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Mirror node correlations tuning synchronization in multiplex networks

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We show that the degree-degree correlations have a major impact on global synchronizability (GS) of multiplex networks, enabling the specification of synchronizability by only changing the degree-degree correlations of the mirror nodes while maintaining the connection architecture of the individual layer unaltered. If individual layers have nodes that are mildly correlated, the multiplex network is best synchronizable when the mirror degrees are strongly negatively correlated. If individual layers have nodes with strong degree-degree correlations, mild correlations among the degrees of mirror nodes are the best strategy for the optimization of GS. Global synchronization also depend on the density of connections, a phenomenon not observed in a single layer network. The results are crucial to understand, predict, and specify behavior of systems having multiple types of connections among the interacting units.

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I. INTRODUCTION

Synchronization is a rhythmic activity exhibited by dynamical units due to the interactions among them [1]. The flashing of fireflies, the ticking of clocks, the synchronized clapping of a large audience, and the pathologically synchronized firing of brain cells during an epileptic attack can all be modeled as networks of dynamical units such that the coupling causes the dynamical units to synchronize [1]. Most of the studies about dynamics on networks have considered either single complex networks, whose nodes are connected following a unique topology, or whose nodes are connected by the same coupling function [2]. In recent years, network science has made a leap towards a better understanding of nature and man made systems by the realization that many real-world complex systems have multiple types of interactions among the same units [3–7]. For instance, the transport system of a country consists of different cities connected by bus, rail, and air networks. The bus network may have a very different connection architecture than the rail and flight networks. Such kinds of systems can be represented better by a framework of multiplex networks. In this interconnected multilayer structure, the activity of one layer affects the activities of other layers and consequently of the entire system. In other words, changes in the interactions or the local dynamics of the nodes in one layer have a crucial impact on the behavior of the same nodes in the other layer. For instance, changes in the network parameter of one layer have been demonstrated to affect the cluster synchronizability of the other layer [8]. Particularly, it has been shown that one can control the global synchronizability (GS) of a system by having optimal rewiring of only one layer of the underlying multiplex networks' architecture [9]. Similarly, synchronization in one layer is shown to be driven by energy transport in other layers [10]. Epidemic spreading and its control require a different strategy for a system having interconnected layers than for a system having only one type of interaction resulting in traditional monolayer networks [11,12]. Neural networks of *Caenorhabditis elegans*, gene-protein interaction, and coauthorship networks are other examples of real-world systems in which the multiplex framework has revealed important properties of the underlying systems [13].

Furthermore, real-world networks possess assortative or disassortative degree-degree correlations (see Fig. 1). These correlations have been demonstrated to affect robustness and controllability of a single layer as well as of the multilayer or multiplex networks [14–16]. Other recent works have demonstrated the importance of degree-degree correlations to the emergence of various properties, such as the giant component in multiplex networks [17] and disease spreading [18] in networks. The impact of degree-degree correlations on the synchronizability of single layer networks has already been investigated [19]. For multilayer networks, it has been shown that connecting two networks through high degree nodes results in an enhanced synchronization of the individual layers [20].

In this paper we investigate the interplay of degree-degree correlation among the nodes in the layers and among the mirror nodes located in different layers onto the GS of the multiplex network. Our analysis demonstrates that, by keeping the architecture of the individual layer unchanged, one can specify the GS in a multiplex network by simply tuning the degree-degree correlation of the mirror nodes. Furthermore, we show that a change in the degree-degree correlation of the mirror nodes causes a more profound impact on the synchrony of the nodes in a layer having positive degree-degree correlations as compared to those having negative or neutral correlations. We further find that GSs in multiplex networks are sensitive to changes in the average degree ($\langle k\rangle$), a phenomenon not observed for single layer networks.

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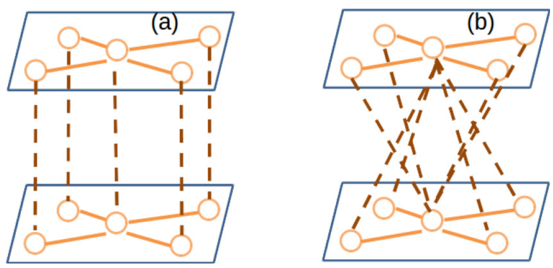

FIG. 1. A schematic depicting the architecture of a multilayer network that is (a) assortative and (b) disassortative.

