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Orthogonal cutting process modelling considering tool-workpiece frictional effect

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Abstract

In this paper a new frictional model of cutting process [1] developed to gain better insight into the mechanics of frictional chatter is presented. The model takes into account the forces acting on the tool face as well as on the tool flank. Nonlinear dynamic behaviour is presented using bifurcation diagrams for nominal uncut chip thickness (feed rate) as the bifurcation parameters. The influence of the depth of cut for different tool stiffnesses have been investigated. Finally, the influence of the tool flank forces on the system dynamics is studied.

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Keywords: tool flank forces; cutting process dynamics; chatter; friction;

1. Introduction

Cutting process generates unwanted vibrations of the tool and the workpiece called chatter. Self-excited chatter vibrations are harmful because can reduce the volumetric efficiency, increase the tool wear, decrease the geometric accuracy of product surface. Recently, a new model has been developed by the authors for the study of frictional chatter in planning/shaping operations [1,2] where the effect of the various operating parameters and the various forces on the resulting dynamics of the system is presented.

In the literature the models of orthogonal cutting are very popular. They usually treat the cutting force as a resultant force deriving from shearing material. However, shearing is not the only phenomenon during machining. Another effect is friction between the tool and the workpiece or the chip which is neglected in most cases. While, the friction force acting on the tool face and the tool flank can also influence cutting system dynamics [2]. The tool face force is considered as a principal cause of tool wear [3–7]. The first meaningful model of orthogonal cutting was developed in the middle of the last century by Merchant [8], but the first strongly nonlinear model of the cutting process was published by Grabec in 1986 [9]. Grabec focussed on frictional chatter wherein both velocity dependent cutting forces as well as dry friction between the chip and the workpiece were considered.

Nomenclature

a_{po}	uncut chip thickness (feed rate)
v_o	cutting velocity
q_o	cutting force coefficient (cutting resistance)
f_x	cutting force in x direction
f_y	cutting force in y direction

This study gave a background of developing meaningful frictional models of cutting process and let us understand the nonlinear response. The process is still ongoing and new models are still being developed wherein various nonlinearities of the cutting forces are introduced. Most researchers focus on cutting forces that depend on the axial width of cut, the chip thickness or the cutting speed but ignore the possibility of the tool losing contact with the workpiece which introduces significant nonlinear behaviour including chaos. Grabec [9] and Wiercigroch and co-workers [10–12] were the first to account for this kind of discontinuous behaviour in their studies of frictional chatter.

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Wahi and Chatterjee [13] studied the effect of the nonlinearity caused by loss of contact between the tool and the workpiece in the study on the regenerative chatter in the turning process. What is more, the forces acting on the tool face and flank are treated as one cutting force. However, Rusinek et. al. [1] have only just treated them separately and pointed out the effect of these on the system behaviour. They have proposed a new model of the cutting process, in which vibrations are generated by a combined action of the velocity-dependent cutting forces and the frictional forces acting on the tool face and tool flank. Nonlinear dependence of the cutting force on the chip thickness has been ignored for simplicity as well as the self-excited Rayleigh term has been used for modelling the smooth nonlinearity in the friction forces as opposed to the complicated form assumed in [9,11,12,14].

In this contribution, we have further investigated the same model as reported in [1] where the forces acting on both the tool face as well as the tool flank are investigated but here the influence of cutting feed rate for various tool stiffness is additionally analysed. However, the main focus of this work is on the nonlinear phenomena caused by velocity dependent cutting forces as well as friction between the various contacting surfaces viz. the tool face and the chip, and the tool flank and the workpiece.

2. Modelling of frictional chatter

A new approach, reported in [1,2], in cutting process modelling is proposed here. A free-body diagram of the tool with forces acting on the flank along with forces on the cutting face is shown in Fig. 1a. On both these surfaces, there is a normal force which acts due to the cutting mechanism on the cutting face (N_1) or the reaction from the workpiece to penetration of the tool on the flank (N_2) and a tangential force due to the friction between the chip and the tool on the cutting face (F_1) and between the workpiece and the tool on the flank (F_2). Forces on the flank come from the formation of a wear flat which is inevitable in any real cutting situation. The model of cutting forces is used to an orthogonal cutting process (shaping with a straight tool), where the tool is presented as a two-degrees-of-freedom spring-mass-damper system with effective modal damping and stiffness in two orthogonal directions videlicet the cutting direction and perpendicular to the workpiece along the tool axis (Fig. 1b). Here, the tool back rake angle is neglected because usually it is very small. The individual forces on the various surfaces are replaced with the help of equivalent forces (F_x and F_y) acting in two orthogonal directions (the horizontal direction x parallel to the workpiece and the vertical direction y perpendicular to the workpiece, 3). The two components of the resultant force from the workpiece and the chip on the tool (F_x and F_y) can be achieved by summing the forces in the vertical and the horizontal directions

