

2-D ELASTODYNAMIC PROBLEM FOR AN INTERFACE CRACK UNDER AN OBLIQUE HARMONIC LOADING

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Abstract. The current study is devoted to solution of the two dimensional elastodynamic problem of a cracked heterogeneous material loaded by harmonic wave. This investigation takes into account the reflection and refraction of oblique waves at an interface between two dissimilar solid half-spaces which contains a crack. The problem is solved by the method of boundary integral equations using an iterative algorithm. The dynamic stress intensity factors are given in the wide range of wave numbers for different properties of the bimaterial with the focus on the effect of the cracks' closure.

1. INTRODUCTION

Structural applications of composite materials have increased due to their excellent stiffness and low weight together with reduced energy consumption. However one of the main weaknesses of composites is the presence of various defects that occur in the region of fiber or matrix bonding. The growth of cracks increases stress in the vicinity of the crack tip, as a consequence it gives the rise to stiffness degradations and overall load bearing capacity of the structure is reduced. Considerable deliberation is paid to the failure analysis as an extremely important tool to improve materials' reliability and reduce the cost, which also helps prevent disasters causing by unpredicted fracture and to build up confidence in safety issues.

A good knowledge of the dynamic response of the composites is essential to achieving in depth understanding of the failure mechanism of the bimetals. A number of studies have contributed to the dynamic analysis of heterogeneous materials involving interface cracks. Babaei and Lukasiwicz [1] investigated the dynamic response of an interface crack between two dissimilar half-spaces under dynamic loading by using dual integral equations, in which an exponential function for the variation of material properties was used. Itou [2] studied the dynamic stress intensity factors around a crack in an interfacial layer between two dissimilar elastic half-planes, and the material properties of the interfacial layer were assumed to be non homogeneous. Ma *et al* [3] discussed the dynamic behaviour of two collinear cracks in composite material under anti-plane incident harmonic stress waves. The effects of crack interaction, wave velocity of materials and frequency of incident waves on dynamic stress intensity factors were investigated. Jiang and Wang considered the dynamic crack propagation in an interfacial layer with spatially varying elastic properties [4]. The problem of a two-dimensional interface crack was solved by Qu [5] for the case of harmonic loading. The effect of the angle of the plane wave incidence was studied for a range of frequencies of the load. Menshykova *et al* considered the case of the shear wave propagating normally to the surface of a linear interface crack neglecting the friction between opposite crack faces [6]. The solution to the two-dimensional problem with interface crack under normal tension-compression loading accounting for the effect of the crack closure was obtained by Menshykova *et al* in [7]. The

detailed description of the iterative algorithm for the problem solution was given and the study of the algorithm convergence was performed in [8].

In this paper, the dynamic crack analysis of two dimensional bimaterial with crack on the interface is presented. For this purpose, the boundary element method was used and the harmonic wave propagation solutions for bimaterials are derived.

2. METHODOLOGY

Let us consider an interface crack located within two-dimensional bimaterial subjected to external harmonic loading. In this case, under investigation is used an unbounded two dissimilar linearly elastic homogeneous isotropic solids. The interface between the half-spaces, Γ^* , acts as the boundary $\Gamma^{(1)}$ for the upper half-space, and the boundary $\Gamma^{(2)}$ for the lower half-space. The planes $\Gamma^{(1)}$ and $\Gamma^{(2)}$ differ by the opposite orientation of their outer normal vectors. It is assumed that surface $\Gamma^{(m)}$ ($m=1, 2$) consists of infinite part $\Gamma^{(m)*}$ and finite part $\Gamma^{(m)cr}$. The bonding interface and the surface of the crack are respectively:

$$\Gamma^* = \Gamma^{(1)} \cap \Gamma^{(2)}, \quad \Gamma^{(cr)} = \Gamma^{(1)cr} \cup \Gamma^{(2)cr}. \quad (1)$$