II. THEORETICAL FRAMEWORK

A two layer multiplex network adjacency matrix can be represented by

$$A = \begin{pmatrix} A^1 & I \\ I & A^2 \end{pmatrix}, \tag{1}$$

where A^1 (A^2) represents the adjacency matrix of the first (second) layer of $N/2$ nodes and I is an identity matrix of order $N/2 \times N/2$; it represents interactions between the two layers.

The Laplacian matrix is defined as $L_{ii} = D_{ii}$ if $i = j$ and $L_{ij} = -A_{ij}$ where $D_{ii} = \sum_{k=1}^{N} A_{ik}$. Since the Laplacian is a positive semidefinite matrix with a zero row sum, its eigenvalues can be arranged as $\lambda_N > \lambda_{N-1} > \cdots > \lambda_2 > \lambda_1 = 0$. The stability of the globally synchronized solution for a coupled dynamical system represented by

$$\dot{x_i} = F(x_i) - \sigma \sum_{j=1}^{N} L_{ij} H(x_j) \tag{2}$$

is determined by the ratio of the largest to the first nonzero eigenvalue of the Laplacian matrix [21,22]. Here, $F(x_i)$ represents the local dynamical evolution of the ith node and remains the same for all the oscillators. x_i is the dynamical variable of the ith node, in general it is an m dimensional vector. $H(x)$ is the coupling function, it is the same for all the nodes, and it can be any linear or nonlinear function of the state vector x. σ is the overall coupling strength. Since the Laplacian is a zero row-sum matrix, there exists a globally synchronized state $x_i(t) = x_j(t)\ \forall\, i,j$ for the above dynamical system. The coupling range in which the globally synchronized solution is stable can be written as $\alpha_1/\lambda_2 < \sigma < \alpha_2/\lambda_N$ [2], where α_1,α_2 are constants determined by the dynamics on the network. Therefore the smaller the value of λ_N/λ_2, the wider the coupling range for which the globally synchronized state is stable, or we say that the network is more synchronizable.

We construct networks with a scale-free (SF) topology by the preferential attachment method, for such networks $p(k) \propto k^{-3}$, where $p(k)$ is the probability of attaching with a node having degree k [23]. Erdös-Renýi (ER) random topologies are generated by connecting every pair of nodes with a probability $\langle k \rangle/N$ [23]. Additionally, the degree-degree correlation coefficient (r) is defined by [24]

$$r = \frac{\left(N_c^{-1} \sum_{i=1}^{N_c} j_i k_i\right) - \left(N_c^{-1} \sum_{i=1}^{N_c} \frac{(j_i + k_i)}{2}\right)^2}{\left(N_c^{-1} \sum_{i=1}^{N_c} \frac{(j_i)^2 + (k_i)^2}{2}\right) - \left(N_c^{-1} \sum_{i=1}^{M} \frac{(j_i + k_i)}{2}\right)^2}, \tag{3}$$

where j_i and k_i are the degrees of two adjacent nodes being connected by the link i and N_c represents the total number of connections in a network. For a given network, in order to vary r, two links of this network are chosen randomly together with their corresponding adjacent nodes. The nodes are ranked according to their degrees. The nodes are reconnected based on their degrees in assortative or disassortative manner with probability p. For constructing an assortative network, out of the four nodes chosen, the higher degree nodes are reconnected among themselves, and the lower degree nodes are reconnected among themselves with probability p. Whereas, for constructing disassortative networks, out of the four nodes, the highest degree node is reconnected with the lowest degree node, and the other two nodes are reconnected among themselves with probability p. Networks of different r values are generated by varying the parameter p [24]. To construct a two layer multiplex network we take two random realizations of a network (SF or ER network) for a given set of parameters (N and $\langle k \rangle$) and multiplex them randomly, i.e., each node in the first layer connects randomly with a mirror node in the second layer in such a manner that one node gets only one interlayer connection. Next, to change the degree-degree correlations (r_m) among the mirror nodes, two interlayer links are broken and rewired in the similar manner as adopted for creating the desired degree-degree correlation for an individual layer as described above. A desired r_m value is obtained by varying p.