$$F_x = N_1 + F_2,\\ F_y = N_2 + F_1. \tag{1}$$

Next the model of frictional chatter is completed with expressions for the various forces (N_1, F_1, N_2, F_2) in terms of the process and material parameters. This model of the cutting force (N_1) is based on the nonlinear characteristics originally introduced by Hastings et al. [3], next adapted by Grabec [9] (Fig. 2(a), green colour) and later ameliorated by Wiercigroch and Krivtsov [12] (red colour). Here the friction forces (F_1 and F_2) are represented by a nonlinear friction model based on the self-exited Rayleigh term for simplicity as reported in [1]. The normal force on the tool flank (N_2) is modelled as contact stiffness between the tool and the workpiece. The parameters of the system are chosen such that the resultant force in the cutting direction from our model (black curve in Fig. 2a) matches the characteristics considered by Wiercigroch and Krivtsov [12]. It can be noticed from Fig. 2b that the force in the vertical direction (F_y) is asymmetric about the $F_y = 0$ axis due to the presence of the force N_2 on the tool flank along with the friction force F_1 on the tool face.

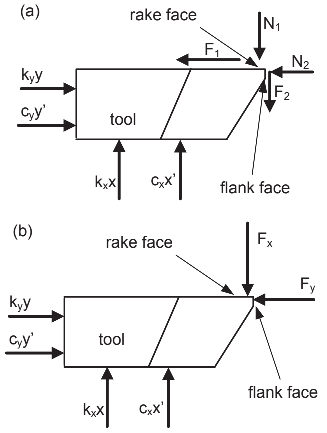

Fig. 1. Free-body diagrams of the cutting tool.

The expressions for the normal forces on the two perpendicular tool surfaces can be written as

$$N_1 = Q_o\, a_p \left(c_1(v_r - 1)^2 + 1\right) H(a_p)H(v_r),\\ N_2 = K_{con}\, a_p\, H(a_p), \tag{2}$$

where Q_o stands for the specific cutting force modulus, a_p is the instantaneous penetration of the tool into the workpiece (uncut chip thickness or feed rate), c_1 is a constant controlling the dependence of the cutting force on the relative velocity between the tool and the workpiece (v_r), K_{con} is the contact stiffness and $H(.)$ represents the Heaviside function. Note that the $H(v_r)$ describes mathematically the loss of contact between the tool and the chip while $H(a_p)$ accounts for the tool coming out of the workpiece. The contact stiffness (K_{con}) is determined in such way to match our force model to the corresponding one given in [12].

Some parameters of the cutting process are depicted more clearly in Fig. 3. The friction forces on the tool face (F_1) and

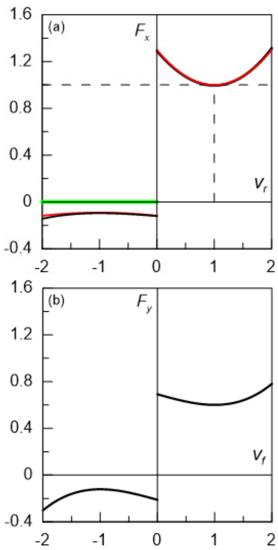

Fig. 2. Cutting force characteristic (a) F_x and (b) F_y [9,12].

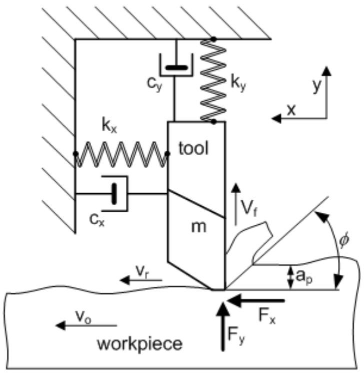

Fig. 3. Physical model of the cutting process.

the tool flank (F_2) are given by

$$F_1 = N_1\mu_y\left(\mathrm{sgn}(v_f) - \alpha_y v_f + \beta_y {v_f}^3\right),$$
$$F_2 = N_2\mu_x\left(\mathrm{sgn}(v_r) - \alpha_x v_r + \beta_x {v_r}^3\right), \quad (3)$$

where μ_x, μ_y denote the static coefficient of friction between the tool and the workpiece, and the tool and the chip, respectively, v_r and v_f are the relative velocities between the tool and the workpiece, and the tool and the chip, respectively and sgn() represents the sign function. Note that, friction is not classical Coulomb model because the friction force depends on the relative velocity (v_r and v_f) as a cubic function. Thus, the friction force is similar to the Sribeck friction model which better describes contact phenomena.

