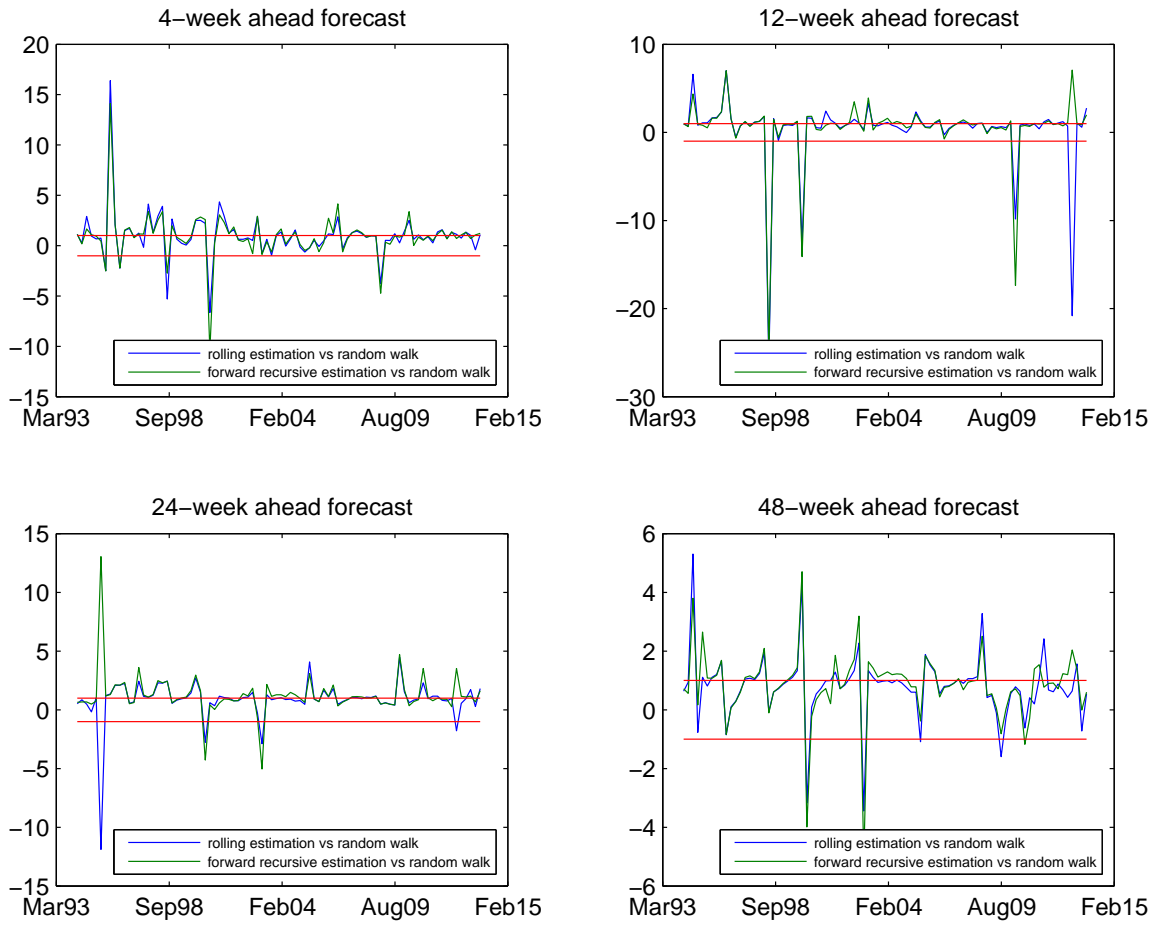


Figure 7: Model relative to random walk: 12-week  $ME$  ratios at Different Horizons using Model Forecasts

*Note:* The model forecasts 12-week  $ME$  to the random walk counterpart ratio is plotted with the red lines =1 and -1.

## References

- Alquist, Ron and Lutz Kilian (2010), “What do we learn from the price of crude oil futures?” *Journal of Applied Econometrics*, 25, 539–573.
- Alquist, Ron, Lutz Kilian, and Robert J. Vigfusson (2013), “Forecasting the price of oil.” In *Handbook of Economics Forecasting* (Graham Elliott and Allan Timmermann, eds.), volume 2, 427–507, North-Holland.
- Bopp, Anthony E. and George M. Lady (1991), “A comparison of petroleum futures versus spot prices as predictors of prices in the future.” *Energy Economics*, 13, 274–282.
- Bowman, Chakriya and Aasim M. Husain (2004), “Forecasting commodity prices: Futures versus judgment.” IMF Working Paper.
- Brennan, M.J. (1958), “The supply of storage.” *The American Economic Review*, 48, 50–72.
- Caporale, Guglielmo, Davide Ciferri, and Alessandro Girardi (2014), “Time-Varing Spot and Futures Oil Price Dynamics.” *Scottish Journal of Political Economy*, 61, 28, URL [http://www.diw.de/documents/publikationen/73/diw\\_{ }01.c.354179.de/dp988.pdf](http://www.diw.de/documents/publikationen/73/diw_{ }01.c.354179.de/dp988.pdf).
- Carlson, Murray, Zeigham Khokher, and Sheridan Titman (2007), “Equilibrium exhaustible resource price dynamics.” *The Journal of Finance*, LXII, 1663–1703.
- Chernenko, Sergey V., Krista B. Schwarz, and Jonathan H. Wright (2004), “The