III. RESULTS

We present results for the impact of changing the degree-degree correlations of the mirror nodes or those on the individual layers on the GS of the entire multiplex network. We find that the presence of correlations among degrees of mirror nodes have a profound impact on the GS of the multiplex networks. Particularly, we demonstrate how one can change the synchronizability of a network just by tuning the degree-degree correlations of the mirror nodes, keeping the architecture of the individual layers unchanged. In order to demonstrate this, we first consider a multiplex network consisting of layers with a scale-free topology. We vary the degree-degree correlations of the mirror nodes and investigate the impact on the GS of the multiplex networks. For several configurations for the topology of the layers with regard to their average connectivity and the intralayer degree correlations (r_{intra}) on decreasing r_m, the value of λ_N/λ_2 decreases, and consequently the GS of the multiplex network increases. Which means that the mirror nodes having dissassortative degree-degree correlation favor synchrony among the nodes in a multiplex network [Figs. 2(a)–2(d)]. This phenomenon can be related with poor synchronizability caused by the overloading of oscillators [25]. Going by the same analogy, the more concentrated the traffic is on the mirror nodes (i.e., when the mirror degrees are assortative), the less robust is the whole network to maintain its synchronous state, leading to reduced synchronizability. A prevalence of mirror nodes having disassortative degrees is thus beneficial for synchronization in the multiplex networks.

The GS of a network is governed strongly by its algebraic connectivity, a measure of the overall connectedness of

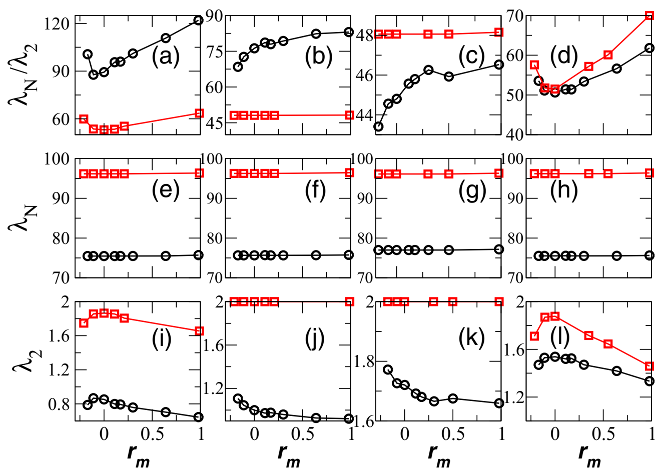

FIG. 2. (a)–(d), (e)–(h), and (i)–(l) indicate changes in λ_N/λ_2, λ_N, and λ_2 with r_m for a multiplex network with different intralayer degree-degree correlations. Layers 1 and 2 are SF networks of size $N_1 = N_2 = 500$. The circle represents $\langle k_1\rangle = \langle k_2\rangle = 6$, whereas the square corresponds to $\langle k_1\rangle = \langle k_2\rangle = 10$. Here $\langle k_1\rangle$ and $\langle k_2\rangle$ denote average degrees of layers 1 and 2, respectively. r_{intra} values for (a)–(c) are 0.20, 0.10, −0.2, respectively. To demonstrate that strong intralayer degree correlations are detrimental for GS, we consider $r_{\text{intra}} = -0.35$ (the circle) and −0.45 (the square) in (d). The network parameters for each column of the second and third rows are the same as in the upper one. Each data point is averaged over 20 network realizations.

the nodes in a network. A network with high algebraic connectivity indicates that it is difficult to break the network into disconnected components [26]. As realized in an earlier work [19], changing the degree-degree correlation has almost no impact on λ_N. The present paper also reflects that changes in λ_N are insignificant for a change in r_m [Figs. 2(e)–2(h)]. It is λ_2 that determines the relationship between GS and r_m. For all the values of r_{intra}, Figs. 2(i)–2(l) exhibit a behavior of λ_2, which is completely in contrast to that of λ_N/λ_2 versus r_m. This behavior complies with one of the earlier findings that an increase in algebraic connectivity is reflected in the better synchronization ability of the nodes [21].