The tool along with its tool-holder assembly is replaced by a two degrees-of-freedom spring-mass-damper system as demonstrated in Fig. 3. The tool can vibrate in x (parallel to the workpiece) and y (perpendicular to the workpiece) directions. The cutting operation considered is a shaping operation so that cutting is orthogonal in nature and hence, the forces out of the $x-y$ plane are negligible during the cutting process. This justifies the choice of two degrees-of-freedom model for the combined machine tool-cutting process dynamics. In this model, the tool is represented by a lumped mass (m) which can move in the x and y directions. Moreover, the interaction between the tool and the tool-holder assembly is replaced by equivalent dampers with damping coefficients c_x, c_y, and also springs with stiffness coefficients k_x, k_y in the x and y directions respectively. With these simplifications, the equations governing the motion of the tool during the orthogonal cutting operation is given by the following ordinary second order differential equations:

$$m\ddot{x} + c_x\dot{x} + k_x x = F_x$$
$$m\ddot{y} + c_y\dot{y} + k_y y = F_y \quad (4)$$

where the forces F_x and F_y are obtained by substituting 2 and 3 in 1. This results in

$$F_x = Q_o\, a_p\left(c_1(v_r-1)^2+1\right)H(a_p)H(v_r)$$
$$+ K_{con}\, a_p\, \mu_x\left(\mathrm{sgn}(v_r) - \alpha_x v_r + \beta_x {v_r}^3\right) H(a_p),$$
$$F_y = K_{con}\, a_p H(a_p) + Q_o\, a_p\left(c_1(v_r-1)^2+1\right)\times \quad (5)$$
$$\mu_y\left(\mathrm{sgn}(v_f) - \alpha_y v_f + \beta_y {v_f}^3\right) H(a_p)H(v_r).$$

The instantaneous penetration of the tool into the workpiece or the uncut chip thickness a_p can be written in terms of the specified uncut chip thickness a_{po} and the tool motion y as

$$a_p = a_{po} - y. \quad (6)$$

The relative velocities between the tool and the workpiece (v_r), and the tool and the chip (v_f) are related to the nominal cutting speed v_o, the shear angle of the workpiece material ϕ and the tool velocities $\dot{x}$, $\dot{y}$ by

$$v_r = v_o - \dot{x}, \quad v_f = v_r \tan\phi - \dot{y}. \quad (7)$$

Note that $tan(\phi)$ in the Eq. 7 is a consequence of the difference in the chip thickness coming out of the tool and the thickness of the workpiece material approaching the tool.

Using a characteristic time scale determined by the natural frequency of vibrations in the x-direction, the nondimensionalized equations of motion is as follows:

$$\ddot{x} + 2z_x\dot{x} + x = f_x, \qquad \ddot{y} + 2z_y\sqrt{\alpha}\dot{y} + \alpha y = f_y, \quad (8)$$

where

$$\alpha = \frac{k_y}{k_x},\ {\omega_x}^2 = \frac{k_x}{m},\ {\omega_y}^2 = \frac{k_y}{m} = \alpha\,{\omega_x}^2,\ z_x = \frac{c_x}{2m\omega_x},\ z_y = \frac{c_y}{2m\omega_y}$$

and the forces are given by

$$\begin{aligned} f_x &= q_o\, a_p \left(c_1 (v_r - 1)^2 + 1\right) H(a_p) H(v_r) \\ &\quad + k_{con}\, a_p H(a_p)\, \mu_x \left(\mathrm{sgn}(v_r) - \alpha_x v_r + \beta_x {v_r}^3\right), \\ f_y &= k_{con}\, a_p H(a_p) + q_o\, a_p \left(c_1 (v_r - 1)^2 + 1\right) H(a_p) \times \\ &\quad H(v_r) \mu_y \left(\mathrm{sgn}(v_f) - \alpha_y v_f + \beta_y {v_f}^3\right), \end{aligned} \tag{9}$$