In order to reference the geometrical positions of the material points a Cartesian coordinate system is used with origin located in the centre of the crack. If there is no force applied to the body, the stress-strain state of both domains will be defined by the dynamic equations of the linear elasticity for the displacement vector $\mathbf{u}^{(m)}(\mathbf{x}, t)$ using Lamé equations:

$$\begin{aligned} (\lambda^{(m)} + \mu^{(m)}) \text{grad div } \mathbf{u}^{(m)}(\mathbf{x}, t) + \mu^{(m)} \Delta \mathbf{u}^{(m)}(\mathbf{x}, t) = \rho^{(m)} \partial_t^2 \mathbf{u}^{(m)}(\mathbf{x}, t), \\ \mathbf{x} \in \Omega^{(m)}, t \in [0, \infty) \end{aligned} \quad (2)$$

where $\lambda^{(m)}$ and $\mu^{(m)}$ and are Lamé elastic constants, $\rho^{(m)}$ is the specific material density and Δ is the Laplace operator, $m=1,2$.

The complex amplitudes of the displacement vector can be represented in terms of scalar and vector potentials, satisfying the inhomogeneous Helmholtz equation:

$$\mathbf{u}^{(m)}(\mathbf{x}) = \text{grad} \Phi^{(m)} + \text{rot} \Psi^{(m)}, \quad \text{div} \Psi^{(m)}(\mathbf{x}) = 0. \quad (3)$$

The incident tension-compression wave of $e^{-i\omega t}$ time dependency, propagating in the oblique direction can be defined by the scalar potential function as given below:

$$\begin{aligned} \Phi_0^{(1)}(\mathbf{x}, t) = A_0 e^{i(k_p(\mathbf{n}, \mathbf{x}) - \omega t)}, \\ \mathbf{n} = (\cos \theta_0, \sin \theta_0) \text{ and } \mathbf{x} \text{ represents the position of vector.} \end{aligned} \quad (4)$$

$$\Phi_0^{(1)}(\mathbf{x}, t) = A_0 e^{ik_p^{(1)}(x_1 \sin \theta_0 - x_3 \cos \theta_0)} e^{-i\omega t}, \quad \Psi_0^{(1)}(\mathbf{x}, t) = 0. \quad (5)$$

The group of reflected and transmitted waves associated with a given harmonic wave is shown in Fig. 1. As a result of their interference, the components of stresses can be expressed as the sum of four waves, two propagating obliquely downwards, and the other two upwards.

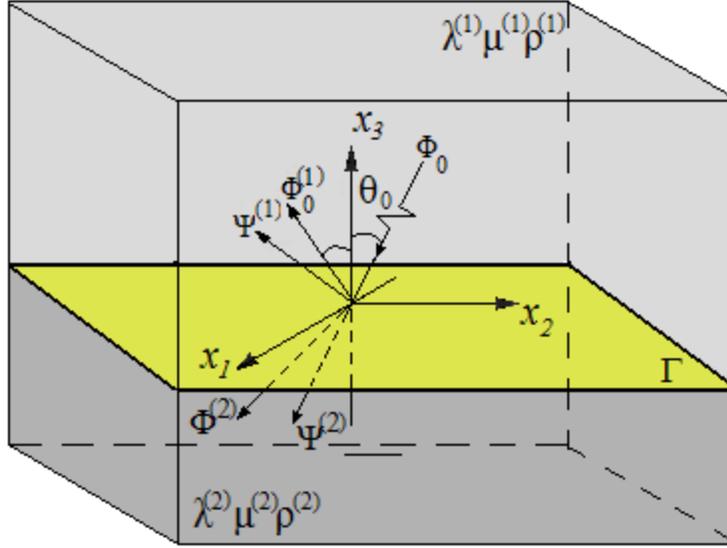


Fig. 1 Refraction and transmission at an interface when wave is incident at an oblique angle