The increase in GS with decreasing r_m, however, is continuous only in case of the multiplex networks which have a small value of intralayer assortativity or disassortativity [Figs. 2(b) and 2(c)]. With an increase in intralayer degree-degree correlations (for instance, at $r_{\text{intra}} = -0.35$ and 0.2), decreasing mirror degree correlations beyond a certain point (at around $r_m = 0$) proves detrimental for the synchronizability of the multiplex networks [Figs. 2(a) and 2(d)]. As we show next, this is due to the fact that algebraic connectivity of both layers is affected adversely for extreme values of positive and negative intralayer degree-degree correlations [27], leading to such GS behavior.

It should be noted that, as the assortativity in the layers of the multiplex network decreases, changes in λ_N/λ_2 become less significant with changing r_m [Figs. 2(a)–2(c)]. We know that increasing the disassortativity of a network decreases the diameter of the network [27]. Therefore, it can be inferred

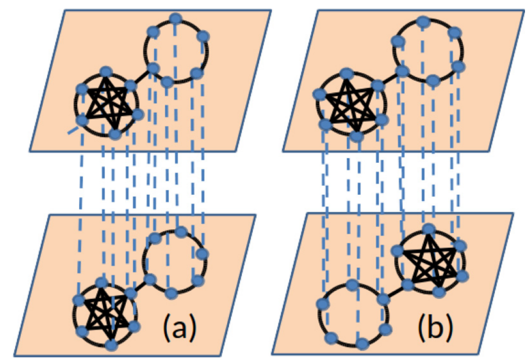

FIG. 3. A schematic depicting the architecture of a multiplex network in which layers have strong positive correlations. (a) and (b) represent assortative and disassortative degree-degree correlations among the mirror nodes.

that the impact of r_m is stronger on networks having larger diameters or average path lengths.

We further investigate the impact of the change in r_m on the GS of dense multiplex networks. For denser networks, changes in the GS with varying r_m are insignificant unless the layers have strong degree-degree correlations. For multiplex networks with strong intra-layer correlations, the GS exhibits a similar behavior as observed for a lower average degree [Figs. 2(a) and 2(d)]. As indicated by λ_2 [the square symbols in Figs. 2(i)–2(l)], for dense layers, the nodes have better connectivity among themselves except for highly assortative or disassortative layers, therefore, changing r_m does not lead to significant changes in the algebraic connectivity of dense multiplex networks.

To have an in-depth understanding of the relation between algebraic connectivity and mirror degree correlations, we investigate the eigenvector corresponding to λ_2, referred to as the Fiedler vector [28]. The sign of the eigenvector entries can be used for partitioning the network into two disconnected components [26]. Using a recursive bisection, we can further partition the network into more components. We construct a network by joining a globally connected network with a ring network of the same size through few links (say 1). We multiplex this network with its replica as in Fig. 3 where the mirror nodes are connected by one link and vary r_m. An individual layer in this multiplex network is roughly similar to a highly assortative network in terms of edge connectivity with very few links joining the dense and sparse parts of the network. For very high and very low values of mirror degree correlations, eigenvector entries are very sparse around 0, thus making it possible to visualize the network consisting of two loosely connected components. One set of nodes corresponds to positive eigenvector entries, and the other set leads to negative eigenvector entries [Figs. 4(a) and 4(c)]. This partitioning is not obvious in Fig. 4(b). Therefore, in such structures, strong correlations among the mirror nodes turn out to be detrimental for GS. This can be concluded by analyzing the λ_2 values. Figures 4(a)–4(c) reveal that the multiplex networks with negative r_m are better globally synchronizable than the networks with positive r_m values. It also is clear from these values that networks with uncorrelated r_m values are optimal for maximizing GS. A close look at

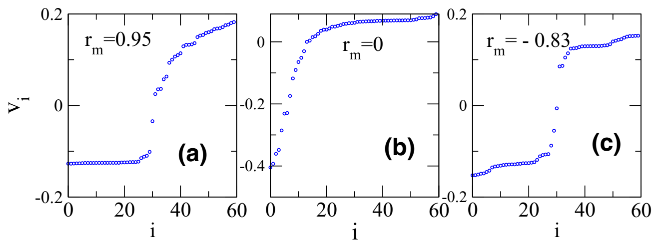

FIG. 4. Entries (v_i) of the eigenvector corresponding to the λ_2 eigenvalue of the Laplacian matrix for different r_m values. The multiplex network size is 60 nodes with the topology given by Fig. 3. Here the nodes are renumbered based on their corresponding v_i values.