where q_o is the nondimensional cutting force coefficient (cutting resistance), a_p is the nondimensional uncut chip thickness (feed rate), $c_1, \alpha_x, \beta_x, \alpha_y, \beta_y$ are the nondimensional constants controlling the nonlinear characteristics of the forces, presented in Fig.2, v_r and v_f is the nondimensional relative velocities between the tool and the workpiece and the tool and the chip, respectively, given by Eq.7 and k_{con} is the nondimensional contact stiffness. Several control parameters in these equations can be changed during a real cutting process. In this study an influence of the nominal uncut chip thickness a_{po} on the system's dynamics is investigated for various α parameter, which means the tool stiffness ratio of x and y direction.

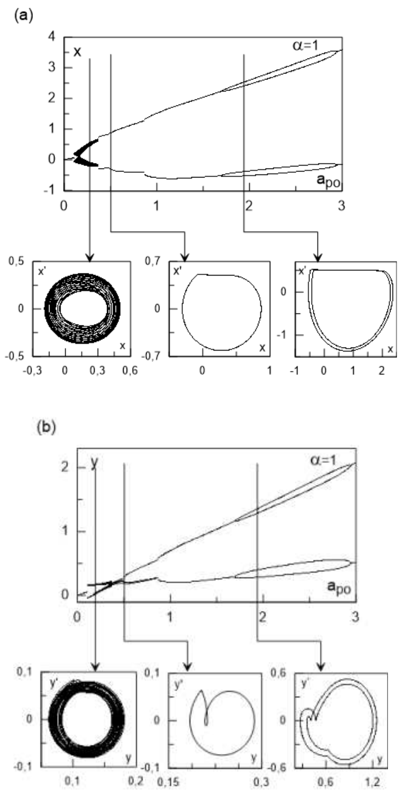

Fig. 4. Bifurcation diagrams for α=1, $x = f(a_{po})$ (a) and $y = f(a_{po})$ (b).

The mathematical description of the cutting process has the discontinuous sign function sgn(.) and the Heaviside function H(.) in Eq.9 which make difficult numerical computations. Therefore, the discontinuities are replaced by their smooth approximations using the hyperbolic tangent and the sigmoid functions given by

$$\mathrm{sgn}(x) = \tanh(\sigma x), \qquad H(x) = \frac{1}{1 + e^{-\sigma x}} \tag{10}$$

where $\sigma = 500$. The above simplification does not have any significant effect on the dynamics of the system but hastens numerical calculations almost 50 times using the MATLAB inbuilt command ode45 with a specified tolerance of 1e-6.

3. Influence of feed rate on system dynamics

A feed rate is one of the most important technological parameter. Therefore, the nominal uncut chip thickness a_{po} is chosen as a parameter in bifurcation analysis. From the practical point of view, altering the feed rate is the simplest way to avoid chatter vibrations. For the bifurcation diagrams, a standard set of nondimensional parameters is used, namely: $z_x = z_y = 0.01$, $v_o = 0.5$, $q_o = 0.9$, $\alpha = 1$, $\alpha_x = \alpha_y = 0.3$, $\beta_x = \beta_y = 0.1$, $\mu_x = \mu_y = 0.5$, $tan(\phi) = 0.45$, $k_{con} = 0.5$, $c_1 = 0.3$. Note here that the choice of the parameters $\alpha_x, \alpha_y, \beta_x, \beta_y, \mu_x, \mu_y, k_{cont}$, and c_1 is dictated by the fact that the forces in the x and the y directions for these parameter values of our model closely match the corresponding forces given in [12]. The other parameters have been directly adapted from [12] as well. The ratio of the stiffness in the x and y directions α is another key parameter and its effect on the system dynamic responses is also studied here. To get an insight into the variation of α on the system dynamics, the parametric investigations are repeated for three different α values, i.e., $\alpha = 1$, 4 and 16. Bifurcation diagrams presented below were constructed by plotting the displacements (x and y) at time instants corresponding to the velocity ($\dot{x}, \dot{y}$) being zero. Curves drawn in blue on the bifurcation diagrams represent the variation of the maximal Lyapunov exponent with the variation of the bifurcation parameter. However, the blue curve is shown only in those diagrams where chaotic vibrations (signified by its value being greater than zero) occur in the parameter range depicted in the figures.