$$\begin{aligned}
\Phi^{(1)}(\bar{\mathbf{x}}, t) &= A_0 e^{ik_p^{(1)}(x_1 \sin \theta_0 - x_3 \cos \theta_0)} e^{-i\omega t} + A^{(1)} e^{ik_p^{(1)}(x_1 \sin \theta^{(1)} + x_3 \cos \theta^{(1)})} e^{-i\omega t}, \\
\Psi^{(1)}(\bar{\mathbf{x}}, t) &= B^{(1)} e^{ik_s^{(1)}(x_1 \sin \gamma^{(1)} + x_3 \cos \gamma^{(1)})} e^{-i\omega t}, \\
\Phi^{(2)}(\bar{\mathbf{x}}, t) &= A^{(2)} e^{ik_p^{(2)}(x_1 \sin \theta^{(2)} - x_3 \cos \theta^{(2)})} e^{-i\omega t}, \\
\Psi^{(2)}(\bar{\mathbf{x}}, t) &= B^{(2)} e^{ik_s^{(2)}(x_1 \sin \gamma^{(2)} - x_3 \cos \gamma^{(2)})} e^{-i\omega t}.
\end{aligned} \tag{6}$$

where $A^{(m)}$ and $B^{(m)}$ are amplitudes of longitudinal and shear waves, respectively; $\omega = 2\pi/T$ is the frequency, T is the period of vibration; $k_p^{(m)} = \omega/c_p^{(m)}$ and $k_s^{(m)} = \omega/c_s^{(m)}$ are the generalized wave number; $c_p^{(m)} = \sqrt{(\lambda^{(m)} + 2\mu^{(m)})/\rho_1}$, $c_s^{(m)} = \sqrt{\mu^{(m)}/\rho_2}$ are the velocities of the longitudinal and shear wave; $\lambda^{(m)}$, $\mu^{(m)}$ are the elastic Lamé constants; ρ_1 and ρ_2 are the densities of materials.

The interaction of the incident wave results in two reflected waves, a longitudinal and a shear wave. Similarly there are two transmitted waves, longitudinal and shear. The angle of reflection of the longitudinal wave is the same as the angle of incidence, but all other angles depend on the materials properties, according to Snell's Law, which can be found in a number of books [9, 10]

$$\frac{\sin \theta_0}{c_p^{(1)}} = \frac{\sin \theta^{(1)}}{c_p^{(1)}} = \frac{\sin \gamma^{(1)}}{c_s^{(1)}} = \frac{\sin \theta^{(2)}}{c_p^{(2)}} = \frac{\sin \gamma^{(2)}}{c_s^{(2)}} \tag{7}$$

where each $\theta^{(1)}$, $\theta^{(2)}$, $\gamma^{(1)}$, $\gamma^{(2)}$ are the reflected and transmitted wave angles.

There are two displacements which must be continuous: the displacement normal to the interface and the displacement parallel to the interface. Similarly, there are two stress components which must be balanced: the stress normal to the interface and the shear stress. Then the boundary conditions of continuity at the interface, Γ^* between two solid half-spaces are [9]:

$$\begin{aligned}
u_1^{(1)}(\mathbf{x}, t) &= u_1^{(2)}(\mathbf{x}, t), \quad u_2^{(1)}(\mathbf{x}, t) = u_2^{(2)}(\mathbf{x}, t), \quad u_3^{(1)}(\mathbf{x}, t) = u_3^{(2)}(\mathbf{x}, t); \\
\sigma_{31}^{(1)}(\mathbf{x}, t) &= \sigma_{31}^{(2)}(\mathbf{x}, t), \quad \sigma_{32}^{(1)}(\mathbf{x}, t) = \sigma_{32}^{(2)}(\mathbf{x}, t), \quad \sigma_{33}^{(1)}(\mathbf{x}, t) = \sigma_{33}^{(2)}(\mathbf{x}, t)
\end{aligned} \tag{8}$$

The components of the displacements are defined as:

$$\begin{aligned}
u_1^{(m)}(\mathbf{x}, t) &= \frac{\partial \Phi^{(m)}(\mathbf{x}, t)}{\partial x_1} - \frac{\partial \Psi^{(m)}(\mathbf{x}, t)}{\partial x_3} \\
u_2^{(m)}(\mathbf{x}, t) &= 0 \\
u_3^{(m)}(\mathbf{x}, t) &= \frac{\partial \Phi^{(m)}(\mathbf{x}, t)}{\partial x_3} + \frac{\partial \Psi^{(m)}(\mathbf{x}, t)}{\partial x_1}
\end{aligned} \tag{9}$$