Fig. 4(a) reveals that half of the nodes have roughly the same magnitude of the corresponding eigenvector entries, whereas the magnitudes of the eigenvector entries corresponding to the other half of the nodes differ a lot among themselves, revealing that the dense parts of both layers form one group, whereas the sparse parts of both layers form the other group. As depicted by Fig. 4(c), eigenvector entries corresponding to the nodes of each group are symmetric around 0. In this case, the dense part of one layer is accompanied by the sparse part of the other layer. A similar analogy can be used to explain the increase and decrease in GS with a decrease in r_m when multiplex networks have high disassortative layers. It should be noted that increasing disassortativity also leads to a decrease in the algebraic connectivity of a network [27]. Therefore if the algebraic connectivity of the layers of a multiplex network is small, r_m values close to zero yield maximum GS.

Multiplex networks manifest a much rich synchronization behavior than exhibited by single layer networks, which is not surprising as there exists another degree of freedom (r_m) associated with the multiplex networks. We find that, in contrast to single layer networks where GS increases with increasing $\langle k \rangle$ [27], the GS of the multiplex networks can exhibit an increase or a decrease depending on r_{intra} and r_m values. For multiplex networks with positive r_{intra} values, the GS exhibits an increase followed by a decrease. However, for disassortative layers depending on r_{intra} and r_m, the GS can show an increase or decrease with an increase in $\langle k \rangle$. Figures 5(a), 5(b) and 5(d) reflect that the GS increases by an increase in $\langle k \rangle$. However, at another value of r_{intra} [Fig. 5(c)], the GS decreases with an increase in $\langle k \rangle$.

The functionality of GS with respect to $\langle k \rangle$ can be understood by tracing λ_N and λ_2 separately. It has been shown that for a single layer network, an increase in $\langle k \rangle$ causes an increase in GS by a process that not only increases λ_N [2], but also increases λ_2 [27]. However, it is λ_2 which dominates the changes in GS as $\langle k \rangle$ increases leading to an enhancement in GS. For the multiplex networks with assortative layers, the increase in GS is due to a very large increase in λ_2. For example, Fig. 5(e) depicts that λ_2 increases from 0.6 to 2 as $\langle k \rangle$ changes from 6 to 14. With increasing $\langle k \rangle$, λ_2 first keeps on increasing and thereafter gets saturated letting λ_N drive the behavior of the GS. The minimal value of λ_N/λ_2 (maximal GS) marks the point where λ_2 saturates for increasing $\langle k \rangle$. For the multiplex networks with disassortative layers, GS may exhibit different behaviors for increasing $\langle k \rangle$ values depending upon how strongly the nodes of the individual layers are correlated. It should be noted that algebraic connectivity of