The bifurcation diagram with the nominal uncut chip thickness (a_{po}) as the bifurcation parameter is presented in Fig. 4-6. All presented cases of the stiffness ratios (α=1, 4 and 16) characterise an increase of vibration amplitude with increasing feed rate. For small values of a_{po}, the system demonstrates only static displacement (single line in bifurcation diagrams) without vibrations. For the case of α=1, at around $a_{po} = 0.15$ quasiperiodic vibrations appear which bifurcate in periodic motion at about $a_{po} = 0.3$. At around $a_{po} = 1.7$ a period doubling bifurcation gives rise to a period-2 motion which again becomes period-1 through an inverse period doubling bifurcation at about $a_{po} = 2.9$. The similar behaviour is presented in case of α=4, but without quasiperiodicity. While for α=16 simpler dynamics is observable because a period-1 motion with "stick-slip" effect is only shown. In all cases y direction has more complex dynamics but the amplitude of vibrations is smaller, specially when α=16.

The presented mode is in concurrence with the well known relationship between the final surface quality and the feed rate that bigger feed rate generates bigger vibrations and then worst surface quality.

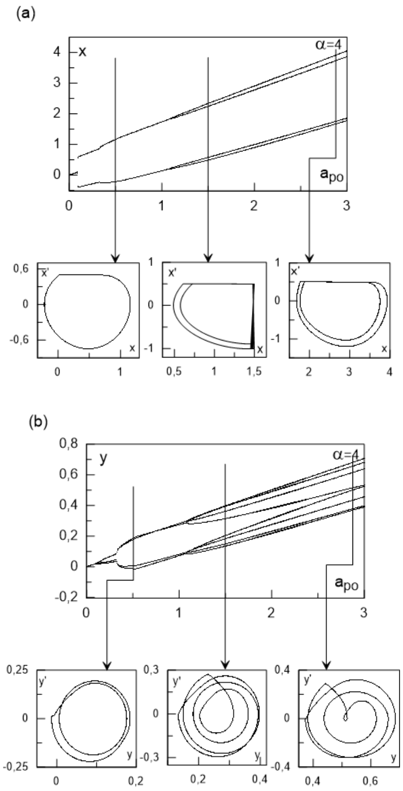

Fig. 5. Bifurcation diagrams for α=4, $x = f(a_{po})$ (a) and $y = f(a_{po})$ (b).

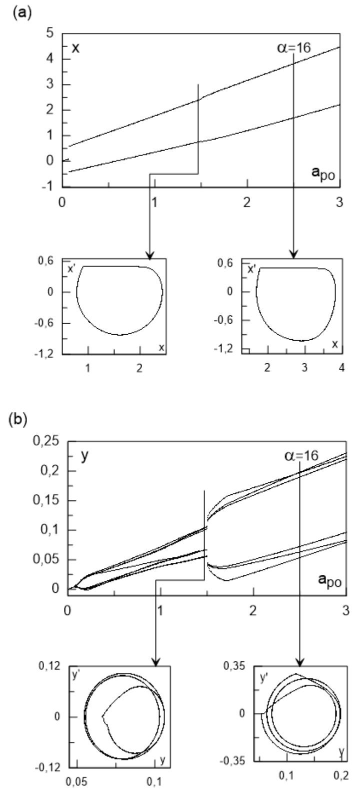

Fig. 6. Bifurcation diagrams for α=16, $x = f(a_{po})$ (a) and $y = f(a_{po})$ (b).

Now, we focus attention to the influence of the forces acting on the tool flank on the system dynamics. The bifurcation diagrams obtained for the full system with those obtained by dropping the friction force and the normal force acting on the tool flank are compared. Substituting $k_{con} = 0$ into 9, we get the system where the forces on the tool flank are neglected ($N_2 = T_2 = 0$) and our new model becomes convergent to models existing in literature, previously developed by Grabec [9] and Wiercigroch and Krivtsov [12]. A study of this case helps us in getting better insights about the role of the tool flank forces on the overall system dynamics.

As far as the feed rate depth is considered, the system behaves classically that means the increase of feed rate causes bigger vibrations (Fig.7-9). However, the system shows chaotic irregular behaviour for smaller nominal depths of cut and this effect is perceptible for the case of $\alpha = 1$ (Fig.7). The Poincaré sections and Lyapunov exponent clearly show that small feed rate causes irregular motions. This implies that for this study the forces on the tool flank are having a stabilizing effect.