Components of the stress can be expressed by the terms of potentials:

$$\begin{aligned}
\sigma_{31}^{(m)}(\mathbf{x}, t) &= 2\mu^{(m)} \frac{\partial^2 \Phi^{(m)}(\mathbf{x}, t)}{\partial x_1 \partial x_3} + \mu^{(m)} \left[\frac{\partial^2 \Psi^{(m)}(\mathbf{x}, t)}{\partial x_1^2} - \frac{\partial^2 \Psi^{(m)}(\mathbf{x}, t)}{\partial x_3^2} \right], \\
\sigma_{32}^{(m)}(\mathbf{x}, t) &= 0, \\
\sigma_{33}^{(m)}(\mathbf{x}, t) &= (\lambda^{(m)} + 2\mu^{(m)}) \frac{\partial^2 \Phi^{(m)}(\mathbf{x}, t)}{\partial x_3^2} + \lambda^{(m)} \frac{\partial^2 \Phi^{(m)}(\mathbf{x}, t)}{\partial x_1^2} + 2\mu^{(m)} \frac{\partial^2 \Psi^{(m)}(\mathbf{x}, t)}{\partial x_1 \partial x_3};
\end{aligned} \tag{10}$$

The appropriate differential equations are substituted into systems (9) and (10). Then, the use of boundary conditions in (8) leads to set of four equations for the amplitudes $A^{(1)}$, $A^{(2)}$, $B^{(1)}$ and $A^{(2)}$ as shown in (11)-(14):

$$A_0 k_p^{(1)} \cos \theta^{(1)} = A^{(1)} k_p^{(1)} \cos \theta^{(1)} + B^{(1)} k_s^{(1)} \sin \gamma^{(1)} + A^{(2)} k_p^{(2)} \cos \theta^{(2)} - B^{(2)} k_s^{(2)} \sin \gamma^{(2)} \tag{11}$$

$$A_0 k_p^{(1)} \sin \theta^{(1)} = -A^{(1)} k_p^{(1)} \sin \theta^{(1)} + B^{(1)} k_s^{(1)} \cos \gamma^{(1)} + A^{(2)} k_p^{(2)} \sin \theta^{(2)} + B^{(2)} k_s^{(2)} \cos \gamma^{(2)} \tag{12}$$

$$\begin{aligned}
A_0 \left(k_p^{(1)} \right)^2 \sin \theta^{(1)} \cos \theta^{(1)} 2\mu^{(1)} \\
= A^{(1)} \left(k_p^{(1)} \right)^2 \sin \theta^{(1)} \cos \theta^{(1)} 2\mu^{(1)} - B^{(1)} \left(k_s^{(1)} \right)^2 \left(\cos^2 \gamma^{(1)} - \sin^2 \gamma^{(1)} \right) \mu^{(1)} \\
+ A^{(2)} \left(k_p^{(2)} \right)^2 \sin \theta^{(2)} \cos \theta^{(2)} 2\mu^{(2)} + B^{(2)} \left(k_s^{(2)} \right)^2 \left(\cos^2 \gamma^{(2)} - \sin^2 \gamma^{(2)} \right) \mu^{(2)}
\end{aligned} \tag{13}$$

$$\begin{aligned}
A_0 \left(k_p^{(1)} \right)^2 \left(\lambda^{(1)} + 2\mu^{(1)} \cos^2 \theta^{(1)} \right) \\
= -A^{(1)} \left(k_p^{(1)} \right)^2 \left(\lambda^{(1)} + 2\mu^{(1)} \cos^2 \theta^{(1)} \right) - B^{(1)} \left(k_s^{(1)} \right)^2 \cos^2 \gamma^{(1)} \sin^2 \gamma^{(1)} 2\mu^{(1)} \\
+ A^{(2)} \left(k_p^{(2)} \right)^2 \left(\lambda^{(2)} + 2\mu^{(2)} \cos^2 \theta^{(2)} \right) - B^{(2)} \left(k_s^{(2)} \right)^2 \cos^2 \gamma^{(2)} \sin^2 \gamma^{(2)} 2\mu^{(2)}
\end{aligned} \tag{14}$$

As a result, the obtained system of equations can define the stresses at the interface generated from harmonic tension - compression wave.