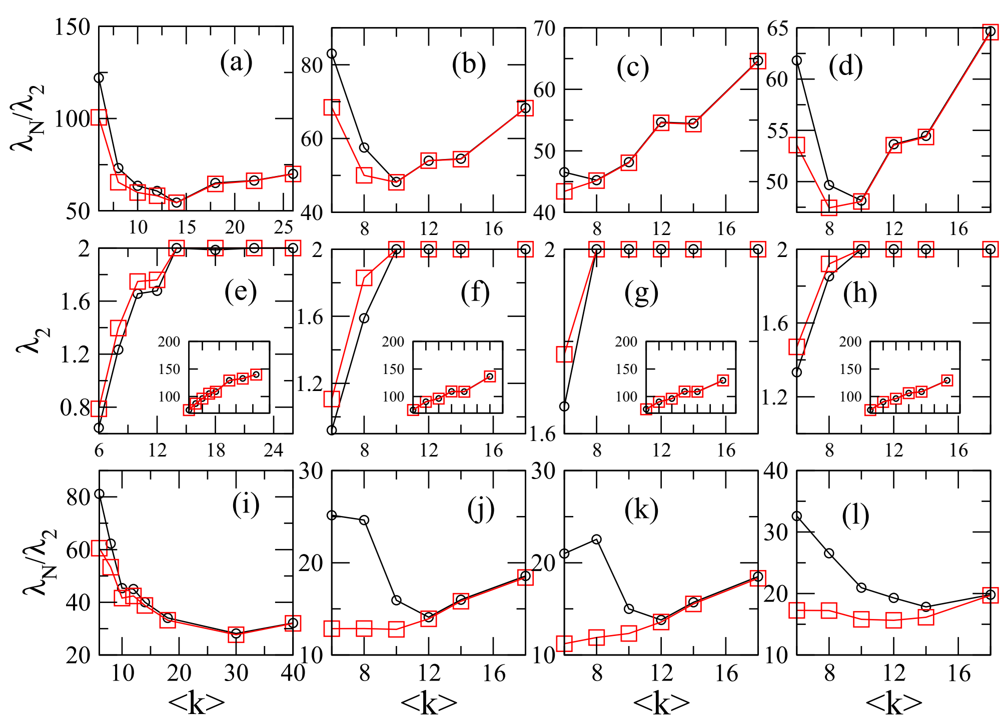

FIG. 5. (a)–(d) and (e)–(h) show changes in λ_N/λ_2, λ_2 with $\langle k \rangle$ at $r_{\text{intra}} = 0.2$ (a) and (e), 0.1 (b) and (f), -0.25 (c) and (g), and -0.35 (d) and (h). Here layers 1 and 2 are SF networks with $N_1 = N_2 = 500$. In (a)–(h) the circles (○'s) and the squares (□)'s correspond to $r_m = 0.98, -0.17$, whereas in (i)–(l) the same corresponds to $r_m = 0.98, -0.98$. In the case of the ER networks, the layers formed are $r_{\text{intra}} = 0.9$ (i), 0.2 (j), -0.6 (k), and -0.9 (l). The inset shows the changes in λ_N with $\langle k \rangle$.

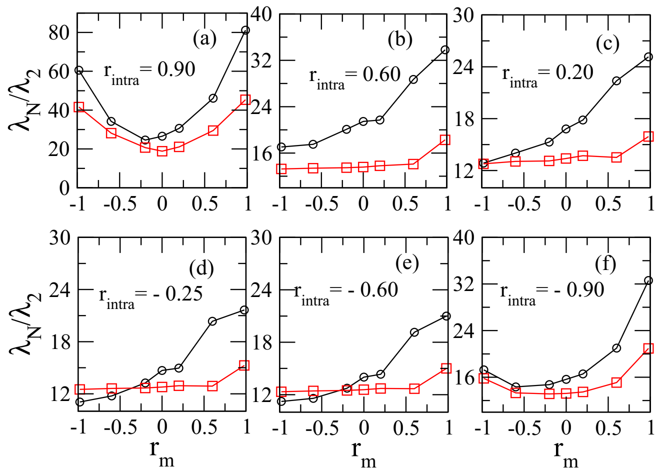

FIG. 6. λ_N/λ_2 with respect to r_m for a network with ER topology layers having different r_{intra} values. Here $N_1 = N_2 = 500$, and the circles (○'s) and the squares (□'s) correspond to $\langle k_1\rangle = \langle k_2\rangle = 6,10$.