4. Conclusions

The influence of the force acting on the tool flank on the dynamics observed during metal cutting is investigated here using a recently developed frictional model of metal cutting. The presented model exhibits a qualitatively different dynamic behaviour as compared to already existing models wherein the tool flank forces are neglected despite the fact that the resultant force characteristics for the new model as well as the already existing model [12] are very close. The system shows a wide variety of responses ranging from periodic, quasiperiodic, subharmonic to even chaotic when the forces on the tool flank are neglected. Therefore, the normal and friction forces on the tool flank have a stabilizing effect on the system dynamics. It has been observed in our study that in general when both the forces acting on the tool flank (normal and friction force) are neglected, the system dynamics is the most complex with chaotic regions. As far as the stiffness ratio α is considered, an increase of α causes stabilizing effect demonstrating by periodic solutions instead of quasiperiodic which are present in case of $\alpha = 1$. According to the presented model, an increase of the uncut chip thickness (feed rate) causes also an increase of chatter vibrations amplitude. That is not too optimistic from practical point of view but some interesting behaviours are found, for instance: quasi - periodic motion, period doubling bifurcation.

Future studies combining the regenerative effects with the frictional ones are required to describe full cutting process dynamics. Then, an experiment will be also planned to check the model correctness and to find stable and unstable regions.

Acknowledgements

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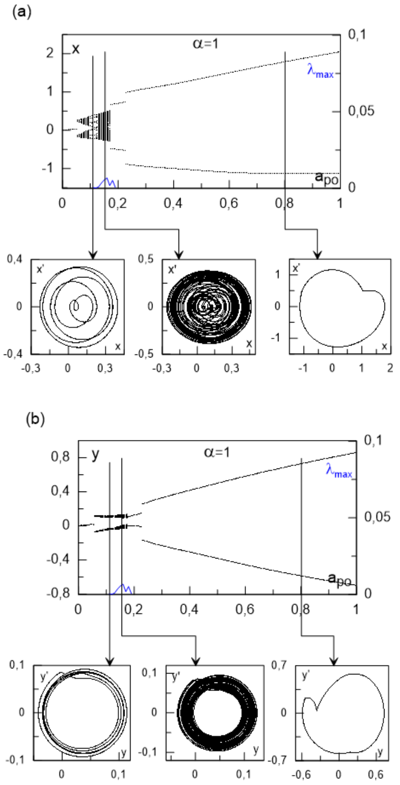

Fig. 7. Bifurcation diagrams for $\alpha = 1$ with neglected tool flank forces ($k_{con} = 0$), $x = f(a_{po})$ (a) and $y = f(a_{po})$ (b).

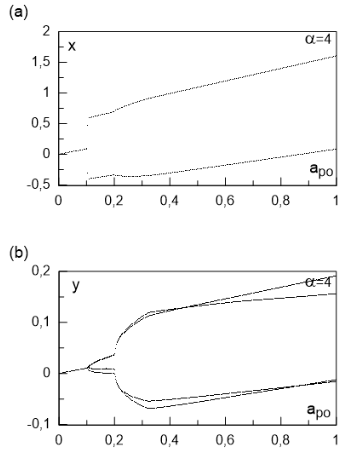

Fig. 8. Bifurcation diagrams for $\alpha = 4$ with neglected tool flank forces ($k_{con} = 0$), $x = f(a_{po})$ (a) and $y = f(a_{po})$ (b).

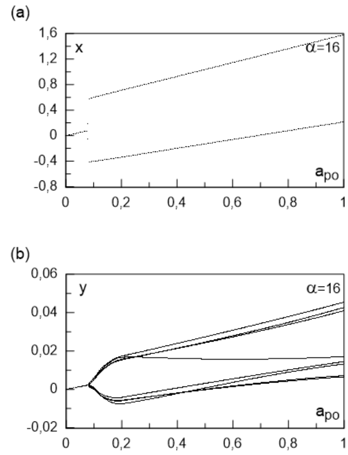

Fig. 9. Bifurcation diagrams for $\alpha = 16$ with neglected tool flank forces ($k_{con} = 0$), $x = f(a_{po})$ (a) and $y = f(a_{po})$ (b).

ropean Regional Development Fund - Project "Modern material technologies in aerospace industry", POIG.01.01.02-00-015/08-00 is gratefully acknowledged.

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