To include the contact interaction of the opposite crack's faces into consideration, the Signorini unilateral constraints must be imposed for the normal components of the contact force and the displacement vectors

$$[u_n(\mathbf{x}, t)] \geq 0, \quad q_n(\mathbf{x}, t) \geq 0, \quad [u_n(\mathbf{x}, t)] q_n(\mathbf{x}, t) = 0, \quad \mathbf{x} \in \Omega, t \in [0; T] \tag{15}$$

where $[\mathbf{u}(\mathbf{x}, t)] = \mathbf{u}^{(1)}(\mathbf{x}, t) - \mathbf{u}^{(2)}(\mathbf{x}, t)$ is the displacement discontinuity vector; and $\mathbf{q}(\mathbf{x}, t)$ is the contact force that arises in the contact region, which is unknown beforehand, changes in time under deformation of the material and must be determined as a part of solution. The contact region also depends on the frequency, magnitude and direction of the external loading complicating the problem even more and making it highly non-linear. The constraints (15) ensure that there is no interpenetration of the opposite crack faces; the normal component of the contact force is unilateral and it is absent for any non-zero opening of the crack. Note that, due to the contact interaction the traction vector at the crack faces, $\tilde{\mathbf{p}}^{(m)}(\mathbf{x}, t)$, is the superposition of the initial traction caused by the incident wave, $\mathbf{g}^{(m)}(\mathbf{x}, t)$, and the contact force, $\mathbf{q}^{(m)}(\mathbf{x}, t)$.

In order to take the contact interaction of the opposite crack faces into consideration, we assume that the tangential components of the displacement discontinuities vector and vector of contact forces satisfy the Coulomb friction law:

$$|q_\tau(\mathbf{x}, t)| < k_\tau q_n(x, t) \Rightarrow \frac{\partial[u_\tau(\mathbf{x}, t)]}{\partial t} = 0, \quad (16)$$

$$|q_\tau(\mathbf{x}, t)| = k_\tau q_n(x, t) \Rightarrow \frac{\partial[u_\tau(\mathbf{x}, t)]}{\partial t} = -\frac{q_\tau(\mathbf{x}, t)}{|q_\tau(\mathbf{x}, t)|} \left| \frac{\partial[u_\tau(\mathbf{x}, t)]}{\partial t} \right|, \quad \mathbf{x} \in \Omega, t \in [0; T]. \quad (17)$$

The opposite crack faces remain immovable with respect to each other in tangential direction while they are held by the friction force. However, as soon as the magnitude of the tangential contact forces reaches a certain limit, depending on the crack friction coefficient (k_τ) and the normal contact forces, the crack faces begin to move and the slipping effect occurs.

3. BOUNDARY INTEGRAL EQUATIONS

With the purpose of solve the initial boundary value problem with unilateral constraints, the Somigliana dynamic identity is used. The components of the displacement field in the upper and lower half-spaces for the cracked body can be written in the following form:

$$\frac{1}{2} u_j^{(m)}(\mathbf{x}) = \int_{\Gamma^{(m)}} p_i^{(m)}(\mathbf{y}) U_{ij}^{(m)}(\mathbf{x}, \mathbf{y}, \omega) d\mathbf{y} - \int_{\Gamma^{(m)}} u_i^{(m)}(\mathbf{y}) W_{ij}^{(m)}(\mathbf{x}, \mathbf{y}, \omega) d\mathbf{y}, \quad \mathbf{x} \in \Gamma^{(m)}, j = \overline{1, 2} \quad (18)$$

where $p_i^{(m)}(\mathbf{x})$ and $u_i^{(m)}(\mathbf{x})$ are components of complex-valued magnitudes of tractions and displacements at the interface, \mathbf{x} is the point of observation, \mathbf{y} is the point of loading and the integral kernel $U_{ij}^{(m)}(\mathbf{x}, \mathbf{y}, \omega)$ is the Green fundamental tensor, which has the form

$$U_{ij}^{(m)}(\mathbf{x}, \mathbf{y}, \omega) = \frac{1}{2\pi\mu^{(m)}} \left(\psi^{(m)} \delta_{ij} - \chi^{(m)} \frac{(y_i - x_i)(y_j - x_j)}{r} \right) \quad (19)$$