a disassortative network is higher than that of an assortative network, therefore if r_{intra} is chosen such that λ_2 is around its maxima, an increase in λ_N with an increase in $\langle k\rangle$ can dominate over an increase in λ_2 thereby leading to a decrease in GS as discussed. Figure 5(c) indicates that, for a multiplex network with $r_{intra} = -0.25$, an increase in $\langle k\rangle$ causes a decrease in GS. Furthermore, depending on the r_{intra} and r_m values, the average connectivity at which λ_N/λ_2 shows minima can change. For example, for $r_{intra} = 0.2$, λ_N/λ_2 attains the minima around $\langle k\rangle = 14$, whereas for $r_{intra} = 0.1$, the highest synchronizable networks are obtained at around $\langle k\rangle = 10$. For positive r_m values, an increase in GS is more significant as compared to negative r_m values [Figs. 5(a), 5(b) and 5(d)]. This can be understood further with the help of the Fiedler vector. A large increase in GS for positive r_m values arises due to a large increase in λ_2 as the connectivity of the lower degree nodes, which are weakly connected, increases with an increase in $\langle k\rangle$ [Fig. 4(a)]. Whereas for negative r_m values, an increase in λ_2 is not prominent as the connectivity of the lower degree nodes is already better than the previous case [Fig. 4(c)]. The major contribution from the multiplex structure is to put a cap on the maximal value of λ_2 that can be achieved for a varying $\langle k\rangle$. In contrast, for single layer networks, λ_2 is proportional to $\langle k\rangle$, and its maximal value ($\lambda_2 = N$) is only achieved when the single layer is connected fully.

Furthermore, we investigate how changes in degree distribution alter the role of degree-degree correlation in determining GS. We find that homogeneous networks (ER networks) follow a similar synchronization behavior due to changes in r_{intra} and r_m as exhibited by heterogeneous networks (SF networks). For moderately correlated layers, negative mirror degree correlation leads to an increase in GS [Figs. 6(b)–6(e)], whereas for strongly correlated layers, neutral or small negative mirror degree correlations are shown to be beneficial for GS [Figs. 6(a) and 6(f)]. For homogeneous networks, r_{intra} has no significant impact on GS unless its magnitude is very high. ER networks show a continuous decrease in λ_N/λ_2 with a decrease in r_m even at $r_{intra} = 0.6, -0.6$ [Figs. 6(b)–6(e)], whereas to find the same phenomenon in SF networks, the maximum r_{intra} values we can achieve are 0.1 [Fig. 2(b)] and -0.2 [Fig. 2(c)]. The requirement of high (dis)assortativity among layers to see a similar effect might be due to the existence of homogeneous degree distribution as it is not as easy to separate the nodes of a homogeneous network into sparse and dense groups as for a SF network having rather heterogenous degree distribution. Furthermore, for a multiplex network with positively correlated ER networks, irrespective of the r_{intra} value, GS shows an increase with an increase in $\langle k\rangle$. As shown in Figs. 5(i)–5(l), increasing $\langle k\rangle$ increases GS until λ_2 reaches its maximum value after which it starts decreasing. It can be attributed to the absence of hubs which were responsible for a high λ_2 value [Fig. 5(c)]. Another point to be noted is that, if homogenous networks form layers of a multiplex network, depending on the value of r_m, the GS of a network may be different even for the same r_{intra} values [Figs. 5(j) and 5(k)]. This is due to the fact that, at positive r_m values, λ_2 dominates over λ_N whereas for negative r_m values the opposite is true. In contrast, if SF networks form the individual layers, dependence of λ_N/λ_2 on $\langle k\rangle$ mainly is determined by r_{intra}. It may be due to the fact that in this case, the minimum achievable r_m value is ≈ -0.17, which is much higher than the case when ER networks form the layers.

IV. DISCUSSION

This paper investigates multiplex networks whose nodes are connected under correlation-based attachment rules for the inter- and intraconnections. Previous works have shown that correlation-based attachments affect dynamical activity of the nodes of an individual layer [13,19]. How these correlation-based preferential attachments affect synchronizability of the nodes in a system having multiple modes of interaction is the prime focus of the current paper. We show that the simultaneous presence of strong correlation among inter- and intralayer nodes should be avoided to achieve good synchronizability. For moderately correlated layers, a strong negative mirror degree correlation leads to an increase in synchronizability of a multiplex network, whereas for strongly correlated layers, moderate degree-degree correlation among the mirror nodes is proved to be the best strategy. We also have investigated the role of the degree distribution on synchronizability. With an increase in the networks connectivity, positive inter- and intralayer degree-degree correlations favor synchronizability as compared to the negative ones. In contrast to single layer networks, multiplex networks provide a much richer framework for specifying synchronizability due to the existence of another parameter, which is the degree-degree correlation of the mirror nodes. The results obtained from this paper provide fundamental understanding towards assessing synchronizability in networks having different types of correlations existing in the same system. They might be crucial for understanding the coherent behavior of complex systems having inherent multiple types of interactions, for instance, neural networks [10].