- information content of forward and futures prices: Market expectations and the price of risk.” *International Finance Discussion Papers* No.808.
- Chinn, Menzie D. and Olivier Coibion (2013), “The predictive content of commodity futures.” *The Journal of Futures Market*, 34, 607–636.
- Coppola, Andrea (2008), “Forecasting oil price movements: Exploiting the information in the futures market.” *The Journal of Futures Markets*, 28, 34–56.
- Cortazar, Gonzalo and Eduardo S. Schwartz (2003), “Implementing a stochastic model for oil futures prices.” *Energy Economics*, 25, 215–238.
- Fama, Eugene F. and Kenneth R. French (1987), “Commodity futures prices: Some evidence on forecast power, premiums, and the theory of storage.” *The Journal of Business*, 60, 55–73.
- Fama, Eugene F. and Kenneth R. French (1988), “Permanent and temporary components of stock prices.” *Journal of Political Economy*, 96, 246–273.
- Figuerola-Ferretti, Isabel and Jesus Gonzalo (2010), “Modelling and measuring price discovery in commodity markets.” *Journal of Econometrics*, 158, 95–107, URL <http://dx.doi.org/10.1016/j.jeconom.2010.03.013>.
- French, Kenneth R. (1986), “Detecting spot price forecasts in futures prices.” *The Journal of Business*, 59, S39–S54.
- Giacomini, Raffaella and Halbert White (2006), “Tests of Conditional Predictive Ability.” *Econometrica*, 74, 1545–1578.

- Hamilton, James D. and Jing Cynthia Wu (2014), “Risk premia in crude oil futures prices.” *Journal of International Money and Finance*, 42, 9–37.
- Herce, Miguel, John E. Parsons, and Robert C. Ready (2006), “Using futures prices to filter short-term volatility and recover a latent, long-term price series for oil.” MIT Center for Energy and Environmental Policy Research Working Paper 06-005.
- Keynes, John Maynard (1930), *A Treatise on Money*. Macmillan, Lodon.
- Malkiel, Burton G. (2003), “The efficient market hypothesis and its critics.” *Journal of Economic Perspectives*, 17, 59–82.
- Moosa, Imad and Nabeel Al-Loughani (1994), “Unbiasedness and time varying risk premia in the crude oil futures market.” *Energy Economics*, 16, 99–105.
- Peck, A.E. (1985), “Economic role of traditional commodity futures market.” In *Futures Markets: Their Economic Role* (A.E. Peck, ed.), chapter 1, 1–81, American Enterprise Institute for Public Policy Research, Washington, D.C.
- Pindyck, Robert S. (1999), “The long-run evolution of energy prices.” *Energy Journal*, 20, 1–27.
- Pindyck, Robert S. (2001), “The dynamics of commodity spot and futures markets: A primer.” *The Energy Journal*, 22, 1–29.
- Reeve, Trevor A and Robert J. Vigfusson (2011), “Evaluating the forecasting performance of commodity futures prices.” International Finance Discussion Papers No.1025.



- Reichsfeld, David A. and Shaun K. Roache (2009), “Do commodity futures help forecast spot prices?” IMF Working Paper.
- Schwartz, Eduardo S (1997), “The stochastic behavior of commodity prices: Implications for valuation and hedging.” *Journal of Finance*, 52, 923–73.
- Silvério, Renan and Alexandre Szklo (2012), “The effect of the financial sector on the evolution of oil prices: Analysis of the contribution of the futures market to the price discovery process in the WTI spot market.” *Energy Economics*, 34, 1799–1808, URL <http://dx.doi.org/10.1016/j.eneco.2012.07.014>.
- Stevens, Jason (2013), “The benefits of storage and non-renewable resource price dynamics.” *Canadian Journal of Economics*, 46, 239–265.
- Tomex, William G. and Roger W. Gray (1970), “Temporal relationships among prices on commodity futures markets: Their allocative and stabilizing roles.” *American Journal of Agricultural Economics*, 52, 372–380.
- Working, Holbrook (1942), “Quotations on commodity futures as price forecasts.” *Econometrica*, 10, 39–52.