Here δ_{ij} is the Kronecker delta; r is the distance between the observation point and the load point. In the two-dimensional case functions $\psi^{(m)}$ and $\chi^{(m)}$ in the frequency-domain are:

$$\begin{aligned} \psi^{(m)} &= K_0(l_2^{(m)}) + \frac{1}{l_2^{(m)}} \left[K_1(l_2^{(m)}) - \frac{c_2^{(m)}}{c_1^{(m)}} K_1(l_1^{(m)}) \right], \\ \chi^{(m)} &= K_2(l_2^{(m)}) - \left(\frac{c_2^{(m)}}{c_1^{(m)}} \right)^2 K_2(l_1^{(m)}). \end{aligned} \quad (20)$$

$K_n(\bullet)$ is the modified Bessel function of the second kind and order n ; $I_1^{(m)} = i\omega r / c_1^{(m)}$ and $I_2^{(m)} = i\omega r / c_2^{(m)}$. The velocities of the longitudinal wave and the transverse shear wave are $c_1^{(m)} = \sqrt{(\lambda^{(m)} + 2\mu^{(m)}) / \rho^{(m)}}$ and $c_2^{(m)} = \sqrt{\mu^{(m)} / \rho^{(m)}}$, respectively.

The differential operator

$$P_{ik}[\bullet, (\mathbf{y})] = \lambda n_i(\mathbf{y}) \frac{\partial[\bullet]}{\partial y_k} + \mu \left[\delta_{ik} \frac{\partial[\bullet]}{\partial \mathbf{n}(\mathbf{y})} + n_k(\mathbf{y}) \frac{\partial[\bullet]}{\partial y_i} \right] \quad (21)$$

is used to obtain the integral kernel $W_{ij}^{(m)}(\mathbf{x}, \mathbf{y}, \omega)$. The operator is applied to the Green fundamental solution $U_{ij}^{(m)}(\mathbf{x}, \mathbf{y}, \omega)$ and after the differentiation the integral kernel obtains the form:

$$W_{ij}^{(m)}(\mathbf{x}, \mathbf{y}, \omega) = \lambda^{(m)} n_i^{(m)}(\mathbf{y}) \frac{\partial}{\partial y_k} U_{kj}^{(m)}(\mathbf{x}, \mathbf{y}, \omega) + \mu^{(m)} n_k^{(m)}(\mathbf{y}) \left[\frac{\partial}{\partial y_k} U_{ij}^{(m)}(\mathbf{x}, \mathbf{y}, \omega) + \frac{\partial}{\partial y_i} U_{kj}^{(m)}(\mathbf{x}, \mathbf{y}, \omega) \right] \quad (22)$$

The complexity of the problem accounting for the effect of crack closure is compounded by the fact that the area of contact varies in time; the size and the form of the contact area is unknown beforehand and must be determine as a part of the solution. As the result, the process is no longer a harmonic process but a steady-state periodic one. Moreover, components of the stress-strain state cannot be represented a function of coordinates multiplied by an exponential function. Hence, the components of the displacement and traction vectors should be expanded into the Fourier series as following:

$$f(\bullet, t) = \frac{f_{0,\cos}(\bullet)}{2} + \sum_{k=1}^{+\infty} (f_{k,\cos}(\bullet) \cos(\omega_k t) + f_{k,\sin}(\bullet) \sin(\omega_k t)), \text{ where} \quad (23)$$

$$f_{k,\cos}(\bullet) = \frac{\omega}{2\pi} \int_0^T f(\bullet, t) \cos(\omega_k t) dt, \quad f_{k,\sin}(\bullet) = \frac{\omega}{2\pi} \int_0^T f(\bullet, t) \sin(\omega_k t) dt. \quad (24)$$