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[1] A. Pikovsky, M. Rosenblum, and J. Kurths, *Synchronization: A Universal Concept in Nonlinear Sciences* (Cambridge University Press, Cambridge, U.K., 2003).
[2] A. Arenas *et al.*, Phys. Rep. **469**, 93 (2008).
[3] M. Kivelä *et al.*, J. Complex Networks **2**, 203 (2014).
[4] M. De Domenico, A. Solé-Ribalta, E. Cozzo, M. Kivelä, Y. Moreno, M. A. Porter, S. Gómez, and A. Arenas, Phys. Rev. X **3**, 041022 (2013).
[5] S. Boccaletti *et al.*, Phys. Rep. **544**, 1 (2014).
[6] P. Shinde and S. Jalan, Europhys. Lett. **112**, 58001 (2015).
[7] C. Sarkar, A. Yadav, and S. Jalan, Europhys. Lett. **113**, 18007 (2016).
[8] S. Jalan and A. Singh, Europhys. Lett. **113**, 30002 (2016); A. Singh, S. Jalan, and S. Boccaletti, Chaos **27**, 043103 (2017).
[9] S. K. Dwivedi, M. S. Baptista, and S. Jalan, Phys. Rev. E **95**, 040301(R) (2017).
[10] V. Nicosia, P. S. Skardal, A. Arenas, and V. Latora, Phys. Rev. Lett. **118**, 138302 (2017).
[11] C. Granell, S. Gomez, and A. Arenas, Phys. Rev. Lett. **111**, 128701 (2013).
[12] G. F. de Arruda, E. Cozzo, T. P. Peixoto, F. A. Rodrigues, and Y. Moreno, Phys. Rev. X **7**, 011014 (2017).
[13] V. Nicosia and V. Latora, Phys. Rev. E **92**, 032805 (2015).
[14] M. E. J. Newman, Phys. Rev. Lett. **89**, 208701 (2002); Phys. Rev. E **67**, 026126 (2003).
[15] S. Nie, X. Wang, and B. Wang, Physica A **436**, 98 (2015).
[16] B. Min, S. D. Yi, K.-M. Lee, and K.-I. Goh, Phys. Rev. E **89**, 042811 (2014).
[17] K.-M. Lee *et al.*, New J. Phys. **14**, 033027 (2012).
[18] G. F. de Arruda *et al.*, Physica D **323**, 5 (2016).
[19] M. Di Bernardo, F. Garofalo, and F. Sorrentino, Int. J. Bifurcation Chaos Appl. Sci. Eng. **17**, 3499 (2007).
[20] J. Aguirre, R. Sevilla-Escoboza, R. Gutiérrez, D. Papo, and J. M. Buldú, Phys. Rev. Lett. **112**, 248701 (2014).
[21] M. Barahona and L. M. Pecora, Phys. Rev. Lett. **89**, 054101 (2002).
[22] L. M. Pecora and T. L. Carroll, Phys. Rev. Lett. **80**, 2109 (1998).
[23] R. Albert and A.-L. Barabasi, Rev. Mod. Phys. **74**, 47 (2002).
[24] R. Xulvi-Brunet and I. M. Sokolov, Phys. Rev. E **70**, 066102 (2004).
[25] T. Nishikawa, A. E. Motter, Y.-C. Lai, and F. C. Hoppensteadt, Phys. Rev. Lett. **91**, 014101 (2003).
[26] M. E. J. Newman, *Networks: An Introduction* (Oxford University Press, New York, 2010).
[27] S. Jalan, A. Kumar, A. Zaikin, and J. Kurths, Phys. Rev. E **94**, 062202 (2016).
[28] M. Fiedler, Czechoslovak Math. J. **23**, 298 (1973).