Since the interpenetration of the contact surfaces is prohibited, the unilateral constraints are imposed and the boundary conditions become nonlinear, the problem requires an iterative solution procedure. The solution of the elastodynamic problem for the cracked body neglecting the effect of the crack closure is obtained during the first part of the algorithm. In the second part the correction of the solution is performed with respect to the following unilateral constraints with friction of the crack faces. The algorithm is applied to the problem of the oblique incidence of a harmonic wave in the bimaterial with an interface crack. During the iterative process the Fourier coefficients are changing till the distribution of the vectors of displacements and contact forces satisfying the constraints (15)–(17) is found.

4. NUMERICAL RESULTS

As a numerical example, a linear interface crack with the length $2R$ is considered in the present study. The materials of the upper and lower half-spaces have the typical properties of steel and aluminum: $E^{(1)} = 207 \text{ GPa}$, $E^{(2)} = 70 \text{ GPa}$; $\nu^{(1)} = 0.25$, $\nu^{(2)} = 0.35$; $\rho^{(1)} = 7800 \text{ kg/m}^3$, $\rho^{(2)} = 2700 \text{ kg/m}^3$.

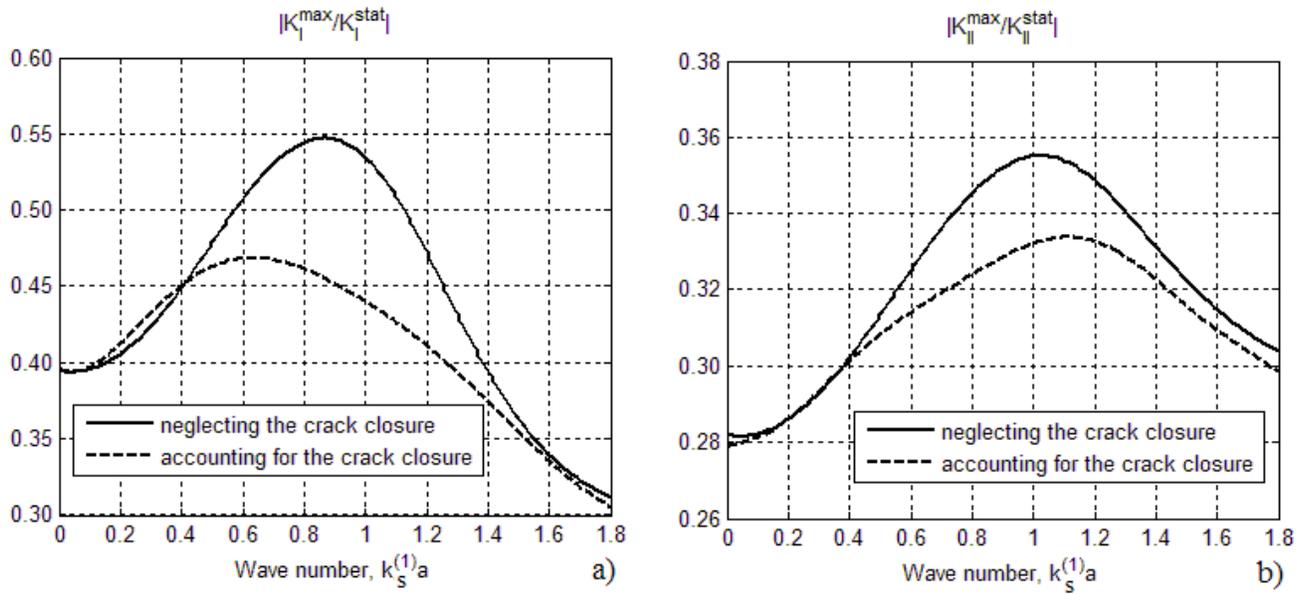
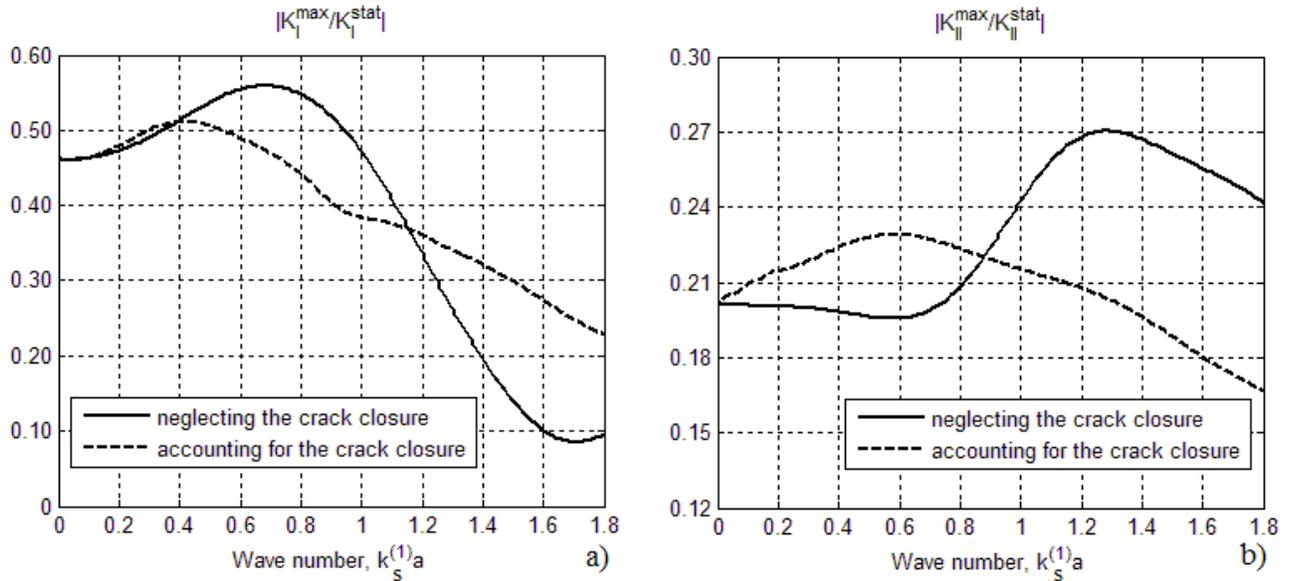


Fig. 2 The stress intensity factors as function of the wave number k_2a for the case $\alpha = 30^\circ$,



(a) the opening mode and (b) the transverse shear mode at the leading crack tip

Fig. 3 The stress intensity factors as function of the wave number k_2a for the case $\alpha = 30^\circ$,
 (a) the opening mode and (b) the transverse shear mode at the trailing crack tip

In Figs. 2(a,b)-3(a,b) the maximum in time values of stress intensity factors (SIF) for opening and the transverse shear modes are given for both crack tips as functions of the wave number, $k_s^{(1)}a$, for the angle of the wave incidence, $\alpha = 30^\circ$. In all cases the values of the SIFs tend to increase first and achieve a maximum and then decrease. There is a significant difference between the values of the stress intensity factors obtained for the case of accounting for the effect of the crack closure and neglecting this effect. The stress intensity factors evaluated for the case of accounting for the crack closure achieve their maximum value at the lower frequencies of loading than those obtained for the case of neglecting the crack closure.

The obtained distribution of stress intensity factors in the vicinity of the trailing crack tip differ from the values in the vicinity of the leading crack tip due to non-symmetry of solution with respect the space and time variables.

5. CONCLUSIONS

The solution of the two-dimensional problem of a cracked heterogeneous solid subjected to oblique incidence of the tension-compression wave was solved in accounting for the effect of the crack closure. The system of boundary integral equations which is used for numerical solution of the considered problem was obtained.

The dynamic stress intensity factors (opening and transverse shear modes) are computed as functions of wave number and compared with those obtained neglecting the crack's closure. The comparison between the results obtained at the trailing and leading tips of the crack are presented.

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